PROCEDURAL STRATEGIES FOR DEPTH-MIGRATION VELOCITY ANALYSIS BY IMAGE-WAVE PROPAGATION IN COMMON-IMAGE GATHERS

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ABSTRACT

We test and compare three iterative approaches for depth migration velocity analysis (MVA) by image-wave propagation in common-image gathers. The first implementation is a global update, aiming at updating the whole model in each iteration, whereas the second one is a layer-stripping algorithm, updating the model one layer at a time, from top to bottom. The third implementation is a mixture of both the first and second approaches. It uses the result of the first iteration from the global update to guide the later iterations of the layer-stripping procedure, because previous investigations found that the first iteration of the global approach already provided valuable model information. However, the further iterations of the global approach presented some convergence issues. We test the three implementations on synthetic data from a rather simple (but representative) model which simulates two sedimentary layers between a water layer at the top and a salt layer at the bottom. Our numerical examples demonstrate that the layer-stripping approach improves convergence. Our examples, also, demonstrate that the pure layer-stripping approach provides a much better result than the global update and improves convergence. The use of the first global result to guide the layer stripping did not help to improve convergence nor model quality. Since MVA by image continuation in CIGs is a rather inexpensive procedure which starts at no a-priori knowledge of the medium, it can be used to build initial models for more sophisticated inversion techniques.

INTRODUCTION

The goal of seismic imaging is to generate a migrated image (in time or depth) which allows to interpret the subsurface geology of the studied area. In order to produce this migrated image, it is necessary to have good velocity information. For a relatively simple medium, assuming lateral homogeneity and flat reflectors, normal moveout (NMO) velocity analysis based on hyperbolic moveout (Dix, 1955) provides a satisfactory velocity model. However, when facing more complex media, it is necessary to use a migration-velocity-analysis (MVA) method. MVA uses common-image gathers (CIGs), which collect images at the same migrated position from different subsets of the data (making use of data redundancy) and are sensitive to the velocity model (Al-Yahya, 1989). According to Sattlegger (1975), the basic idea of velocity analysis in a CIG is that if the migration velocity is correct, redundant data should be migrated to the same depth position, which means that reflection events should be flat across the CIG. In contrast, if the migration velocity is not correct, redundant data will be migrated to different position, which means that reflection events will present a residual moveout (RMO) in the CIG (see also Zhu et al., 1998).

A substantial amount of research works has been dedicated to MVA as a way of obtaining a velocity model for migration. RMO analysis has become the preferred tool for MVA because of its simplicity and clearness. Schleicher et al. (2008) developed a method for RMO analysis using a finite-difference implementation of a partial differential equation (PDE) named image-wave equation (Hubral et al., 1996).

PDEs of this type describe the displacement of events under the variation of the migration velocity (Goldin, 1994; Fomel, 1994, 1997, 2003). The idea of Schleicher et al. (2008) is to use an image-wave equation to propagate events inside a single CIG until they become flat. Thus, for each event it is possible to find a velocity value which flattens it. Then, the data would be migrated again, but now with these velocity values. Since this is an approximation, the process must be iterated until an acceptable velocity model is reached.

The huge advantage of MVA by image-wave propagation is its capacity to build an image starting with a model as simple as a constant velocity model, as done by Schleicher et al. (2008). Their implementation of this method deals only with time-migration, which works well in media with moderate lateral velocity variations. Schleicher et al. (2008) also discussed the theory for a CIG image-wave equation for depth migration, but did not implement it.

Gomes (2016) presented an implementation and first numerical tests of the depth version of the CIG image-wave equation (see also Gomes et al., 2016a,b). In his procedure, the whole model is updated at each iteration, in a similar way to the time-domain procedure of Schleicher et al. (2008). Gomes (2016) found that the first iteration results, starting at a constant-velocity initial model, were encouraging, already providing a glimpse of the structure of the model, with reflector depths and layer velocities not too far from the true ones. However, further iterations did not work satisfactorily, converging only slowly or not at all towards the true model.

In this work, we present a layer-stripping procedure to update the velocity model. Our numerical tests on the model of Gomes (2016) demonstrate that such a procedure improves the convergence of the method. This new procedure changes the interpretation of the velocity values. In the global update approach of Gomes (2016), velocities represent inverse averages over the medium slowness. Therefore, an additional step is required to convert these velocities into layer velocities. In our layer-stripping approach, velocities are actual interval velocities that can be directly used to update the velocity model.

THEORY AND METHOD

First and foremost it is necessary to understand what an image-wave is. According to Hubral et al. (1996), an image-wave is not a physical wave, but it behaves in an analogous way. Just as physical waves propagate as a function of time, image-waves propagate as a function of migration velocity. Because of this conceptual similarity, it is possible to derive image-wave equations using simple plane-wave considerations. The idea is that different migrated images are like snapshots of the image-wave in different "instants" of migration velocity.

Starting from the equation of Al-Yahya (1989) that describes the migrated position of a horizontal reflector in a homogeneous medium as a function of migration velocity, Schleicher et al. (2008) derived the following partial differential equation (PDE) that kinematically describes the depth displacement of events in a CIG, i.e., the image-wave equation in a CIG,

$$\frac{\partial p}{\partial z} + \frac{vz}{h^2 + z^2}\frac{\partial p}{\partial v} = 0 \,, \tag{1}$$

where p is the migrated wavefield, or in other words, the "image-wave", v is migration velocity, h is half offset and z is the migrated pseudo-depth. Equation 1 has the form of the 1D one-way wave equation and it can be discretized with a finite-difference scheme such as the one devised by Schleicher et al. (2008).

The CIG migrated with an initial velocity c_0 is the initial condition to image-wave propagation. When applying the finite-difference scheme to this CIG, it will be continued from the starting velocity to a new one without the need of another migration of the multi-coverage data. In the velocity continuation process, the starting velocity (or reference velocity) is the constant velocity used to migrate the data in the first iteration. If the continuation velocity is equal to the velocity of the event, its curve will become flat and if not, it will have a residual moveout. Thus, by continuing a CIG it is possible to vary the moveout of events. Our intention is to flatten every identifiable event so we can extract velocity and depth information associated with them.

