GLOBAL OPTIMISATION FOR AVO INVERSION: A GENETIC ALGORITHM USING A TABLE-BASED RAY-THEORY ALGORITHM

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ABSTRACT

Amplitude Variation with Offset (AVO) inversion provides estimates of the P-wave velocity, S-wave velocity and density of a stratified medium. Global optimisation is desirable for the inversion to account for the multi-parametric behaviour of the AVO inversion which is strongly affected by the initial estimates of the model rock properties. We investigate the dependency between P-wave, S-wave velocity and density in the recovered parameters using empirical relations as constraints. In inversion schemes, forward modelling is often the most time-consuming process. To reduce the computation time, we have implemented a genetic algorithm using a table-based ray-theory algorithm to allow for a significant amount of vertically inhomogeneous models in the global search. Our results show that the genetic algorithm is capable of recovering the physical model parameters with good agreement for examples using the empirical constraints. However, it sometimes converged to solutions which were far from the correct answer. Still, those solutions represented good models to explain the observed dataset, exemplifying the non-uniqueness of the problem. The forward modelling algorithm has shown excellent performance to be used in global optimisation schemes, because it allows the use of a vast number of members in the population of the genetic algorithm.

INTRODUCTION

The methodology of Amplitude Variation with Offset (AVO) inversion has been widely used in the industry as a direct way to determine hydrocarbon indicators. AVO inversion allows for a quantitative interpretation of the amplitude of seismic data in order to estimate the rock properties. For the forward modelling part of AVO inversion schemes, one classically calculates the reflection coefficients for plane waves as a function of incident angle (offset) using the formulas of Knott (1899) and Zoeppritz (1919). For the reverse model, Rosa (1976) derived and verified the ill-posed nature of the inversion of Zoeppritz equations for rock properties, meaning that multiple combinations of input can produce almost the same output. Besides, due to its multi-parametric formulation, the solution space is very complex with many local minima, which makes it difficult to find correct solutions.

For those reasons, Stoffa and Sen (1991), Mallick (1995) and others suggest employing global optimisation schemes to treat the AVO inversion problem. However, the process of global optimisations requires many forward models which considerably increases the computation time. Many of the works published in the literature use forward modelling algorithms based on the wave equation, which demand great computational power.

This study is based on the premise that unconsolidated sediments with small rock-property changes between layers can be modelled with ray theory without suffering from an unacceptable loss of resolution. For computational ease, we have developed a very fast forward modelling algorithm based on ray theory.

This algorithm makes use of precomputed tables storing the ray quantities in order to speed up the computation of synthetic seismograms for global optimisation. Our resulting genetic algorithm is based on this *table-based ray theory*. It relies on findings of Stoffa and Sen (1991) and Sen and Stoffa (1992) and uses the empirical relationships of Gardner et al. (1974) and Castagna et al. (1985) as constraints to verify the dependency among the parameters.

SYNTHETIC COMMON-MIDPOINT SEISMOGRAM: FORWARD MODELLING

The forward modelling procedure implemented in this work is a so-called *table-based ray tracing* algorithm. It assumes an Earth model composed of n horizontal isotropic layers. The parametric equations to compute the two-way traveltimes and source-receiver offsets as a function of a constant ray parameter are given by (Slotnick, 1959)

$$x = 2\sum_{i=1}^{n} \frac{h_i p v_i}{(1-p^2 v_i^2)^{1/2}} \, , \tag{1}$$

$$t = 2\sum_{i=1}^{n} \frac{h_i}{v_i(1-p^2 v_i^2)^{1/2}} \, , \tag{2}$$

where h_i and v_i are, respectively, the layer thickness and velocity of the ith layer. Moreover, p denotes the ray parameter defined by

$$p = \frac{\sin(\theta_i)}{v_i} \, . \tag{3}$$

Here θ_i is the angle between the seismic ray and the vertical in the ith layer. Using a sonic log or other vertical velocity information, we use equations 1 and 2 to build three different two-dimensional tables (*offset, traveltime* and *reflection coefficient*) before starting the inversion procedure.

To initiate the process, we calculate the 2D traveltime table. for this purpose, we transform the P-wave velocity, supposed to be given as a function of depth, to the time domain. The same applies to the S-wave velocity and density functions. We then determine the ray parameter in accordance with the incident angle in the first medium varying from 0° to 89° in steps of 1°. This results in 90 columns in the resulting traveltime table. The rows are associated with the t_o times in increments of the time sample interval.

