3D SHIFTED HYPERBOLA

I. Abakumov, B. Schwarz, and D. Gajewski

email: *abakumov_ivan@mail.ru*
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ABSTRACT

The shifted hyperbola was first introduced almost 40 years ago as an alternative to the conventional NMO hyperbola used in stacking seismic multi-channel data. Since its introduction, it has sparked increasing interest in the late 1980s and early 1990s and still remains a topic of active research. In previous works it has been shown that the shifted hyperbola has unique properties, which make it a feasible alternative to conventional NMO stacking. In addition to the appealing fact, that, due to the independence of the zero-offset reference traveltime, the shifted hyperbola generally leads to stretch-free NMO correction, it was also demonstrated that, for the same reason, its moveout correction can be implemented in a highly parallel fashion. For multi-layered inhomogeneous media, previous authors have found that the shifted hyperbola provides high accuracy, which in certain situations can surpass the conventional stretch-prone NMO approach. Despite all its successes, to our knowledge, the shifted hyperbola has never been convinvincingly extended to 3D acquisitions, which form todays standard tool to infer earth structure with the seismic method. To close this gap, in this work, we introduce a formulation of the shifted hyperbola that is valid in three dimensions. Similar to its 2D counterpart, the new approximation allows an efficient implementation and does not cause the undesired effect of wavelet stretch. A numerical 3D example indicates that the new 3D shifted hyperbola, is more accurate than the conventional 3D NMO, and hence, bears the potential of an improved stacked volume and more reliable stacking parameters.

INTRODUCTION

Accurate traveltime approximations for large offsets are important for many tasks in seismic processing (Aleixo and Schleicher, 2010). Due to its ability to accurately describe the reflection moveout in multilayered inhomogeneous media, the shifted hyperbola approximation suggested by Malovichko (1978), de Bazelaire (1988) and Castle (1994) has gained a lot of attention in the seismic community. Previous studies have shown, that the conceptually different parameterization of the shifted hyperbola provides an alternative view on traveltime moveout, that can lead to improved stacked sections compared to the conventional normal moveout (NMO) approximation (see, e.g., Thore et al., 1994). Moreover, it was demonstrated to remain accurate in the case of transversely isotropic media with a vertical symmetry axis (VTI media, Siliqi and Bousquié, 2000; Ursin and Stovas, 2006). Stovas and Fomel (2012) have further improved the accuracy of the shifted hyperbola by extending the approximation to the phase and group-phase domains.

Unlike the conventional NMO, the shifted-hyperbola moveout is independent of the zero-offset reference traveltime, which has numerous advantages. Firstly, the shifted-hyperbola moveout can be calculated outside of the time loop, which significantly speeds up the computation in grid-based implementations of the parameter estimation (see, e.g., de Bazelaire, 1988). Also, stacking with the shifted hyperbola generally does not suffer from the effect of wavelet stretch arising from the conventional NMO correction. In addition, due to the different mechanisms to account for overburden heterogeneity, the semblance of the shifted hyperbola and conventional NMO are shaped differnetly, which impacts the convergence behavior of currently employed search algorithms (Schwarz and Gajewski, 2017). As a result, systematic differences of the estimated attributes can be observed, which can be exploited in various ways, e.g. for the desgin of improved diffraction filters (Schwarz and Gajewski, 2017). Nowadays, 3D seismic surveys have become a standard exploration and exploitation tool (Vermeer, 2002). While the conventional NMO hyperbola has successfully been extended to 3D seismic acquisitions (Levin, 1971; Grechka and Tsvankin, 1998), to our knowledge there exists no convincing extensions of the shifted hyperbola to three dimensions. To close this gap and to make the aforementioned advantages accessible for the 3D community, we introduce two shifted hyperbola approximations, which are readily applicable to 3D multi-channel seismic data.