Just like Schleicher et al. (2008) and Gomes (2016), we use semblance panels quite similar to the ones used in conventional NMO analysis (e.g., Yilmaz, 1987) to evaluate the flatness of a CIG after the velocity continuation. In our work flow, to pick velocities and depths of events, we use primarily the semblance panel. When events in the semblance panel are not well focused, we combine their information with a visual inspection of continued CIGs to obtain the best possible results.

One restriction of the image-wave equation (1) is that is has been derived under the assumption of a constant migration velocity. This means that the velocity-continuation procedure will always work with an average velocity.

After preliminary tests, Schleicher et al. (2004) observed that the mean velocity which determines the depth-migrated event positioning through image continuation in a medium with vertical velocity variation is the inverse of the slowness average. For a model with flat layers, this implies that the interval velocity V_{I}^{j} in the jth layer can be calculated from average velocities according to (Gomes et al., 2016b)

$$V_{\mathrm{I}}^{j} = \frac{(z_j - z_{j-1})}{(z_j - z_0)/V_{\mathrm{M}}^{j} - (z_{j-1} - z_0)/V_{\mathrm{M}}^{j-1}} , \tag{2}$$

where z_j and z_{j-1} are the depths of two consecutive events below depth z_0 and V_{M}^{j} and V_{M}^{j-1} are the corresponding average velocities associated with these two events. This inversion to interval velocities is required only for the global update strategy. When executing layer-stripping, this is not necessary because velocity values already represent local averages, which can be assumed to closely resemble layer velocities.

Another consequence of the constant-velocity assumption is the need for a constant initial reference velocity value, V_{R}, to start the image-wave propagation, even if the migration was actually carried out with an inhomogeneous migration velocity model. Our numerical tests indicate that the value of V_{R} is not critical to the behaviour of the procedure. The extracted velocity information for each event is the difference between the velocity that flattens the event in the CIG and the reference velocity.

Since the procedure starts with a constant-velocity migration, the velocity update in the first iteration can be performed with the extracted values of the flattening velocities V_{F}^{j}, where superscript j indicates the number of the reflection event in the CIG. At later iterations, an additional correction is necessary, because the velocity model is already heterogeneous. In time migration, the velocity-updating formula from residual migration (Rothman et al., 1985) can be used (Schleicher et al., 2008). In the global update strategy for depth migration, slowness averages need to be updated according to (Gomes et al., 2016b)

$$1/V_{\mathrm{M},i+1}^{j} = 1/V_{\mathrm{M},i}^{j} + 1/V_{\mathrm{F}}^{j} - 1/V_{\mathrm{R}} , \tag{3}$$

where $V_{\mathrm{M},i+1}^{j}$ is the new average velocity value after updating the one from the previous iteration, $V_{\mathrm{M},i}^{j}$. In contrast, the layer-stripping strategy directly updates interval velocities V_{F}^{j} according to

$$V_{\mathrm{I},i+1}^{j} = V_{\mathrm{I},i}^{j} + V_{\mathrm{F}}^{j} - V_{\mathrm{R}} , \tag{4}$$

where $V_{\mathrm{I},i+1}^{j}$ and $V_{\mathrm{I},i}^{j}$ are interval-velocity values in two subsequent iterations. Moreover, V_{F}^{j} is the velocity value that flattens the jth event and V_{R} is the reference velocity used to start the velocity continuation. Note that if in the first iteration of both strategies, the initial model has constant velocity, i.e., $V_{\mathrm{I},0} = V_{\mathrm{M},0} = V_{\mathrm{R}}$, then flattening velocities are already the desired updated velocities.

In other words, the global-update iterative algorithm of this depth-remigration approach, from Gomes et al. (2016a), closely follows the time-remigration algorithm of Schleicher et al. (2008), with just replacing the velocity-update formula in accordance with the migration domain. More specifically, the steps of the global-updating procedure are listed in the left column of Table 1.

Actually, Gomes (2016) showed that updating average velocities in step 4 using equation 4 instead of equation 3 led to slightly better results.

In our layer-stripping approach, we determine interval velocities one layer at a time. We iterate for a given layer until its event in the CIG is sufficiently flat and only then proceed to the next deeper event, continuing in this manner until all events are flattened. Thus, only local velocities are updated, allowing us to use equation 4 directly. Since this procedure reduces the velocity error of shallower layers before proceeding to deeper layers, error accumulation should be reduced, favoring convergence at deeper layers. Our numerical examples confirm this expectation. The steps of the layer-stripping algorithm are listed in the right column of Table 1.

Global Update	Layer Stripping
1. Migrate the data using a constant velocity model and sort the migrated data into CIGs.	1. Migrate the data using a constant velocity model and sort the migrated data into CIGs.
2. Apply the image continuation process by solving equation 1.	2. Apply the image continuation process by solving equation 1.
3. From the continued CIGs, determine flattening velocities and depths for all events.	3. From the continued CIGs determine flattening velocities and depths for the current event.
4. Update average velocities in the previous model using equation 3 (not necessary in first iteration).	4. Update the interval velocity using equation 4.
5. Calculate the corresponding new interval velocities using equation 2.	5. Migrate the data with this new velocity model.
6. Migrate the data with this new velocity model.	6. Repeat steps 2 to 5 until current event is satisfactorily flat.
7. Repeat steps 2 to 6 until events are satisfactorily flat on CIGs.	7. Go to next event.
	8. Repeat steps 2 to 7 until all events are satisfactorily flat.

Table 1: Iterative algorithms for the global and layer-stripping velocity updating procedures.

EXAMPLES

To test our layer-stripping MVA strategy and compare it to the global update approach of Gomes et al. (2016a) we applied both strategies to the same model used in Gomes et al. (2016a) in their first example. This model is depicted in Figure 1. Using this model, we also tested an approach which uses the global-update first result to guide the layer-stripping procedure. This model consists of two sedimentary layers with velocities 2000 m/s and 3000 m/s embedded between a water layer with a flat bottom and a salt layer with rather strong topography. The model extension is 25 850 m. Since there are no reflectors below the top-of-salt, salt velocity cannot be extracted from seismic reflection data. Following the same approach as Gomes et al. (2016a), for the global update we flood the model of each iteration below the determined position of the last reflector with the original salt velocity of 4500 m/s. For the layer-stripping tests, we observed it is best not to use the salt velocity since it increases errors as we proceed to later iterations.

