RSO: A NEW MULTIPARAMETER STACKING OPERATOR FOR AN/ISOTROPIC MEDIA

C. Vanelle, M. Bobsin, P. Schemmert, B. Kashtan, and D. Gajewski

email: *claudia.vanelle@zmaw.de*
keywords: *traveltimes, multiparameter stacking, anisotropy, converted waves*

ABSTRACT

Multiparameter stacking has become a standard tool for seismic reflection data processing. Although different traveltime operators exist, whose accuracy depends on the offset and reflector curvature, neither of these can account for anisotropy. We introduce a new stacking operator, the so-called 'recursive stacking operator' (RSO), which is derived from evaluating Snell's law at a locally spherical interface in an anisotropic medium. Examples show that the new method performs well for the whole range of reflector curvatures from nearly planar reflectors to the diffraction limit.

INTRODUCTION

Over the past years, a number of multiparameter stacking operators have been introduced as an extension of the CMP stacking technique. Examples of such operators are the common reflection surface stack (CRS, Müller, 1999), multifocusing (MF, e.g., Landa et al., 2010), and the shifted hyperbola (de Bazelaire, 1988). These operators describe the traveltime surface for a reflected event in the short offset limit. The accuracy of the individual methods differs and depends not only on the considered offset but also on the reflector curvature. Neither of these existing operators considers seismic anisotropy.

Recently, a new stacking operator for monotypic waves in isotropic media was introduced by Vanelle et al. (2010). It was derived from Snell's law for a spherical interface and leads to an implicit expression for the traveltime surface. It can be applied recursively and we therefore refer to it as the 'recursive stacking operator' (RSO). In this work, we suggest an extension of this operator to account for anisotropy.

After deriving the new operator, we demonstrate that it leads to reliable results for a wide range of reflector curvatures from nearly planar reflectors to the diffraction limit.

METHOD

We consider a spherical reflector in a homogeneous medium. The radius of the reflector is R, with its centre at the location $(x_c, 0, H)$, as shown in Figure 1a. The coordinates x_1 and x_2 are those of a source and a receiver, respectively, both at the depth z=0 and y=0. The angle θ defines the reflection point at $\vec{r} = (R\sin\theta, 0, H - R\cos\theta)$. The ray/group velocities of the down- and upgoing ray segments are $v_i(\vartheta_i)$ with the group angles ϑ_i (see Figure 1b).

The traveltimes t_i of the down and upgoing ray segments are given by

$$t_i^2 = \frac{(x_i - x_c - R\sin\theta)^2 + (H - R\cos\theta)^2}{v_i^2(\vartheta_i)} \quad , \tag{1}$$

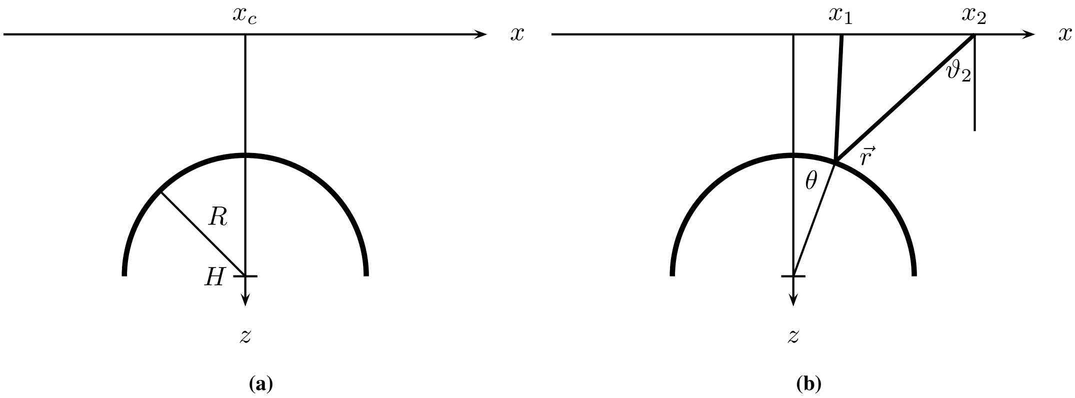

Figure 1: Reflector geometry (a) and acquisition (b) for the spherical reflector. The reflection point $\vec{r}$ is defined by the angle θ. The angles ϑ_i are the ray/group angles.

or, in midpoint and half-offset coordinates (x_m, h):

$$t_1^2 = \frac{(x_m - h - x_c - R\sin\theta)^2 + (H - R\cos\theta)^2}{v_1^2(\vartheta_1)} \quad ,$$
$$t_2^2 = \frac{(x_m + h - x_c - R\sin\theta)^2 + (H - R\cos\theta)^2}{v_2^2(\vartheta_2)} \quad .$$