Once the traveltime table has been computed, the corresponding 2D tables for offset and traveltime can easily be determined using equations 1 and 2 respectively. Since the velocity function is sampled in equal increments of the sample rate, in the same way, the 2D tables are, the computation of the incident angle for each cell in the 2D table is possible with the help of equation 3.

With the incidence angle known, the next step is to compute the 2D table of reflection coefficients R_P at each interface. For this purpose, we utilise Shuey's approximation (Shuey, 1985), given by

$$R_P(\theta) = A + B\sin^2(\phi) + C\sin^2(\phi)\tan^2(\phi) \, , \tag{4}$$

where θ is the incidence angle at the interface under consideration, and ϕ is the average of the incidence and transmission angles. Also, the coefficients A, B and C are defined as

$$\begin{aligned} A &= \frac{1}{2}\left(\frac{\triangle V_P}{V_P} + \frac{\triangle\rho}{\rho}\right) \, , \\ B &= \frac{1}{2}\frac{\triangle V_P}{V_P} - 2\left(\frac{V_S}{V_P}\right)^2\left(\frac{2\triangle V_S}{V_S} + \frac{\triangle\rho}{\rho}\right) \, , \\ C &= \frac{1}{2}\left(\frac{\triangle V_P}{V_P}\right) \, . \end{aligned} \tag{5}$$

In these expressions, $\triangle V_P = V_{P2} - V_{P1}$, $\triangle V_S = V_{S2} - V_{S1}$, $\triangle\rho = \rho_2 - \rho_1$ are the differences between the P and S-wave velocity and density values, respectively, across the interface and V_P, V_S and ρ are their arithmetic averages.

In order to build a synthetic common-midpoint (CMP) section, we define which are the offsets to model and start a row-by-row search inside the *offset table* looking for the requested offsets. Once the cell is found