2D SHIFTED HYPERBOLA

The most basic traveltime approximation is the conventional normal moveout (Mayne, 1962)

$$t(x) = \sqrt{t_0^2 + \frac{x^2}{v_{\mathrm{nmo}}^2}}, \tag{1}$$

where t_0 corresponds to the zero-offset traveltime, x is the source-receiver offset and v_{nmo} denotes the normal moveout (NMO) velocity. Being a simple hyperbola with a single stacking parameter v_{nmo}, the NMO approximation is only accurate for small offsets. In order to find a more accurate traveltime approximation, several authors (Malovichko, 1978; de Bazelaire, 1988; Castle, 1994) have suggested the so-called shifted hyperbola. It is a non-hyperbolic two-parametric traveltime approximation

$$t(x) = t_0 - t_p + \sqrt{t_p^2 + \frac{x^2}{v_a^2}}, \tag{2}$$

where the parameter t_p is the focusing time and the parameter v_a is the average velocity. The heterogeneity coefficient S establishes the link between (t_p, v_a) and (t_0, v_{nmo}) (see, e.g., Castle, 1994):

$$t_p = \frac{t_0}{S}, \qquad v_a^2 = S v_{\mathrm{nmo}}^2. \tag{3}$$

In the isotropic layered model, S is related to the ratio of velocity momentums (Malovichko, 1978). In the VTI medium, S may be expressed as a function of the anisotropy parameter η (Siliqi and Bousquié, 2000; Ursin and Stovas, 2006; Aleixo and Schleicher, 2010). To achieve a better fit Castle (1994) proposed that S should be defined as a function of offset. In this case, the heterogeneity coefficient S controls the deviation of equation (2) from the hyperbola. de Bazelaire (1988) demonstrated that for small offsets, the average velocity v_a may be replaced by the near-surface velocity v_0, resulting in a single-parameter approximation:

$$t(x) = t_0 - t_p + \sqrt{t_p^2 + \frac{x^2}{v_0^2}}. \tag{4}$$

So as follows from expressions (2) and (4), there exist two flavors of the shifted hyperbola, one solely utilizing a shift in time, the other in addition allowing for a variable velocity parameter.

3D SHIFTED HYPERBOLA

In 3D seismic surveys, the sources and the receivers are distributed in a measurement surface, rather than a line. In this case, the offset $\mathbf{x}$ becomes a two-dimensional vector $\mathbf{x} \equiv \{|\mathbf{x}|\cos\xi; |\mathbf{x}|\sin\xi\}$. The 3D NMO equation reads:

$$t(\mathbf{x}) = \sqrt{t_0^2 + \frac{|\mathbf{x}|^2}{v_{\mathrm{nmo}}^2(\xi)}}, \tag{5}$$

where the NMO velocity depends on the azimuth of the CMP line ξ (Levin, 1971). The azimuthal dependence of v_{nmo} typically has a simple elliptical form (Grechka and Tsvankin, 1998). Equation (3) establishes the link between the focusing time t_p, the heterogeneity coefficient S and the NMO velocity.

Since the NMO velocity depends on the angle ξ, both the focusing time and the heterogeneity coefficient are functions of the angle ξ,

$$t_p(\xi) = t_0 \frac{v^2_{\rm nmo}(\xi)}{v_0^2}, \qquad S^{-1}(\xi) = \frac{v^2_{\rm nmo}(\xi)}{v_0^2}, \tag{6}$$

and inherit the elliptical behavior. This result has the following interpretation. The conventional NMO can be derived in the effective auxiliary medium, whereas the one-parametric shifted hyperbola has a physically sound interpretation in the optical auxiliary medium. Since both the optical and effective approaches are not sufficient for the 3D case (Abakumov et al., 2017), individual effective or optical media are considered for each angle ξ. Thereby, the 3D counterpart of the one-parametric shifted hyperbola (4) reads

$$t(\mathbf{x}) = t_0 - t_p(\xi) + \sqrt{t_p^2(\xi) + \frac{|\mathbf{x}|^2}{v_0^2}}. \tag{7}$$

Since the above-mentioned arguments remain valid for the two-parametric approximation, the general 3D shifted-hyperbola approximation, i.e. the 3D extension of expression (2), can be written as

$$t(\mathbf{x}) = t_0 - t_p(\xi) + \sqrt{t_p^2(\xi) + \frac{|\mathbf{x}|^2}{v_a^2(\xi)}}. \tag{8}$$

Note, that the approximations (5) and (7) have three stacking parameters, and the approximation (8) has five stacking parameters. In the following, we investigate the accuracy of the two 3D extensions of the shifted hyperbola in comparison with the established conventional 3D NMO approach. In order to prevent confusion, we refer to the 3D version of the one-parameter equation (7) with abbreviation "SH-1" and to expression (8) with "SH-2".