For this model, we generated synthetic data using the ray-tracing modeling tool Cshot, distributed as part of Seismic Un*x. Since we decided to study the acoustic case, only primary reflections of P waves were modeled, neither taking geometrical spreading nor absorption into account. The acquisition geometry is regular, with 50 common offset sections, spaced at 100 m each, the smallest offset being 50 m and largest being 4950 m.

To compare the implementations, we started both the global update and the layer-stripping iterative procedures with a constant-velocity migration using a homogeneous initial velocity model of 2000 m/s, in accordance with the first step of both iterative procedures. The second and third steps of the first iteration are still fully identical for both implementations. For the velocity analysis, we selected 52 CIGs at every 500 m to be continued for velocities between 1400 m/s and 4000 m/s, starting at the reference velocity of 2000 m/s. Figure 2 shows the semblance panel from the continuation process for the CIG at position 7525 m (briefly referred to as CIG 7525).

From the panel in Figure 2, one reason becomes clear why the global update approach can run into difficulties. In panels like this one, it is very hard to identify and pick the semblance peaks of all three reflection events. While there are better focused panels at other CIGs, generally, picking errors will occur and lead to velocity errors. Looking into the continued CIGs themselves can help to resolve ambiguities but makes the process more cumbersome. Figure 3 shows the continued CIG 7525 at three velocities that approximately flatten the three events for short offsets. In the global update approach, these flattening

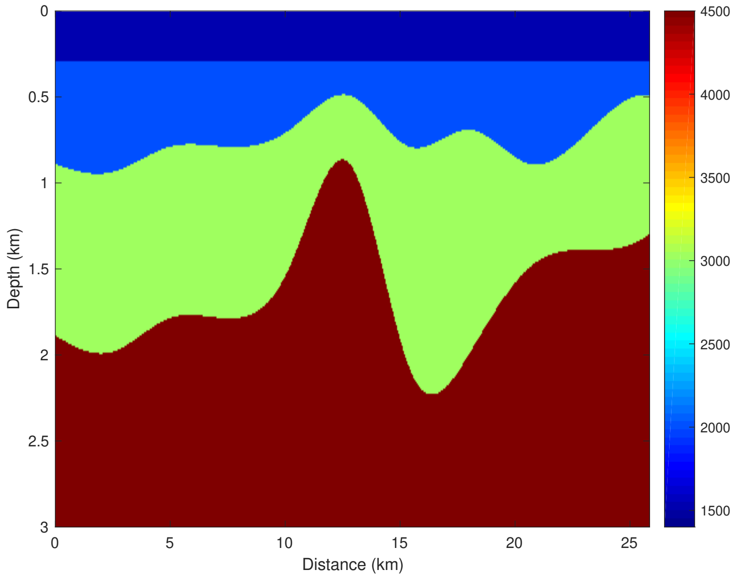


Figure 1: Velocity model used to generate the seismic data.

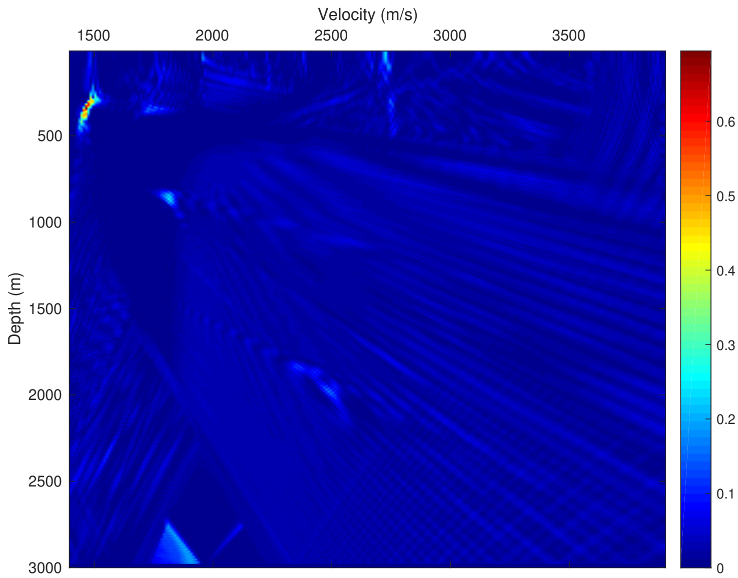
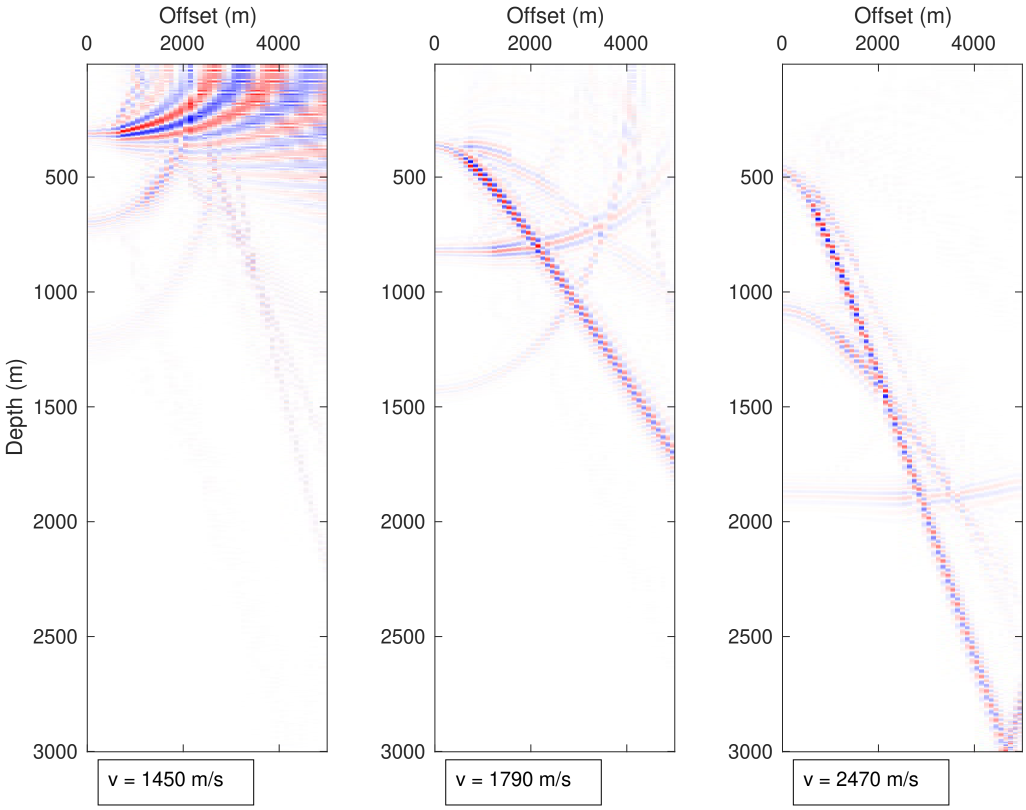


