GENERALIZED SCREEN PROPAGATOR REVISITED

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ABSTRACT

We revisit modeling and migration in the frequency-wavenumber (e.g. plane-wave) domain, able to account for lateral velocity variations. A generalized higher-order screen propagator to be used in modeling and migration in the frequency-wavenumber domain has been developed. The new formulation has been compared with other known screen propagators and its superior behavior has been demonstrated.

INTRODUCTION

For laterally homogeneous media, modeling and migration in the frequency-wavenumber (e.g., plane-wave) domain are carried out basically by the multiplication, at each depth step, of a simple propagator factor. That simple scheme, not valid for lateral velocity variations, needs to be adapted able for that media. Such adaptations are referred in the literature as phase-screen or thin-slab propagator methods. Gazdag and Sguazzero (1984) introduced the phase-shift plus interpolation technique (PSPI) which can handle larger lateral variations in velocity. However, this method relies on the use of multiple reference media. Another approach is to employ the Generalized Screen (GS) operator (Le Rousseau and de Hoop, 2001), which is restricted to weaker lateral velocity variations. Stoffa et al. (1990) derived the split-step Fourier (SSF) operator, which can handle lateral variations but is limited in accuracy to near-vertically propagating waves. The SSF operator is of Born type but is stable also for large velocity contrasts (however degrading in accuracy). To include waves propagating at non-vertical angles, Huang et al. (1999) introduced the Extended Local Born Fourier (ELBF) propagator. Chen and Ma (2006) proposed a second (and higher-order) version of that operator. However, despite being able to handle larger angles more accurately, such operators still suffer from the underlying Born assumption, which can be critical in cases of larger contrasts. In this paper we therefore propose a new generalized screen propagator which can handle both larger propagation angles as well as velocity contrasts beyond those limited to the Born approximation.

Screen propagators

We consider the 1-way wave-equation approach to modelling and migration. A homogenous earth model is used to illustrate the basic approximations behind various screen propagators. We limit our discussion to 2-D wave propagation, but a generalization to 3-D is straightforward. The exact propagator kernel can be written on the form (e.g. between extrapolation depths z and $z + \Delta z$)

$$A(k_x, \omega) = \exp[ik_z \Delta z], \quad k_z = \sqrt{(w/c)^2 - k_x^2} = \sqrt{k^2 - k_x^2}, \quad k^2 - k_x^2 \geq 0. \tag{1}$$

Next, we introduce a background or reference velocity c_0, and rewrite Equation (1) as follows:

$$A(k_x, \omega) = \exp\left[(ik_{z0}\Delta z)\sqrt{1 + \frac{\gamma}{1 - k_x^2/k_0^2}}\right], \quad k_{z0} = \sqrt{(w/c_0)^2 - k_x^2} = \sqrt{k_0^2 - k_x^2}, \quad \gamma = (c_0/c)^2 - 1\ . \tag{2}$$

In case $c > c_0$, we assume that evanescent waves are removed in Equation (2), e.g. $k_0^2 - k_x^2 \geq 0$. The following series expansion can then be employed in the phase:

$$ik_{z0}\Delta z\sqrt{1+\frac{\gamma}{1-k_x^2/k_0^2}} = ik_{z0}\Delta z + ik_{z0}\Delta z\sum_{n=1}^{\infty}\left(\begin{array}{c}1/2\\ n\end{array}\right)\left\{\frac{\gamma^n}{[1-k_x^2/k_0^2]^{n-1/2}}\right\}\ , \tag{3}$$

where we have made use of the generalized binomial coefficients defined by

$$\left(\begin{array}{c}\alpha\\ n\end{array}\right) = \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}\ , \quad \alpha\in C,\ \ n\geq 0\ . \tag{4}$$

Combination of Equations (2) and (3) gives the kernel approximation

$$A(k_x,\omega) \approx \exp[ik_{z0}\Delta z]\left[1+(ik_0\Delta z)\left\{\sum_{n=1}^{\infty}\left(\begin{array}{c}1/2\\ n\end{array}\right)\frac{\gamma^n}{[1-k_x^2/k_0^2]^{n-1/2}}\right\}\right]\ . \tag{5}$$

