# SEQUENTIAL FULL-WAVEFORM INVERSION OF REFRACTED AND RAYLEIGH WAVES

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## ABSTRACT

This study presents advances in elastic multi-parameter full-waveform inversion (FWI) for shallow seismic applications. Rayleigh waves are attractive for near-surface characterisation since they exhibit a high signal-to-noise ratio in field data recordings and they are easily excited by using hammer blows. Rayleigh waves are highly sensitive to the S-wave structure of the subsurface. FWI fails to reconstruct the P-wave velocity model due to its low sensitivity to Rayleigh waves which dominate the misfit. In order to properly incorporate the P-waves we initially perform a mono-parameter FWI of the refracted waves. This is followed by a multi-parameter inversion of the full wavefield. In a synthetic study we show that our new sequential approach is able to reconstruct more accurately the shallow P-wave velocity and improves the resolution of the shallow S-wave velocity and density structure.

## INTRODUCTION

Shallow seismic applications are highly attractive for various investigations, including geotechnical and hydrogeophysical characterisation of the subsurface. Vertical hammer blows allow the excitation of Rayleigh waves and compressional P-waves. Rayleigh waves are highly sensitive to the S-wave structure of the subsurface, whereas the refracted P-waves can be used to invert the P-wave velocity. Full-waveform inversion (FWI; Tarantola, 1984) of these wave types allows us to derive information of the elastic parameters and reconstruct accurate subsurface models that describe our measured data. However, in most cases the amplitudes of the Rayleigh waves are much higher than the amplitudes of the compressional P-waves. In these particular cases FWI fails to properly incorporate the contribution of the P-waves, as the misfit is dominated by the high-amplitude Rayleigh waves. To compensate for this effect we suggest a new sequential FWI strategy, in which we initially invert using the low-amplitude refracted P-waves and then subsequently the full wavefield including the Rayleigh waves.

#### METHOD AND THEORY

In the framework of 2D elastic FWI, both the forward modeling (Bohlen, 2002) and inversion (Köhn et al., 2011) are done in the time-domain using finite differences. The inversion problem is solved by a conjugate gradient method. The gradients are computed in the time domain with the adjoint method. As a misfit function we use the least-squares misfit of the normalized wavefields (Groos et al., 2014). Further preconditioning includes a semi-circular source tapering and a multi-scale stepwise inversion of seismic signals (Groos, 2013).

The new sequential FWI strategy, called SFWI, consists of a two-stage strategy. In the first stage we invert only the refracted P-waves to obtain an initial P-wave velocity model. In the second stage, we apply a three-parameter (reconstruct the P-wave velocity  $(v_p)$ , S-wave velocity  $(v_s)$  and density  $(\rho)$ ) FWI using the full wavefield, including both wave types.

The main advantage of SFWI is that we are increasing the contribution of P-waves. More precisely, FWI will try to minimize the misfit between modelled and observed seismograms through the amplitude fitting of both Rayleigh and refracted waves, despite the fact that the amplitudes of the Rayleigh waves are much higher than the amplitudes of the P-waves.

## SYNTHETIC TESTS

In this section we present a synthetic case study and compare the results of conventional FWI (stage 2, without an initially updated  $v_p$  model) with SFWI (stage 1 and 2). The acquisition geometry of the synthetic study consists of a linear profile of 36 multi-component (vertical and horizontal inline) geophones, with an equidistant spacing of 1 m. Nine vertical hammer blows act as sources, with an equidistant spacing of 5 m. The grid spacing is chosen to be 0.15 m, which results in a model space of 50.4 m in the horizontal and 18 m in the vertical direction. The range of the frequencies used is from 10 up to 70 Hz in steps of 10 Hz.

The true  $v_p$ ,  $v_s$  and  $\rho$  models (figure 1) are based on a geological situation found on an airfield in Rheinstetten in western Germany. The  $v_p$  model consists mainly of two layers with a semicircular lowvelocity anomaly in the upper layer. Equivalent structures are assumed for the density (Groos, 2013). The S-wave velocity is increasing linearly with depth. In all three models we observe a discontinuity at 5.97 m depth, representing the groundwater table followed by a homogenous half-space. The second column of figure 1 depicts the initial models of the  $v_p$ ,  $v_s$  and  $\rho$  parameters. The starting models consist of a smooth gradient of increasing velocity/density over depth, while the water table is estimated at 9 m depth.