Figure 2: Semblance panel of the continued CIG 7525.

velocities become the new values for the average velocities at these depths. In the layer-stripping approach, only the better focused peak for the first event is used to define the layer velocity down to that depth.

By performing this analysis at each of the 52 CIGs selected at every 500 m, we created a new velocity model. In the global update approach, the whole model is updated at once. For the layer-stripping approach, in each iteration, only the current layer we are working on is updated and the velocity found is used to flood the rest of the model (from that layer to the bottom of the model). Away from the selected CIGs, the velocity model is filled by linear extrapolation. Practically all velocity models shown here have been smoothed in order to use them for Kirchhoff migration at the beginning of each iteration.



(a) (b) (c)

Figure 3: Continued CIG 7525 for three velocities that flatten each one event.

Global update implementation

In the global update approach (Figure 4), as explained before, each iteration updates the whole model. Figure 4 shows the results of the first three iterations. White dashed lines represent the true position of interfaces and black lines represent interface position as obtained by the velocity analysis. It is clear that already after the first iteration (Figure 4a), we have an acceptable overall impression of the velocity model. The second iteration (Figure 4b) improves the layer velocities and the positioning of the second interface. However, another iteration worsens velocities in the second and third layer (Figure 4c). Further iterations (not shown here) did not help to improve the results. These observations are consistent with the findings of Gomes et al. (2016a), who also note a failure of convergence of the global updating procedure.

Layer-stripping implementation

In the layer-stripping approach (Figure 5), only one layer is updated at a time. After accepting the result (using to some kind of convergence criterion) of a given layer, the algorithm proceeds to the next one. Our convergence criterion is that we accept layer velocity and interface position if they do not change considerably between two consecutive iterations. For that reason, we need at least two iterations for each layer to confirm convergence.

Figure 5a and 5b show the result of the first and second iteration, working on the first layer. Again, white dashed lines represent the true position of interfaces and black lines represent interface position given by the velocity analysis.

Only after fixing the velocity and reflector position of the first layer (Figure 5b), the algorithm proceeds to the second layer (Figure 6a). The velocity and interface of the second layer converged after six more iterations (Figure 6). In other words, Figure 6f shows the result after a total of eight iterations. We see that

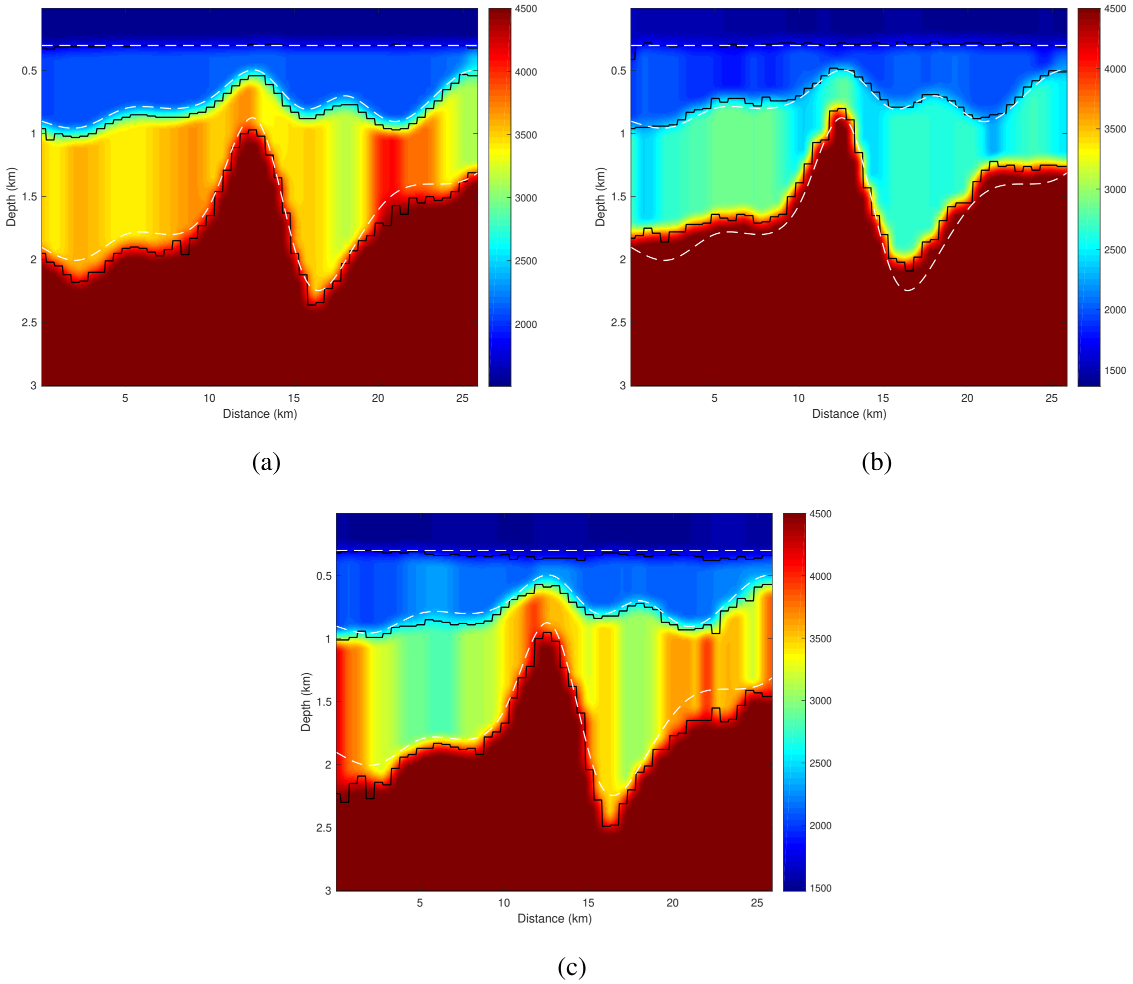


Figure 4: Evolution of models at each consecutive iteration for the global update implementation. (a) first iteration, (b) second iteration, (c) third iteration.

