AUXILIARY ANISOTROPIC MEDIUM

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ABSTRACT

Traveltime moveout is a key ingredient of traditional and modern stacking schemes and, in addition to the constructive summation of redundant data components, has versatile application such as trace interpolation, regularization, or velocity model building. It has been shown that higher-order traveltime operators provide a better fit for curved target structures and complex overburdens. In addition, multidimensional stacking schemes have been shown to optimally utilize multi-channel data redundancy. Derivations of higher-order multidimensional moveout approximations require a simplified model to fit seismic data from a heterogenous overburden, i.e., an auxiliary medium and an analytical description of the reflector. The existing mechanisms to account for the overburden heterogeneity, either by a shift in velocity (effective medium), or by a shift of the reference time (optical medium), could not yet be extended to the 3D case. To fill that gap, we suggest an auxiliary anisotropic medium, which in the 3D case allows to simulate wavefronts of complex shape. We show that this anisotropic auxiliary medium naturally incorporates properties of effective and optical auxiliary media. The auxiliary anisotropic medium and a locally analytical description of the reflector shape constitute the 3D simplified model, which enables the derivation of 3D extensions of the existing 2D multidimensional moveout approximations.

INTRODUCTION

The concept of an auxiliary medium is widely used for the derivation of moveout approximations, which still form an important ingredient of most processing workflows. It suggests that in the presence of heterogeneity the actual subsurface model is replaced by an auxiliary medium of constant velocity. The advantage of this replacement lies in the fact that in the auxiliary medium the rays are straight and the wavefront elements are circular, which significantly simplifies the ray propagation and provides an appealing geometrical interpretation of the reflection process. The auxiliary medium also allows the use of analytical operators in processing and imaging, which is a practical value.

Two types of auxiliary media are known in the 2D case. The heterogeneous medium is either replaced by an effective medium – a homogeneous medium of "average" velocity, or by an optical medium – a homogeneous medium of near-surface velocity. The effective and optical media are used in the derivation of reflection moveout approximations in common-midpoint (CMP) gathers: the classical normal moveout (NMO) hyperbola (e.g., Mayne, 1962) and the shifted hyperbola introduced by de Bazelaire (1988). While the classical NMO is based on the velocity-shift mechanism (effective medium) to account for heterogeneity of the subsurface, the shifted hyperbola utilizes a time shift, typical for the optical representations. Schwarz and Gajewski (2017) investigated the mechanisms by which multidimensional stacking operators account for heterogeneity of the overburden and showed that the described duality can be extended to these approaches. They found that the common-reflection-surface (CRS, Jäger et al., 2001), non-hyperbolic CRS (Fomel and Kazinnik, 2013) and implicit CRS (Schwarz et al., 2014) utilize the velocity-shift mechanism, while multifocusing (Gelchinsky et al., 1999) is based on the time-shift method. In addition, Schwarz and Gajewski (2017) proposed a recipe to transform time shifts into velocity shifts and vice versa.

However, this well-established theory of auxiliary media breaks down when the 3D case is considered. In the 3D case, wavefronts have two principal curvatures and, hence, can not be accurately approximated by purely spherical wavefronts. In order to overcome this problem, we propose a new auxiliary medium which accounts for heterogeneity of the subsurface by a special type of anisotropy that allows for a convenient derivation of 3D multidimensional moveout approximations.