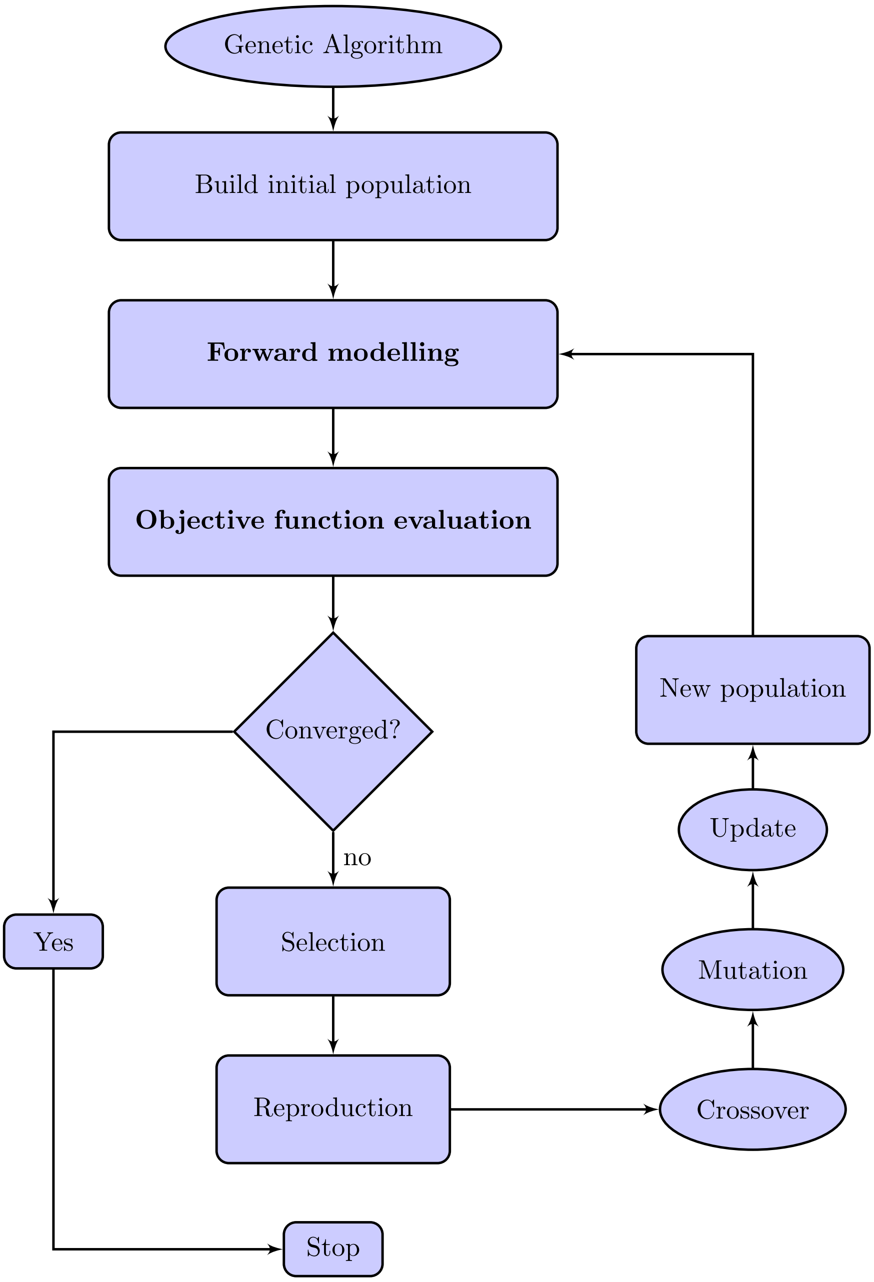

Figure 1: Flowchart of the genetic algorithm.

for the specified offset, we go into the same cell position inside the *traveltime* and *reflectivity tables* to place the reflection coefficient from that position at the correct time arrival inside the trace.

Because the algorithm is very simple, it provides the potential of being adapted in the future to study more complex effects such as the stretch from normal moveout (NMO) correction, NMO without stretch (ray-trace NMO correction), array effects affecting the amplitude of the wave received by the streamer, extraction of AVO attributes, polar anisotropic media, etc.

Table 1: Model parameters to test the inversion scheme.

Layers	Thickness (m)	Vp (m/s)	Vs (m/s)	Density (g/c^3)
1	1000	2000	551.72	2.07
2	50	2800	1241.38	2.25
3	50	2300	810.34	2.14
4	500	3000	1413.80	2.29

GLOBAL OPTIMISATION APPROACH: GENETIC ALGORITHM

The genetic algorithm (see a generic flowchart in Figure 1) is a technique used to perform global optimisations based on the idea of simulating a natural selection process (Holland, 1975). For this purpose, the algorithm starts at an initial pseudo-random population of a potential solution to the problem. Using the forward-modelling equations and, possibly, additional constraints, the genetic algorithm constantly modifies this initial population in order to reach local minima positions. Since the procedure starts at a pseudo-random population of a potential solution, it is expected that after some iterations the genetic algorithm guides the population to the best-fit positions.

After each minimisation step of the iteration, the algorithm needs to generate a new population to initiate the next step. For this purpose, it *selects* a percentage of the present population for the reproduction scheme in order to carry over information that has been successful in the sense of the problem. The principles of *crossover* and *mutation* describe the two classic techniques to produce a new initial population. Crossover is used to exchange genetic information between the members of the final populations of the previous step, and mutation serves to randomly change the genetic information in order to achieve a better exploration of the solution space. The selection of the surviving members of the population requires measuring their fitness, i.e., the quality each potential solution achieves in explaining the data to be optimised. In our implementation, we use the objective function of Porsani et al. (2000) given by

$$h = \frac{2y^T x}{y^T y + x^T x} \; , \tag{6}$$

where, x and y are the observed and modelled data in the time domain and x^T, y^T are their transposes. In order to constrain the inverse problem, we use the dependency of the S-wave velocity and density on the P-wave velocity as described by the Gardner (Gardner et al., 1974) and Castagna (Castagna et al., 1985) relations. In this way, we determine S-wave velocity and density as functions of the P-wave velocity.

RESULTS

We used a synthetic model to study the performance of the global optimisation method for AVO inversion and its relationships with the prior information, i.e., the Gardner et al. (1974) and Castagna et al. (1985) constraints, and also to analyse the performance of the investigated forward modelling algorithm. Table 1 contains the physical parameters of our simple synthetic model. We chose the thicknesses of the two inner layers to be small enough to allow for interaction between the wavelets.

We modelled a synthetic data set with the table-based ray-theory algorithm using the parameters from Table 1, a zero-phase Ricker wavelet with a central frequency of 40 Hz, and a time sample rate of 2 ms. These parameters were also used in the modelling for the inversion. In order to perform the global optimisation, we generated an initial population of 1000 models. For each of these models in the population, we modelled a synthetic CMP gather. All CMP gathers modelled during each iteration of the genetic algorithm are NMO corrected.

The genetic algorithm used a *selection rate* of 50%, i.e., the best 500 models after a minimisation step were used to generate the modified population of again 1000 models for the next iteration. The mutation, update, and crossover probabilities were parameters used in the genetic algorithm are given by $P_{mutation}$ = 0.1, P_{update} = 0.47, and $P_{crossover}$ = 0.90, respectively. The maximum number of allowed iterations was 200.