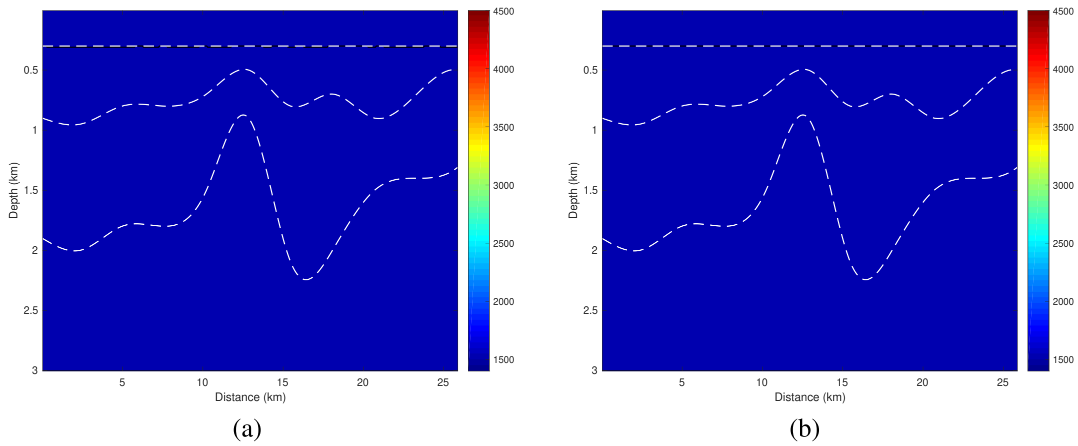


Figure 5: Evolution of models for the layer-stripping implementation: Results of the (a) first and (b) second iteration, which act on the first layer only.

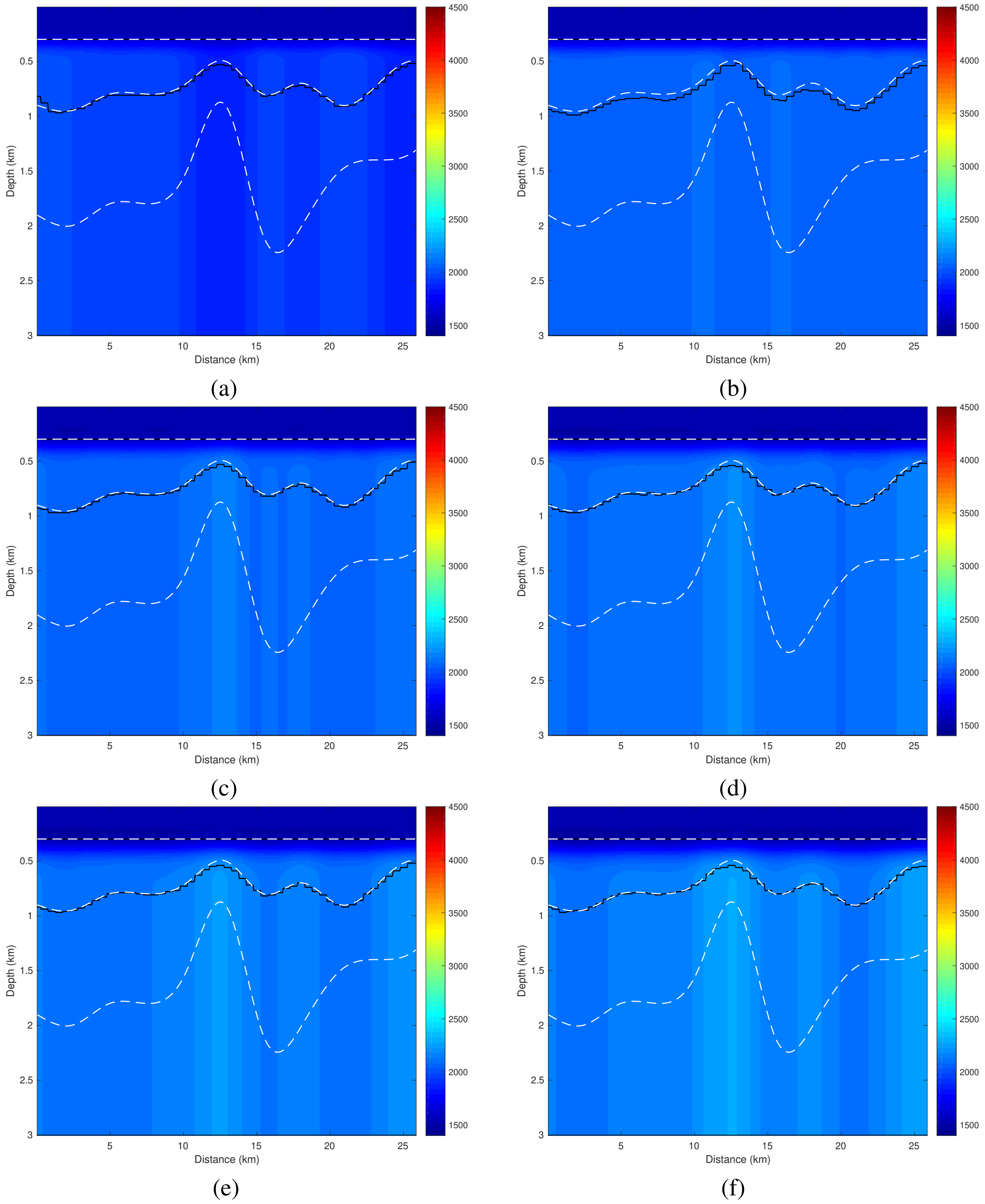


Figure 6: Evolution of models for the layer-stripping implementation: Results of the third to eighth iteration, which act on the second layer only.

both layer velocity and bottom interface of the second layer match the true values quite well.