The phase screen propagator given by Equation (5) corresponds to the generalized high-order propagator proposed by Chen and Ma (2006). Its first-order version (e.g. setting n=1 in Equation (5)), gives the Extended Local Born Fourier (ELBF) propagator of Huang et al. (1999). The main problem with operators derived from Equation (5) is the underlying assumption of a Born model, since the correction terms are derived with respect to a propagator kernel of the background medium. In order to construct a more robust propagator approximation, we propose an alternative approach. First, we rewrite Equation (3) further as follows:

$$ik_{z0}\Delta z\sqrt{1+\frac{\gamma}{1-k_x^2/k_0^2}} = ik_0\Delta z + ik_0\Delta z\sum_{n=1}^{\infty}\left\{\left(\begin{array}{c}1/2\\ n\end{array}\right)\left\{1+\left[-1+(1-k_x^2/k_0^2)^{-n+1/2}\right]\right\}\gamma^n\right\}\ . \tag{6}$$

We now combine Equations (2) and (6) to obtain

$$\begin{aligned} A(k_x,\omega) &= \exp\left[ik_0\Delta z\sum_{n=1}^{\infty}\left(\begin{array}{c}1/2\\ n\end{array}\right)\left\{-1+(1-k_x^2/k_0^2)^{-n+1/2}\right\}\gamma^n\right] \\ &\times\ \exp\left[i\left\{k_{z0}+k_0\sum_{m=1}^{\infty}\left(\begin{array}{c}1/2\\ m\end{array}\right)\gamma^m\right\}\Delta z\right] \\ &\approx \left[1+ik_0\Delta z\sum_{n=1}^{N}\left(\begin{array}{c}1/2\\ n\end{array}\right)\left\{-1+(1-k_x^2/k_0^2)^{-n+1/2}\right\}\gamma^n\right] \\ &\times\ \exp\left[i\left\{k_{z0}+k_0\sum_{m=1}^{M}\left(\begin{array}{c}1/2\\ m\end{array}\right)\gamma^m\right\}\Delta z\right]\ , \end{aligned} \tag{7}$$

where we have replaced the infinite summations with finite ones, of orders (number of terms) N and M, respectively. It is to be observed that the correction terms in Equation (7) have been derived with respect to a propagator kernel which already includes the 'thin-lens' terms. As a consequence, the resulting kernel is expected to be more robust to large velocity jumps. If we consider the lowest or first order version of the new generalized screen propagator, namely, setting $M = N = 1$ in Equation (7), we have

$$A(k_x,\omega) = \left[1+\frac{1}{2}ik_0\Delta z\left\{-1+\left(1-\frac{k_x^2}{k_0^2}\right)^{-1/2}\right\}\gamma\right]\exp\left[i\left\{k_{z_0}+\frac{k_0}{2}\gamma\right\}\Delta z\right]\ . \tag{8}$$

In the limit of *almost vertically travelling* waves, i.e., $k_x \to 0$, the above expression reduces to

$$A(k_x,\omega) = \exp\left[i\left\{k_{z_0}+\frac{k_0}{2}\gamma\right\}\Delta z\right]\ , \tag{9}$$

which is known in the literature as the Split-Step Fourier (SSF) operator (Stoffa et al., 1990). We can now establish the new generalized 1-way screen propagator advocated for in this paper (monochromatic waves and order N). Introducing the notations

$$k_0 = \frac{\omega}{c_0(z_j)}, \quad k_{z0} = \sqrt{\left(\frac{\omega}{c_0(z_j)}\right)^2 - k_x^2}, \quad \text{and} \quad \gamma(x, z_j) = \left[\frac{c_0(z_j)}{c(x, z_j)}\right]^2 - 1\,, \tag{10}$$

where $c(x, z_j)$ and $c_0(z_j)$ are, respectively, the 'true' laterally varying medium velocity at level z_j, and the reference velocity. We consider propagation of pressure $p(x, z_j)$, at depth z_j to the pressure $p(x, z_j + \Delta z)$ at depth $z_j + \Delta z$, both with the same horizontal coordinate, x. That is given by

$$p(x, z_j + \Delta z, \omega) = \mathcal{F}_{k_x}^{-1}\left\{A(k_x, \omega)\mathcal{F}_x\left[p(x, z_j, \omega)\right]\right\}\,. \tag{11}$$