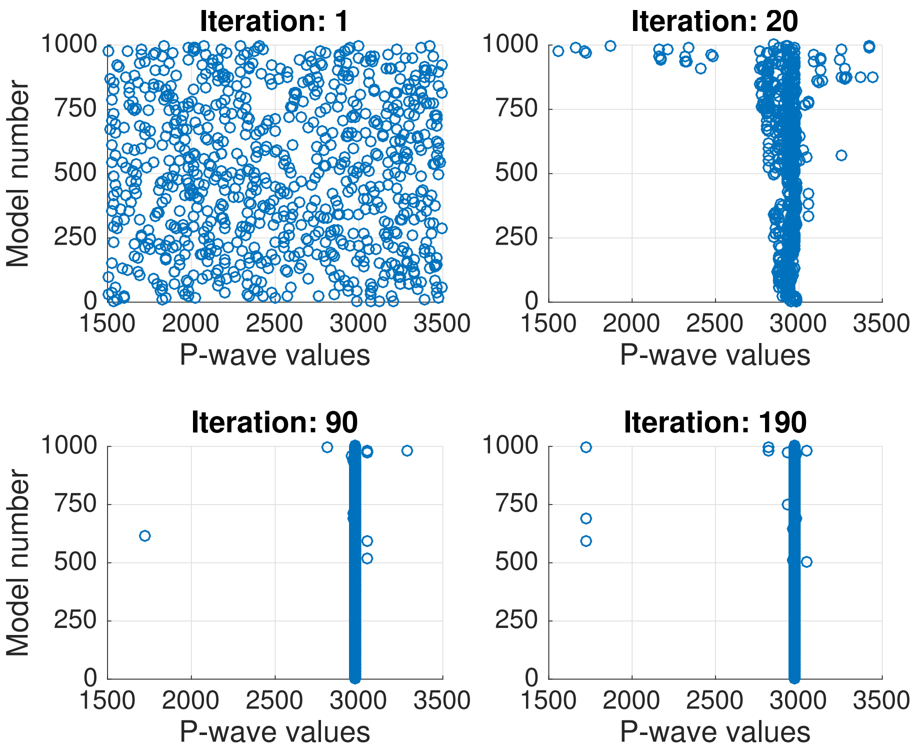

Figure 2: Evolution of the genetic algorithm solutions for the P-wave velocity parameter in the fourth layer when running with the Gardner et al. (1974) and Castagna et al. (1985) constraints.

Table 2: Recovered model parameters of the fourth layer using the genetic algorithm with the Gardner and Castagna constraints.

Parameters	Synthetic Model Parameters	Recovered Model (without constraints)	Relative Error (%)
P-wave (m/s)	2000	2002	-0.10
	2800	2699	3.61
	2300	2394	-4.09
	3000	2658	11.40
S-wave (m/s)	551.72	778	-41.01
	1241.38	1327	-6.90
	810.34	1106	-36.49
	1413	1358	3.95
Density (g/c^3)	2.07	1.535	25.83
	2.25	1.739	22.71
	2.14	1.541	28.01
	2.29	1.934	15.31
Correlation Coef.	1	0.999847412	