MULTIAZIMUTH REFLECTION EXPERIMENT

In order to compare the 3D NMO and the 3D shifted hyperbola moveout approximations, we consider the so-called Complex model (see Figure 1a). The Complex model consists of an analytical reflector below an inhomogeneous overburden as it can typically be found in the Gulf of Mexico. Such a model is complicated enough to possess all effects of realistic 3D media and at the same time allows the numerical computation of reflection traveltimes. The Complex model is characterized by the depth of the reflection point of the zero-offset ray, which in this case is approximately equal to 1.0 km. The acquisition consists of 36 CMP lines oriented in different azimuthal directions, defined by the angle ξ. For each line, the "theoretical" (computed numerically to high precision) values of $v_{\rm NMO}(\xi)$, $t_p(\xi)$ and $S(\xi)$ were evaluated (blue crosses in Figures 1b-d). These values perfectly fit the sinusoidal approximation (red line in Figures 1b-d), i.e., clearly show elliptical behavior.

We use the "theoretical" values $v_{\rm NMO}(\xi)$, $t_p(\xi)$ and $S(\xi)$ to compute the approximated 3D traveltimes. Figures 2a-c illustrate the relative errors of 3D NMO, SH-1, and SH-2. As is apparent from these figures, for small offsets, approximation SH-1 provides results overall superior to those gained with the conventional NMO expression, however, its accuracy decreases with offset. For the chosen range of offsets, equation SH-2 demonstrates almost perfect accuracy. Finally, we compare the accuracy of the estimated stacking parameters. For the finite spread (the spread length was equal to the depth of the reflection point), we found the best-fit stacking parameters $\tilde{v}_{\rm nmo}(\xi)$ for NMO, $\tilde{t}_p(\xi)$ for SH-1, and $[\tilde{t}_p(\xi), \tilde{v}_a(\xi)]$ for SH-2. In the next step, in order to facilitate a quantitative comparison, we use equations (3) and (6) to transform the attributes to NMO velocities. Through this we gain $\tilde{v}_{\rm nmo}(\tilde{t}_p)$, and $\tilde{v}_{\rm nmo}(\tilde{t}_p, \tilde{v}_a)$. As can be concluded from Figure 3, the new 3D shifted hyperbola SH-2 not only provides the best fit, but also the most accurate stacking parameters.

CONCLUSIONS

The estimation of parameters of the NMO velocity ellipse is a common procedure in 3D CMP processing. We believe that this processing step can strongly benefit from the improvement of fit of the proposed 3D