For the third layer, another five iterations were needed (Figure 7), bringing the total number of iterations to thirteen. Figure 7e exhibits the final model after thirteen iterations, showing a very accurate velocity model of the subsurface down to the top of the salt layer.

While the match between the true and recovered velocities and interface positions at the third layer is not as perfect as for shallower events, we recognize great improvement over the global update result of Figure 4. The quality of the velocity model in Figure 7e appears to be sufficient to serve as an initial model for more sophisticated inversion techniques.

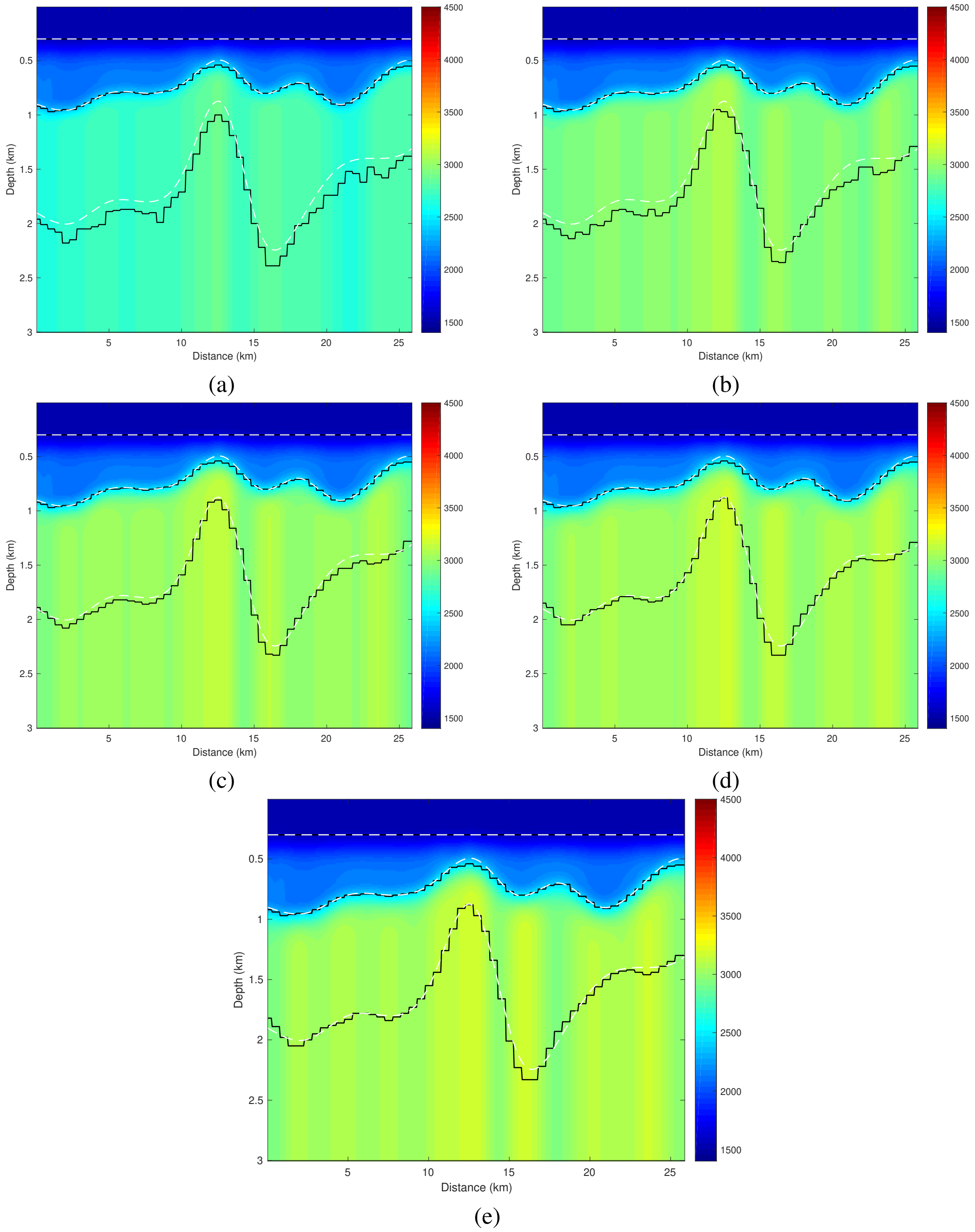


Figure 7: Evolution of models for the layer-stripping implementation. Results of the ninth to thirteenth iteration, which act on the third layer only. Part (e) shows the final model obtained with the layer-stripping algorithm.

Hybrid approach: Layer-stripping guided by global update

Because of the rather good result after the first iteration of the global updating procedure (see again Figure 4a), we wanted to see if it was possible to reduce the number of iterations in the layer-stripping updating by using this information. For this purpose, we tried to guide the layer-stripping procedure with the velocities from the first global iteration. This hybrid approach works in the following way: We start with the first