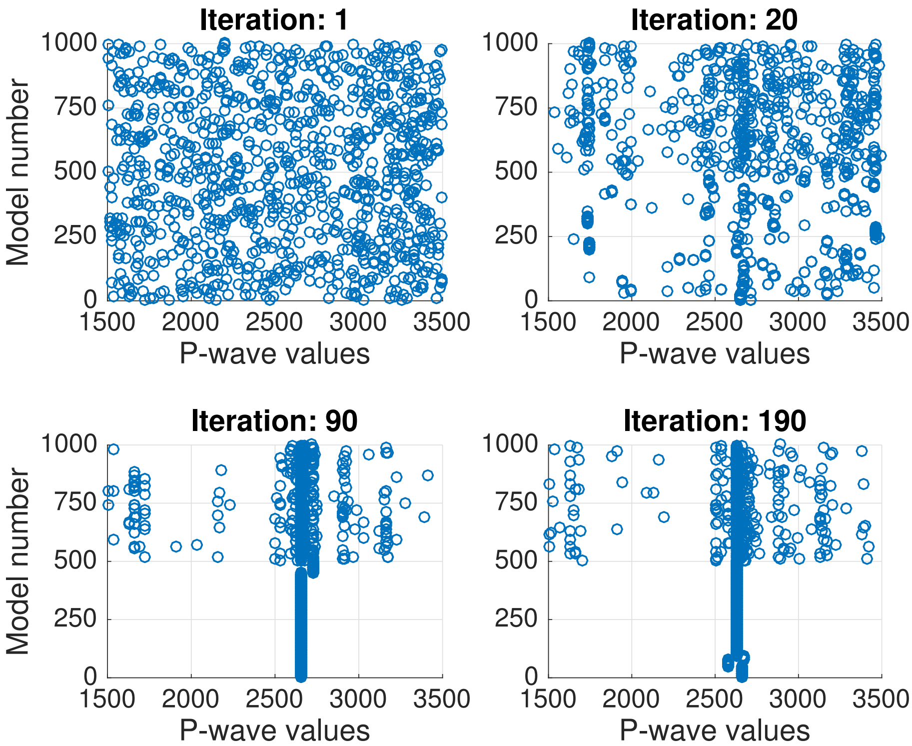

Figure 3: Evolution of the genetic algorithm solutions for the P-wave velocity parameter in the fourth layer when running without the Gardner et al. (1974) and Castagna et al. (1985) constraints.

To study the quality of the inversion, we concentrate on the parameters of the fourth layer of the model in Table 1. Figure 2 shows the evolution of the genetic algorithm solutions for the inverted P-wave velocity parameter in the fourth layer when running with the Gardner et al. (1974) and Castagna et al. (1985) constraints. In Table 2, we have compiled the corresponding recovered parameter values for the model that provided the best result of the objective function.

As we can see in Figure 2, when using the constraints, the algorithm is guiding, already after a few iterations, all the members of the population close to the correct position in the solution space. Table 2 demonstrates that the recovered parameters are very close to those of the true model, and the correlation coefficient between the original data and the data from the recovered model is very high. This confirms that the genetic algorithm was able to find an excellent model.

We then repeated the experiment without making use of the relationships of Gardner et al. (1974) and Castagna et al. (1985) that describe the S-wave velocity and density as a function of the P-wave velocity. Figure 3 shows the corresponding evolution of the population of the genetic algorithm without these constraints, and Table 3 compiles the recovered model parameters.

We can see in Figure 3 that without the additional constraints to restrict the solutions, the algorithm is leading a significant part of the population to particular solutions that are not very close to the correct one. Nonetheless, these solutions are good answers to the objective function of the inverse problem. As we can see from Table 3, the parameters of the best inverted model in this case are rather far away from the true parameters. Nonetheless, comparing the correlation coefficients, we recognise that this model is still almost as good in fitting the data as the best inverted model that uses the constraints. Even with more than 20% of relative error in the average, the best model recovered without the constraints still produces a very similar CMP gather to that from the true model. This highlights the fundamental issue of the AVO inversion problem, which is its non-uniqueness. As a consequence, the underlying inverse problem without constraints is ill-posed, as verified by Rosa (1976).

Let us emphasise the very fast nature of our implementation of this genetic algorithm. In the process of

Table 3: Recovered model parameters of the fourth layer using the genetic algorithm without the Gardner and Castagna constraints.

Parameters	Synthetic Model	Recovered Model (with constraints)	
		Parameters	Relative Error (%)
P-wave (m/s)	2000	2000	0.00
	2800	2788	0.43
	2300	2280	0.87
	3000	2971	0.97
S-wave (m/s)	551.72	552.71	-0.07
	1241.38	1231	0.84
	810.34	793	2.14
	1413	1389	1.75
Density (g/c^3)	2.07	2.069	0.01
	2.25	2.247	0.13
	2.14	2.134	0.24
	2.29	2.284	0.25
Correlation Coef.	1	0.999887645	

globally optimising the objective function with and without the constraints on the parameters, the algorithm generated 301.000 forward synthetics in 25 minutes. This points towards its potential to remain feasible after generalisation to more general situations.

CONCLUSIONS

We have presented a table-based ray-tracing forward-modelling algorithm that is very fast in calculating the traveltime and reflection coefficient vs. offset information for many vertically inhomogeneous models. By providing good performance with restricted computer power, this algorithm has shown excellent potential to be used in a genetic global optimisation scheme for AVO inversion. In the inversion procedure, parameter constraints were helpful to reach the correct global minimum. When applied without constraints, the algorithm converged incorrect results that satisfy the objective function with quite a low residual. This is a consequence of the non-uniqueness of the AVO problem, which makes the inversion ill-posed.

In future research, we hope to formulate the genetic algorithm inside the Bayesian framework to provide solid statistical information about the distribution of the solutions during the iterations. It is expected to be able to visualise distinguishable concentrations of particular solutions which could be useful to determine possible scenarios for the recovered parameters, in this way allowing to account for the non-uniqueness of the problem.

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