With the help of Equation (7), we have

$$\begin{aligned} p(x, z_j + \Delta z, \omega) &= \mathcal{F}_{k_x}^{-1}\Bigg\{ \exp(ik_{z0}\Delta z)\Bigg[\mathcal{F}_x\left[\tilde{p}(x, z_j, \omega)\right] \\ &+ \quad (ik_0\Delta z)\sum_{n=1}^{N}\begin{pmatrix} 1/2 \\ n \end{pmatrix}[-1 + (1 - (k_x^2/k_0^2)^{-n+1/2}\mathcal{F}_x\left[\gamma^n(x, z_j)\tilde{p}(x, z_j, \omega)\right]\Bigg]\Bigg\}\,, \end{aligned} \tag{12}$$

where

$$\tilde{p}(x, z_j, \omega) = \exp\left[ik_0\Delta z\sum_{m=1}^{M}\begin{pmatrix} 1/2 \\ m \end{pmatrix}\gamma^m(x, z_j)\right]p(x, z_j, \omega)\,. \tag{13}$$

Note that, in Equations (12) and (13) we are allowed to set the order M larger than N. The idea behind this hybrid algorithm is that the 'thin-lens' term in Equation (13) should be made as accurate as possible since the computational time will be virtually unaffected. However, the choice of order in Equation (12) (e.g., value of N) determines the effective computational time to a large extent. To make the implementation more robust, the factor $[1 - k_x^2/k_0^2]^{-1/2}$ can be approximated by a Taylor series expansion as proposed by Huang et al. (1999).

NUMERICAL FORWARD MODELING EXAMPLE

The main purpose of the simulations was to demonstrate the ability of the new generalized screen propagator to handle large jumps in velocities. A simple 2-D type of velocity model was employed as shown in Fig.1a. In the forward modeling computations we used a spatial sample interval of $10m$ and a temporal sample interval of $0.004s$. The lateral dimension of the model was defined by 1024 samples and the total extrapolation depth by 160 samples. A centered surface source with a Ricker zero-phase wavelet (center frequency of 20Hz) was employed.

In the computations, the band between $0.5Hz$ and $60Hz$ was used. The velocity of the background was set to $2000m/s$ and that of the embedded anomalous structure to $6000m/s$ (cf. Fig.1a). Thus this velocity contrast is very strong and far beyond that of the Born approximation. The high-velocity target had a lateral dimension of 1000m and a thickness of 480m, and the depth down to its top surface was 780m. Figure 1b shows the travel time contours computed in the velocity model in Fig.1a, using a finite-difference solution of the Eikonal equation proposed by Vidale (1988). This result can be employed to check the phase accuracy of the various screen propagators tested. In the calculations we used a reference velocity equal to that of the background, e.g. $c_0 = 2000m/s$. In this example we tested only a first-order version of the new generalized screen-propagator (e.g. setting $N = 1$ in Equation (12)). To further improve the accuracy we chose $M = 4$ in Equation (13).

The performance of this new propagator was compared with the SSF (Stoffa et al., 1990) and the first order ELBF (Huang et al., 1999). The forward extrapolated results obtained are shown in Figs.1c-e. Due to the symmetry, we only plot the computed response to the right of the source position and within a range of $1000m$ (above that range the amplitudes are negligible). This zone is defined by the two vertical dotted lines in Fig.1b. The distance is then measured relative to the source location. We can easily see from Fig.1e that the first order ELBF propagator does not perform well for a large velocity contrast due to the inherent

Born assumption (observed instability and the propagated wave field arrives much later than the exact solution). Both the SSF propagator (cf. Fig.1d) and the new generalized screen propagator (cf. Fig.1c) represent stable solutions. However, their phase accuracies are quite different as illustrated in Fig.1f. In case of the SSF method, the absolute relative travel time error (e.g. after picking travel times and compare with Eikonal solution in Fig.1b) is from 10% and above (upper curve). Alternative use of the new proposed method reduces this error significantly down to less than 3% for near-vertically traveling waves (e.g. lower curve in Fig.1f).

The generalized screen propagator derived in this paper is tailored for large positive contrasts in velocity. This follows from the series expansion in Equation (3) with its underlying assumption of$|\gamma| < (<) 1$. Thus, if large contrasts exist in a given model, the minimum velocity should be chosen as the reference velocity for each extrapolation step. However, if more moderate velocity variations are present, the average velocity can also be used.