# RESULTS

We present the results of the conventional multi-parameter FWI (figure 1, column 3) and compare it with the new sequential FWI approach (figure 1, column 4).

### **Conventional FWI**

With conventional FWI we basically apply a multi-parameter inversion without a previously updated  $v_p$  model (only stage 2). The inversion exhibits a good reconstruction of the  $v_s$  model since the wavefield is dominated by the high-amplitude Rayleigh waves, which are mainly sensitive to the S-wave velocity structure. For this reason, FWI fails to update the true depth of the groundwater table. We observe that the  $v_s$  model in the upper 4 meters is well resolved with the exception of some source-related artefacts. The  $v_p$  model is updated slightly only for the upper 3 meters, which is most probably due to the cross-talk from  $v_s$ . The low sensitivity of the Rayleigh waves to  $v_p$  and the strong deviation between the initial and true  $v_p$  models do not allow a good reconstruction of the P-wave velocity. The density is very poorly reconstructed and a clear interface related to the water table is not present.

#### Sequential FWI

The final models from SFWI present a higher correlation to the true models than the conventional FWI. In stage 1, the misfit is sufficiently decreased and the depth of the groundwater table is updated to 5.73 m, which is very close to the actual depth of 5.97 m. The P-wave velocity above the groundwater table could not be well reconstructed. This is achieved in stage 2 by the FWI of the Rayleigh waves. Specifically, the  $v_p$  is updated up to approximately 6 meters depth and the discontinuity due to the groundwater table is present. The low velocity anomaly is also visible in  $v_p$ . After further research we found that this is not the result of a cross-talk from the  $v_s$  model. The information from the updated  $v_p$  model is reflected also in  $v_s$  and  $\rho$  models, where the FWI algorithm incorporates properly the information obtained from  $v_p$  and updates the depth of the water table to the other parameters. In terms of the  $v_s$  parameter, the source-related artefacts do not dominate the upper 3 meters, as in the case of conventional FWI, and the obtained  $v_s$  model highly resembles the true  $v_s$  model. Finally, also the reconstructed density model provides a much higher correlation to the true density, as opposed to conventional FWI where no information could be safely extracted. However, we found that the information from the low-velocity anomaly is due to cross-talk with the  $v_s$  model.



**Figure 1:** Model space of the elastic properties. The rows of the figure represent the  $v_p$  model,  $v_s$  model and density, respectively. The columns represent the true and starting models and the final results of the conventional and sequential FWI strategies. Red stars represent the source locations and the dotted black line the absorbing boundaries (PMLs).

#### **Comparison of seismograms**

In order to verify the quality of the reconstructed models we compare the seismograms at the last frequency step (70 Hz) of the vertical and horizontal inline component for both the conventional and sequential FWI.

In figure 2 we compare the displacement seismograms for four different offsets of the observed data (true model, black) with the data obtained by conventional FWI (red) and SFWI (blue). In both components we can observe that in SFWI the first arrivals are fitted exceptionally well. This high fitting of the first arrivals is reflected in the model space (Figure 1) by the good reconstruction of the depth of the groundwater table. The Rayleigh waves are fitted by both methods equally well. Finally, SFWI reached a lower misfit value than conventional FWI at all frequencies.

## CONCLUSIONS

Elastic FWI fails to reconstruct the P-wave velocity model due to its low sensitivity to the Rayleigh waves which dominate the misfit. To overcome this problem we suggest a two-stage sequential approach for which initially only the information from the refracted waves is used to reconstruct the  $v_p$  model. In this first stage, the actual depth of the groundwater table and the P-wave velocity of the upper half-space have been resolved. This information is then given as an input to stage 2 and enhances the multi-parameter FWI. The P-wave velocity of the upper layer converges closely to the true model in stage 2. The artefacts in the vicinity of the sources which were present in the reconstructed model of  $v_s$  using conventional FWI are reduced in the case of SFWI. The new sequential approach we presented shows the potential of FWI to recover the elastic properties of the subsurface in great detail, resolving previous problems and allowing a better convergence and lower misfit over the full wavefield.



**Figure 2:** Normalized displacement seismograms for 4 different offsets over time calculated from the observed data (black), FWI (red) and SFWI (blue) for both horizontal inline ( $d_x$ , left) and vertical ( $d_z$ , right) components. The time is clipped to 0.4 s instead of the full recording time of 1 s.

#### PUBLICATIONS

Additional results were published by Athanasopoulos and Bohlen (2016, 2017).

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