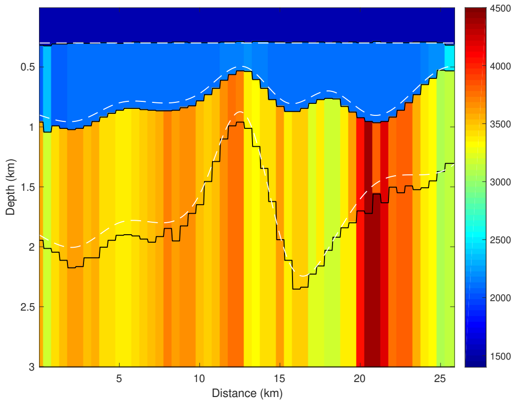
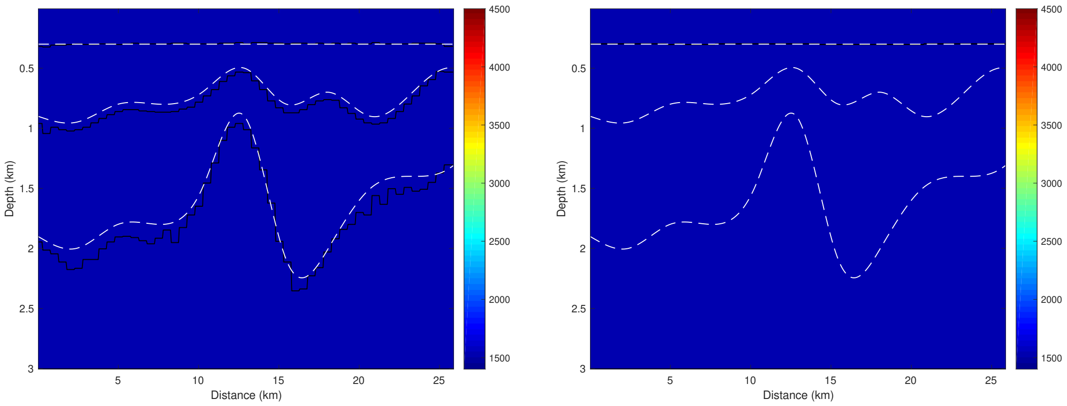


Figure 8: Model with initial velocities for the hybrid approach, obtained from the first iteration of the global update.



(a) (b)

Figure 9: Evolution of models for the hybrid approach: (a) Initial model from the first global iteration. (b) Results of the first iteration, acting on the first layer only.

result of the global update technique without salt-velocity flooding (Figure 8) and smooth it horizontally layer by layer. These layer velocities will give the head start for the work in each specific layer, one at a time, as done for layer-stripping. Note that we make no use of the estimated layer depths of the model in Figure 8. As before, white dashed lines represent the true position of interfaces and black solid lines represent interface position given by the velocity analysis.

Figure 9a shows the model built with the velocity of the first layer of the model in Figure 8. This model was used to start the layer-stripping procedure experiment (starting with a migration with this model). Then, a single iteration was sufficient to determine the first layer. The resulting model is shown in Figure 9b.

In the next step, we fixed the result of the first layer and used the velocity of the second layer from the smoothed version of the model in Figure 8 to build a new model (Figure 10a). This model is then used to migrate the data and start the second iteration, the result of which can be seen in Figure 10b. To define the second layer, five iterations were necessary (see parts (b) through (f) of Figure 10).

Finally, we fixed the result of the second layer and applied the velocity of the third layer from the smoothed version of Figure 8 to build a new model (Figure 11a) and initiate the iterations for the third layer. The results of the next four iterations are shown in parts (b) to (e) of Figure 11.

As one can observe, there are areas in the third layer where the velocity does not converge. This probably happens because of too large velocity errors in third-layer result in the initial model (Figure 8). Note that in the first iteration of the global approach, deeper layers will accumulate all errors from shallower layers, resulting in rather poor estimates of the layer velocities in deeper regions of the model. As a consequence, the initial velocities in the third layer present too strong variations, causing the velocity there