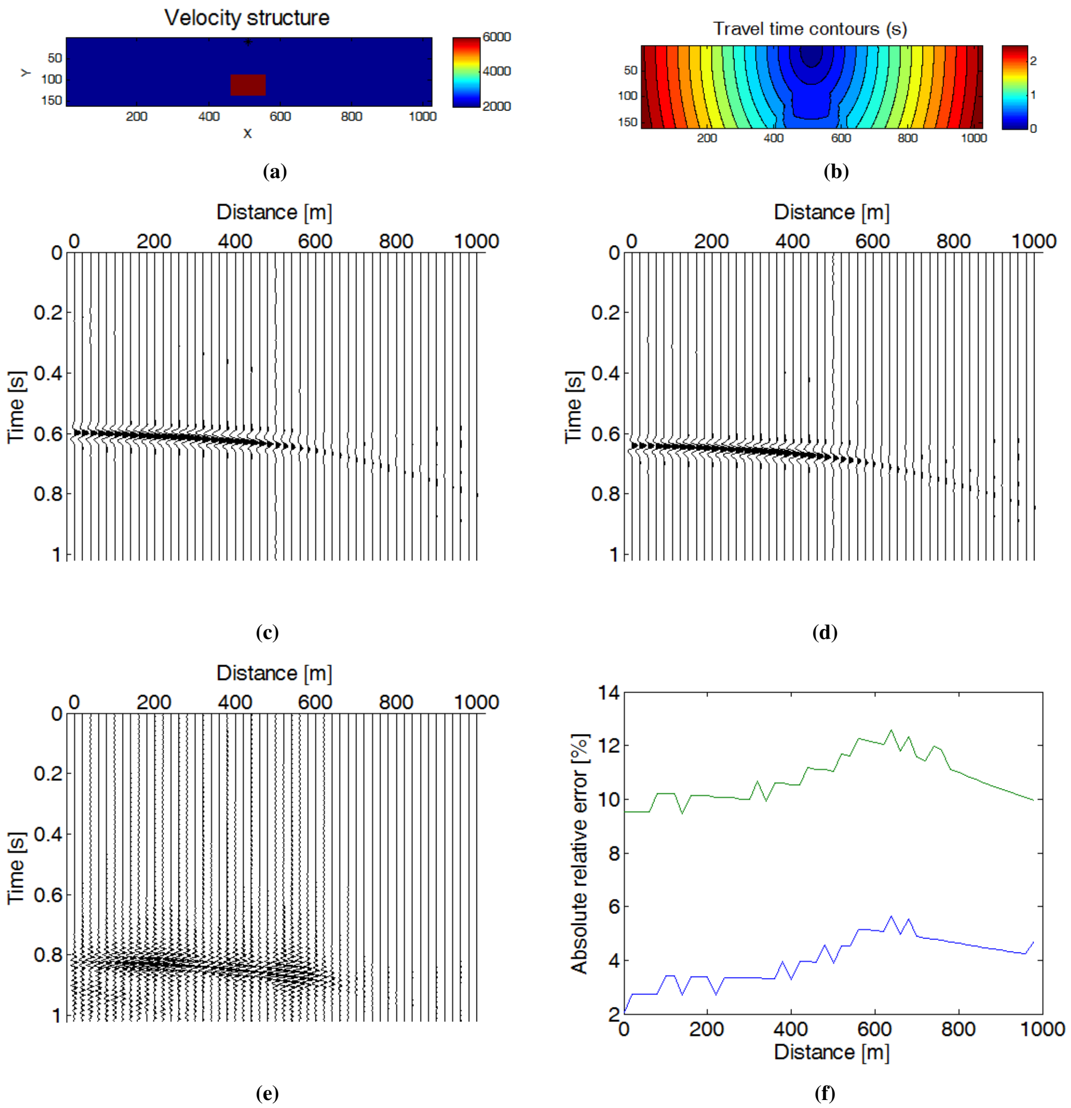

Figure 1: (a) 2-D velocity model; (b) Corresponding travel time contours; (c) First-order new phase screen propagator; (d) Split-step Fourier (SSF) propagator; (e) First-order Extended Local Born Fourier (ELBF) propagator; (f) absolute relative travel time errors (upper curve SSF and lower new method). Reference velocity used in the extrapolations: $c_0 = 2000 m/s$.

CONCLUSIONS

We propose a generalized higher-order screen propagator to be used in modeling and migration in the frequency-wavenumber domain. The relations of the new method with with other known screen propagator methods are discussed. First numerical applications indicate the superior performance and good potential of the proposed formulation.

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