AUXILIARY MEDIA

The geometrical approach is commonly used for the derivation of multidimensional stacking operators. The derivation is usually based on a simplified model of the subsurface: an analytical reflector in an auxiliary constant-velocity medium. The simplified model is related to "reality" (a curved reflector below a heterogeneous overburden) through the two hypothetical (NIP and normal) experiments (Hubral, 1983). The NIP experiment enables to define the properties of auxiliary media. In the NIP experiment, a fictitious source S is placed at the reflection point of the central ray. The source S generates the wavefront that arrives at the central point x_0 at the time $t_0/2$, with the emergence angle α and with the radius of curvature $R_{\rm NIP}$ (see Figure 1). The near-surface velocity at the point x_0 is assumed to be v_0. The goal is to find an image source S^* determined by the parameters $\tilde{\alpha}$, $\tilde{R}_{\rm NIP}$ in the auxiliary medium of constant velocity $\tilde{v}$ that generates an identical wavefront (or an identical moveout $\Delta\tilde{t}_{\rm NIP} = \tilde{t}_{\rm NIP}(x) - \tilde{t}_0$). To achieve this goal, we have to equate the first and second-order spatial derivatives of the traveltime of the NIP wave in the auxiliary medium and in the heterogeneous medium

$$\frac{\partial\tilde{t}_{\rm NIP}}{\partial x} = \frac{\partial t_{\rm NIP}}{\partial x}; \quad \frac{\partial^2\tilde{t}_{\rm NIP}}{\partial x^2} = \frac{\partial^2 t_{\rm NIP}}{\partial x^2} \quad \Leftrightarrow \quad \frac{\sin\tilde{\alpha}}{\tilde{v}} = \frac{\sin\alpha}{v_0}; \quad \frac{1}{\tilde{v}}\frac{\cos^2\tilde{\alpha}}{\tilde{R}_{\rm NIP}} = \frac{1}{v_0}\frac{\cos^2\alpha}{R_{\rm NIP}}, \tag{1}$$

and consider the following obvious relationship

$$\tilde{t}_0 = \frac{2\tilde{R}_{\rm NIP}}{\tilde{v}}. \tag{2}$$

The system of equations (1,2) can be solved with respect to $\tilde{t}_0$, $\tilde{v}$, $\tilde{\alpha}$ and $\tilde{R}_{\rm NIP}$ under one of the following conditions:

- $\tilde{\alpha} = \alpha$. Solution of the system leads to the optical medium (see Figure 1a and Table 1). In this case, the wavefronts are locally identical, the moveouts coincide up to the time shift $\delta t = \tilde{t}_0 - t_0$.

- $\tilde{t}_0 = t_0$. Solution of the system leads to the effective medium (see Figure 1c and Table 1). In this case, the wavefronts do not coincide, but the moveouts are locally identical.

However, we may also want to fulfill both conditions

- $\tilde{\alpha} = \alpha, \quad \tilde{t}_0 = t_0$. In this case, the solution of the system is possible only under the assumption of an anisotropic auxiliary medium. In the anisotropic auxiliary medium the velocity varies with the direction (see Figure 1b and Table 1). The advantage of this approach is that both the wavefronts and the moveouts are locally identical.

In the normal wave experiment, a fictitious exploding reflector element around the NIP is considered (see Figure 2). Similarly to the NIP experiment, equalization of the spatial derivatives of the traveltime of the normal wave enables to link the radius of curvature of the normal wave in the auxiliary medium $\tilde{R}_{\rm N}$ and in the heterogeneous medium $R_{\rm N}$ (see Table 1). In the auxiliary media, the normal wavefront is generated by the circular reflector with the origin at the center of curvature of the normal wavefront $\tilde{R}_{\rm N}$ and the radius $\tilde{R}$:

$$\tilde{R} = \tilde{R}_{\rm N} - \tilde{R}_{\rm NIP}. \tag{3}$$

The thus obtained simplified models are used for the derivation of higher-order 2D multidimensional moveout approximations, e.g., multifocusing (optical medium) and implicit CRS (effective medium).