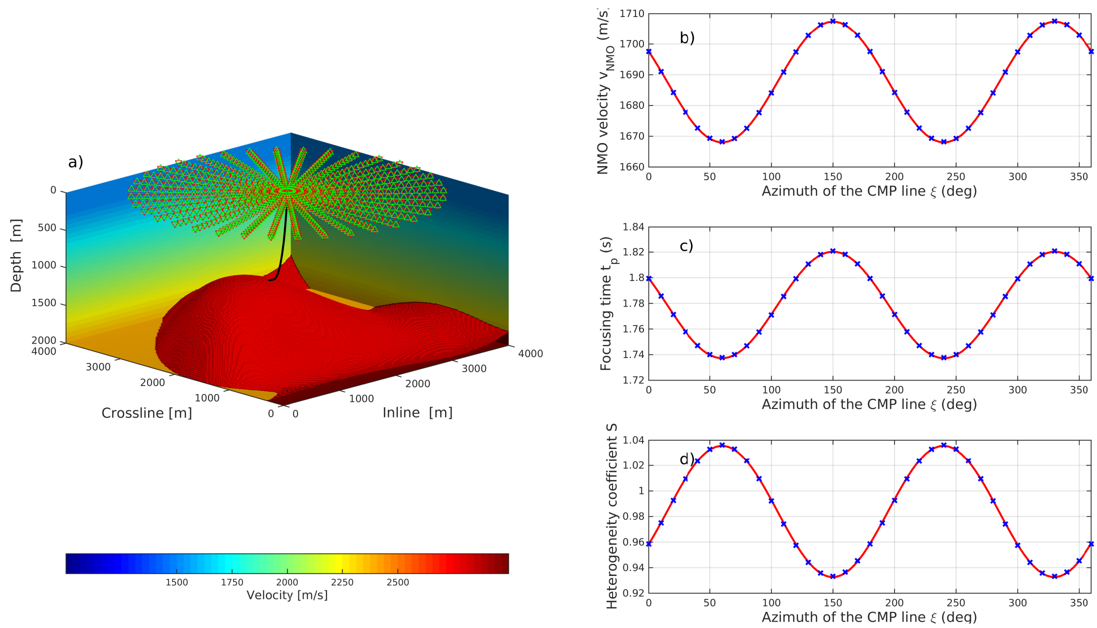

Figure 1: Illustration of the Complex model (a). The model consists of the constant velocity part ($v_0 = 1500\ m/s$, $z \leq 250\ m$) simulating the water layer, and the constant-gradient velocity part ($v = v_0 + \gamma(z - z_0)$, $z_0 = 250\ m$, $\gamma = 0.5\ s^{-1}$, $z > 250\ m$) simulating the sedimentary layer. The reflector (red surface) simulates the top of the salt body. The reflector is described by the fourth order polynomial function of lateral coordinates. The black line indicates the trajectory of the central ray. The depth of the reflection point is approximately equal to $1.0\ km$. Such a model allows the estimation of the "theoretical" NMO velocity (b), focusing time (c) and heterogeneity coefficient (d) (blue crosses). The red line indicates the result of sinusoidal interpolation.

shifted hyperbola moveout approximation. The 3D shifted hyperbola is independent of the zero-offset reference time, and hence inherits all attractive features of the 2D shifted hyperbola. Due to its improved accuracy, compared to the conventional 3D NMO approach, the new approximation bears the potential to provide an improved stacked volume and more reliable stacking parameters, which directly influence subsequent processing steps such as velocity determination and migration in time and depth.

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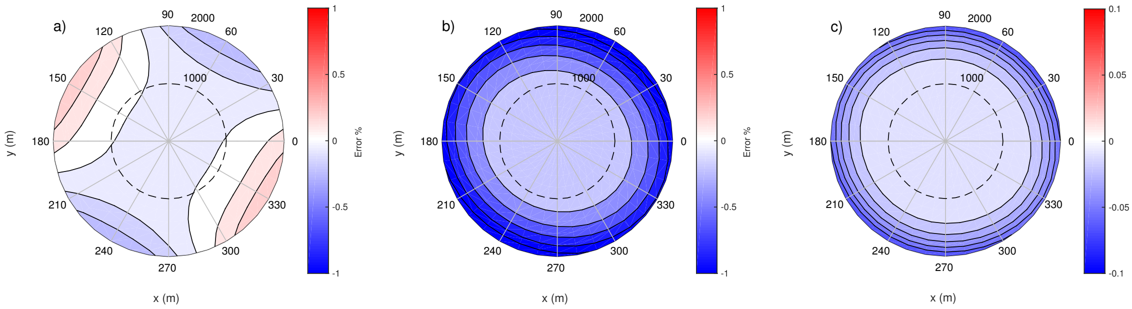

Figure 2: Relative errors of traveltime approximations plotted as a function of $|\mathbf{x}|$ and azimuth angle ξ: $0 \leq |\mathbf{x}| \leq 2000\ m$, $0^0 \leq \xi \leq 360^0$. (a) – NMO, (b) – SH-1, (c) – SH-2. Note the different color scale in (c).

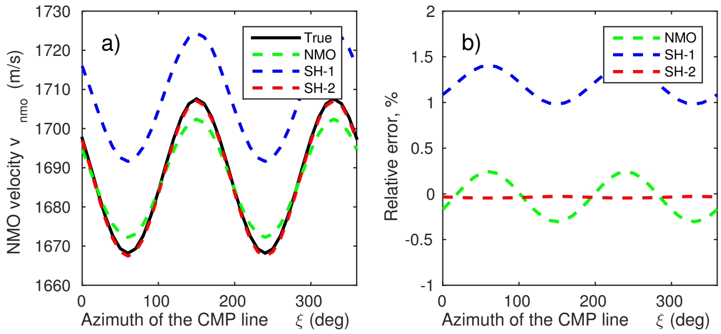

Figure 3: (a) Comparison of the "theoretical" v_{nmo} (black) and the NMO velocities estimated from the best-fit stacking parameters of NMO (green), SH-1 (blue), and SH-2 (red). (b) Relative errors of the estimated NMO velocities.

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