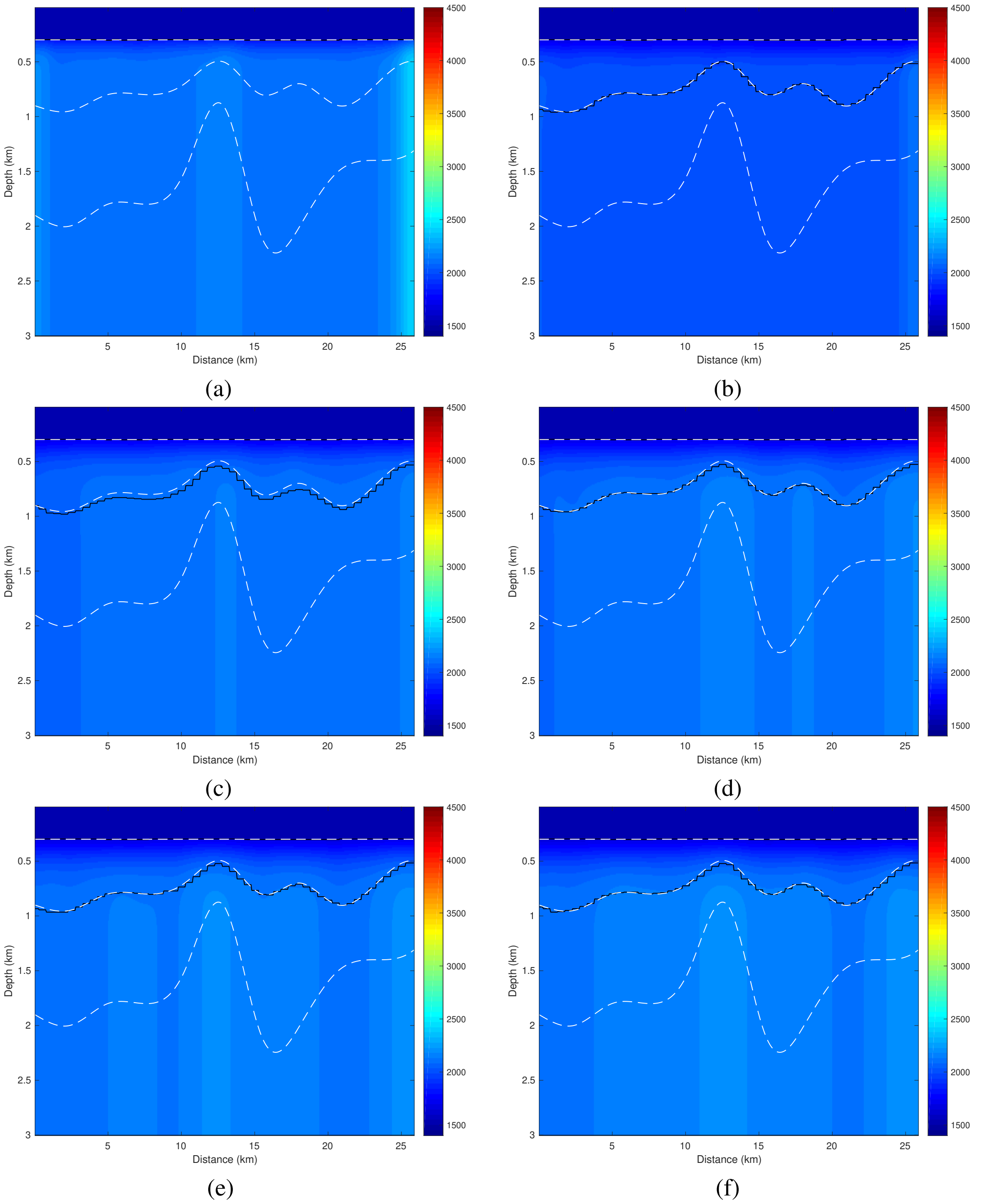


Figure 10: Evolution of models for the hybrid approach: (a) Initial model for the third iteration, after insertion of the second-layer velocities from the first global iteration (figure 8) into the model after the second iteration (Figure 9b). (b) to (f) Results of the third to seventh iterations, acting on the second layer only.

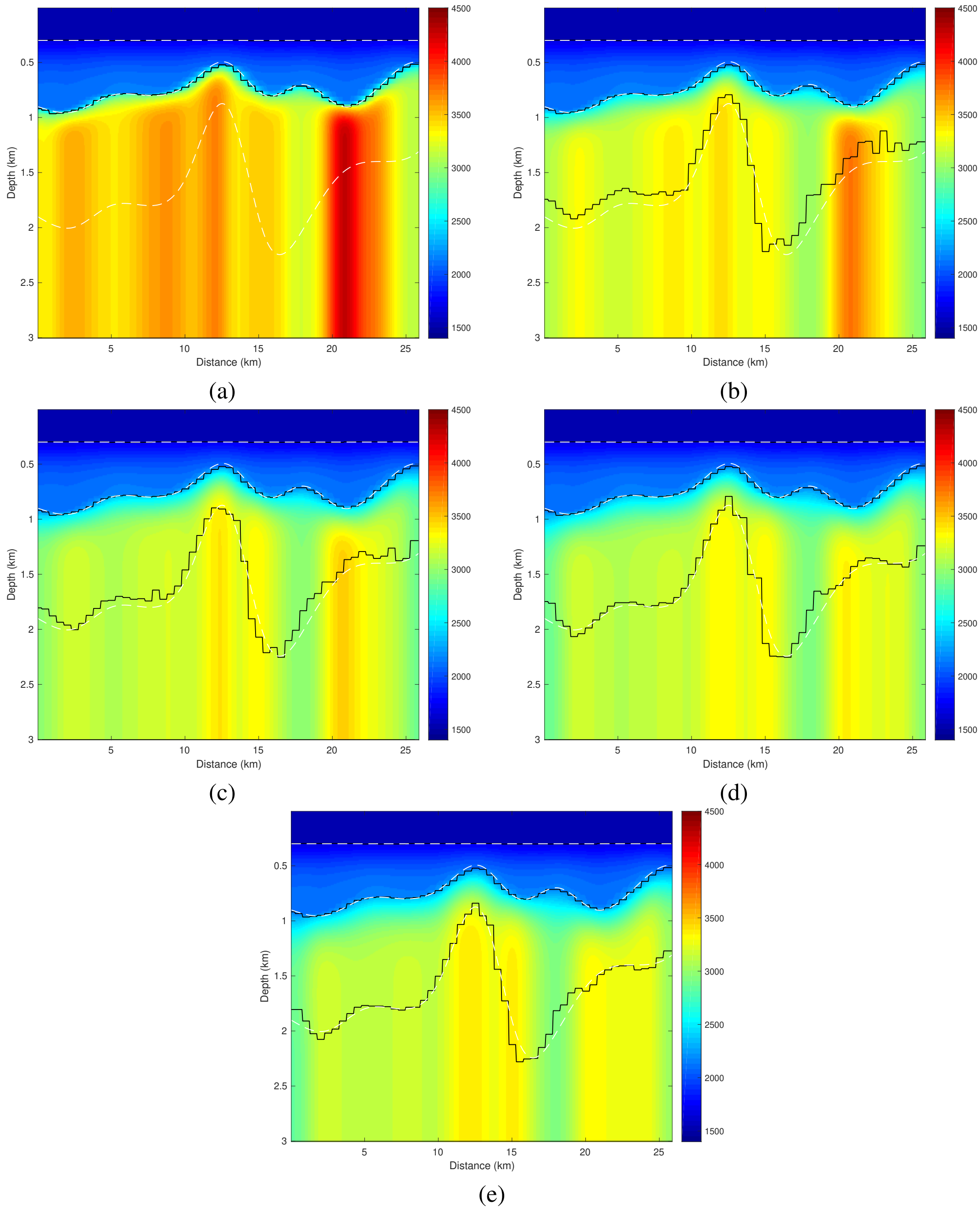


Figure 11: Evolution of models for the hybrid approach: (a) Initial model for the seventh iteration, after insertion of the third-layer velocities from the first global iteration (figure 8) into the model after the sixth iteration (Figure 10f). (b) to (e) Results of the seventh to tenth iterations, acting on the third layer only.

to diverge. Thus, we refrained from pursuing further iterations.

CONCLUSIONS

In this work, we have investigated a layer-stripping strategy for depth MVA with velocity continuation in CIGs and compared it to the earlier global update procedure. We also tested a mixture of these two approaches, using the first result of the global update to guide the layer-stripping experiment. Whereas the global update strategy ideally updates average-slowness values in the whole model, the layer-stripping approach updates local, i.e., interval velocities one layer at a time. Our numerical experiments demonstrate that the layer-stripping approach solves the convergence issues presented by the global approach. The reason is that the former only proceeds to deeper reflection events after shallower layers have been sufficiently well defined, propagating less errors to deeper layers. Our numerical experiments comparing the two strategies demonstrate that this leads not only to better convergence but also to an improved final velocity model over the global velocity updating strategy. A hybrid approach using the velocities from the first global update to guide the layer-stripping procedure is only useful for the shallowest layers.

It is to be stressed that MVA by image continuation in CIGs is a rather inexpensive procedure which starts at no a-priori knowledge of the medium. Therefore, it can be used to build initial models for more sophisticated inversion techniques.

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