Table 1: Properties of auxiliary media

Auxiliary medium	**Optical**	**Anisotropic**	**Effective**
Velocity	$\tilde{v}=v_0$	$\frac{1}{\tilde{v}^2(\vartheta)}=\frac{\sin^2\vartheta}{v_{x'}^2}+\frac{\cos^2\vartheta}{v_{z'}^2}$ $v_{x'}^2=v_0^2 R_{\mathrm{NIP}}/\tilde{R}_{\mathrm{NIP}}$ $v_{z'}^2=v_0^2$	$\tilde{v}=\frac{v_{\mathrm{NMO}}}{\sqrt{1+\frac{v_{\mathrm{NMO}}^2}{v_0^2}\sin^2\alpha}}$ $v_{\mathrm{NMO}}=\sqrt{\frac{2v_0 R_{\mathrm{NIP}}}{t_0\cos^2\alpha}}$
Parameters of NIP experiment	$\tilde{t}_0=\frac{2R_{\mathrm{NIP}}}{v_0}$ $\tilde{\alpha}=\alpha$ $\tilde{R}_{\mathrm{NIP}}=R_{\mathrm{NIP}}$	$\tilde{t}_0=t_0$ $\tilde{\alpha}=\alpha$ $\tilde{R}_{\mathrm{NIP}}=\frac{t_0 v_0}{2}$	$\tilde{t}_0=t_0$ $\sin\tilde{\alpha}=\frac{\tilde{v}}{v_0}\sin\alpha$ $\tilde{R}_{\mathrm{NIP}}=\frac{t_0\tilde{v}}{2}$
CMP moveout	$\left(t+\tilde{t}_0-t_0\right)^2=\tilde{t}_0^2+\frac{4h^2}{v_0^2}$	$t^2=t_0^2+\frac{4h^2}{v_{\mathrm{NMO}}^2}$	$t^2=t_0^2+\frac{4h^2}{v_{\mathrm{NMO}}^2}$
Parameters of Normal experiment	$\tilde{R}_{\mathrm{N}}=R_{\mathrm{N}}$ $\tilde{R}=\tilde{R}_{\mathrm{N}}-\tilde{R}_{\mathrm{NIP}}$	$\tilde{R}_{\mathrm{N}}=R_{\mathrm{N}}$ $\tilde{R}=\tilde{R}_{\mathrm{N}}-\tilde{R}_{\mathrm{NIP}}$	$\tilde{R}_{\mathrm{N}}=\frac{\tilde{R}_{\mathrm{NIP}}}{R_{\mathrm{NIP}}}R_{\mathrm{N}}$ $\tilde{R}=\tilde{R}_{\mathrm{N}}-\tilde{R}_{\mathrm{NIP}}$

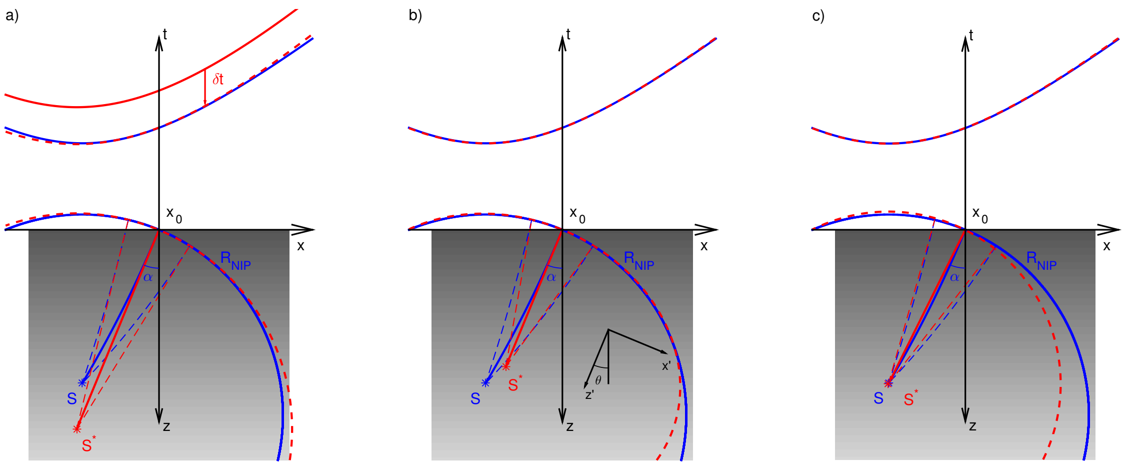

Figure 1: NIP experiment in the inhomogeneuos medium (blue rays and wavefronts) and its interpratation by optical (a), anisotropic (b) and effective (c) auxiliary media. Note that in the effective medium (c) the actual and image sources do not generally coincide.

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In the 2D case, the circular wavefront approximation is valid, because an arbitrary wavefront is locally defined by one curvature. However, in the 3D case, an arbitrary wavefront has two principal curvatures (see Figure 3a) and can not be accurately approximated by a spherical wavefront. Hence, isotropic (optical or effective) auxiliary media can not "focus" the arbitrary wavefront (see Figure 3b). In contrast, the anisotropic auxiliary medium, which in the 3D case is defined as (Abakumov, 2016)

$$\frac{1}{\tilde{v}^2(\vartheta,\varphi)}=\frac{\sin^2\vartheta\cos^2\varphi}{v_{x'}^2}+\frac{\sin^2\vartheta\sin^2\varphi}{v_{y'}^2}+\frac{\cos^2\vartheta}{v_{z'}^2}; \tag{4}$$

$$\tilde{R}_{\mathrm{NIP}}=\frac{t_0 v_0}{2}, \qquad v_{x'}^2=v_0^2/\left(\tilde{R}_{\mathrm{NIP}}k'^{11}_{\mathrm{NIP}}\right), \qquad v_{y'}^2=v_0^2/\left(\tilde{R}_{\mathrm{NIP}}k'^{22}_{\mathrm{NIP}}\right), \qquad v_{z'}^2=v_0^2, \tag{5}$$

focuses the wavefront with the arbitrary curvature matrix $\mathbf{K'}_{\mathrm{NIP}}$ at the depth $\tilde{R}_{\mathrm{NIP}}$ (see Figure 3c). In the 3D case, the condition (3) transforms to

$$\mathbf{K'}_{\mathrm{R}}^{-1}=\mathbf{K'}_{\mathrm{N}}^{-1}-\mathbf{K'}_{\mathrm{NIP}}^{-1}. \tag{6}$$

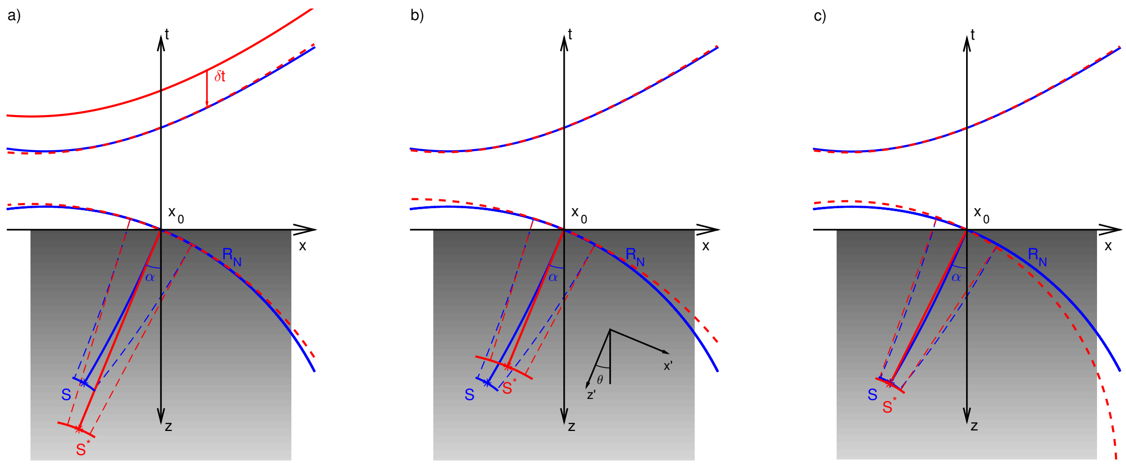

Figure 2: Normal experiment in the inhomogeneuos medium (blue rays and wavefronts) and its interprata-tion by optical (a), anisotropic (b) and effective (c) auxiliary media.

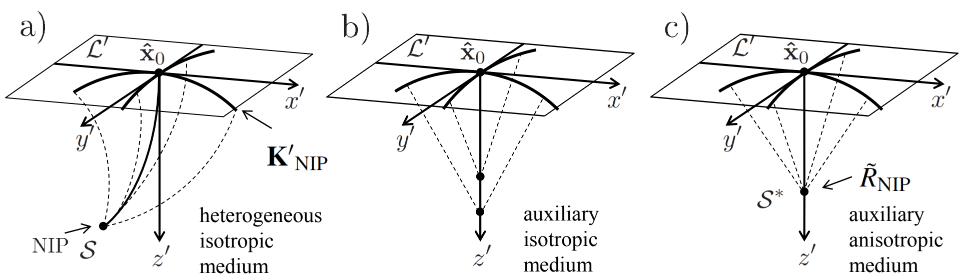

Figure 3: NIP experiment in the 3D case. The fictitious source S in a heterogeneous medium generates the wavefront with the curvature matrix $\mathbf{K}'_{\mathrm{NIP}}$ (a). In the case of an auxiliary isotropic medium (b), the NIP wavefront can not be "focused" at one point. Consideration of the auxiliary anisotropic medium (c) overcomes this problem. Notice that the wavefront is drawn in the system $\mathcal{L}'$ - the special ray-centered coordinate system whose x' and y'-axes coincide with the principal directions of curvature of the NIP wavefront.

DERIVING 3D MOVEOUT APPROXIMATIONS

In the 3D case, the traces of the CMP bin are used to be stacked along the hyperbolic trajectories

$$t^2 = t_0^2 + \frac{4|\mathbf{h}|^2}{v_{\mathrm{NMO}}^2(\xi)}, \tag{7}$$

with the NMO velocity depending on the direction of the profile ξ (e.g., Levin, 1971). Similarly, the 3D shifted hyperbola moveout approximation (Abakumov et al., 2017)

$$\left(t + \tilde{t}_0(\xi) - t_0\right)^2 = \tilde{t}_0^2(\xi) + \frac{4|\mathbf{h}|^2}{v_0^2} \tag{8}$$

includes the focusing time $\tilde{t}_0(\xi)$ which varies with the profile direction. The directional dependence of the stacking parameters indicate that the individual effective or optical medium is considered for each angle ξ. In contrast to that, the CMP moveout in the anisotropic auxiliary medium

$$t^2 = t_0^2 + 4\mathbf{h}^{\mathbf{T}}\mathbf{W}\mathbf{h}, \qquad \mathbf{W} = \frac{t_0}{2v_0}\mathbf{R}\mathbf{K}_{\mathrm{NIP}}\mathbf{R}^{\mathbf{T}}, \tag{9}$$

is uniquely defined by the parameters of the auxiliary anisotropic medium. Note that equation (9) is formally identical to the 3D elliptical NMO equation (see Grechka and Tsvankin, 1998) and also coincides with the CRS operator in the CMP gather. Equations (4)-(6) form the 3D simplified model, which allows the model-based derivation of higher-order 3D multidimensional moveout approximations, such as 3D nonhyperbolic CRS (Fomel and Kazinnik, 2013), and 3D implicit CRS (Abakumov, 2016).

CONCLUSIONS

We have presented a new type of auxiliary medium which utilizes anisotropy to account for 3D heterogeneity of the subsurface. Unlike conventional isotropic auxiliary media, the anisotropic auxiliary medium can be naturally extended to the 3D case. The findings of this study systematize the existing stacking operators and provide the model for the geometrical derivation of multidimensional stacking operators. The auxiliary anisotropic medium can also be applied to other important problems, e.g., the interpretation and derivation of 3D time migration operators.

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