The sum of t_1 and t_2 must fulfil Snell's law, i.e., $\partial(t_1 + t_2)/\partial\theta = 0$. The derivatives of t_1 and t_2 with respect to θ are

$$\frac{\partial t_i}{\partial\theta} = \frac{1}{2\,t_i}\frac{\partial t_i^2}{\partial\theta} = \frac{R}{v_i^2\,t_i}\left[H\sin\theta - (x_i - x_c)\cos\theta\right] - \frac{t_i}{v_i}\frac{\partial v_i}{\partial\theta} \quad , \tag{2}$$

where

$$\frac{\partial v_i}{\partial\theta} = \frac{\partial v_i}{\partial\vartheta_i}\frac{\partial\vartheta_i}{\partial\theta} \quad . \tag{3}$$

From the geometry of the ray paths shown in Figure 1b, we have that

$$\tan\vartheta_i = \frac{x_i - x_c - R\sin\theta}{H - R\cos\theta} \quad , \tag{4}$$

which leads us to

$$\frac{\partial\vartheta_i}{\partial\theta} = \frac{R}{v_i^2\,t_i^2}\left(R - H\cos\theta - (x_i - x_c)\sin\theta\right) \quad , \tag{5}$$

and finally to

$$\begin{aligned}\frac{\partial t}{\partial\theta} \;=\; & \underbrace{\left[\frac{H}{v_1^2 t_1} + \frac{H}{v_2^2 t_2} + \frac{x_1 - x_c}{v_1^3 t_1}\frac{\partial v_1}{\partial\vartheta_1} + \frac{x_2 - x_c}{v_2^3 t_2}\frac{\partial v_2}{\partial\vartheta_2}\right]}_{A} R\sin\theta \\ & + \underbrace{\left[\frac{H}{v_1^3 t_1}\frac{\partial v_1}{\partial\vartheta_1} + \frac{H}{v_2^3 t_2}\frac{\partial v_2}{\partial\vartheta_2} - \frac{x_1 - x_c}{v_1^2 t_1} - \frac{x_2 - x_c}{v_2^2 t_2}\right]}_{B} R\cos\theta \\ & + \underbrace{\left[-\frac{R}{v_1^3 t_1}\frac{\partial v_1}{\partial\vartheta_1} - \frac{R}{v_2^3 t_2}\frac{\partial v_2}{\partial\vartheta_2}\right]}_{C} R \quad .\end{aligned} \tag{6}$$

Introducing the abbreviations A, B, and C as shown above this equation can be shortened to

$$A \sin\theta + B\cos\theta + C = 0 \quad . \tag{7}$$

Its solution is

$$\sin\theta = -\frac{AC}{A^2+B^2} \pm \frac{B}{A^2+B^2}\sqrt{A^2+B^2-C^2} \quad , \tag{8}$$

where the negative sign must be chosen, as will be shown below.

If the velocities do not depend on direction, equation (7) simplifies considerably. Since the quantity C vanishes we find that

$$\tan\theta = -\frac{B}{A} \quad \Rightarrow \quad \sin\theta = -\frac{B}{\sqrt{A^2+B^2}} \quad . \tag{9}$$

The sign of $\sin\theta$ is negative because the cosine is positive for a reflection from the sphere. Equation (8) collapses to

$$\sin\theta = \pm\frac{B}{\sqrt{A^2+B^2}} \quad . \tag{10}$$

Comparing the coefficients of these expressions lets us recognise that the negative sign must be chosen in (8).

Note that until here, all expressions are exact. Since the angles ϑ_i and thus the velocities v_i and traveltimes t_i implicitly depend on θ, equation (8) cannot be directly solved for θ. We can, however, apply (8) in a recursive fashion using θ_0 as initial angle to obtain an update for θ from (8), which can then be used to compute the traveltimes t_i with (1). Further iterations can be applied to enhance the accuracy.

Special case: converted waves in an isotropic medium

Expressing the x_i by midpoint and half-offset coordinates (x_m, h) and substituting A and B into (7), we obtain for converted waves that

$$\tan\phi = \frac{(x_m - x_c)\left(v_2^2\,t_2 + v_1^2\,t_1\right) - h\left(v_2^2\,t_2 - v_1^2\,t_1\right)}{H\left(v_2^2\,t_2 + v_1^2\,t_1\right)} \quad , \tag{11}$$

where v_1 and v_2 are the P- and S-wave velocities. For zero offset, the angle is

$$\tan\phi_0 = \frac{x_m - x_c}{H} \quad . \tag{12}$$

With the help of ϕ_0, we can also write $\tan\phi$ as

$$\tan\phi = \tan\phi_0 - \frac{h\left(v_2^2\,t_2 - v_1^2\,t_1\right)}{H\left(v_2^2\,t_2 + v_1^2\,t_1\right)} \quad . \tag{13}$$

Special case: monotypic waves in an isotropic medium

In this case, the velocities v_1 and v_2 coincide and equation (11) reduces to

$$\tan\phi = \frac{(x_m - x_c)\,(t_2 + t_1) - h\,(t_2 - t_1)}{H\,(t_2 + t_1)} = \tan\phi_0 - \frac{h\,(t_2 - t_1)}{H\,(t_2 + t_1)} \quad , \tag{14}$$

where the zero-offset angle is again

$$\tan\phi_0 = \frac{x_m - x_c}{H} \quad . \tag{15}$$

Special case: polar anisotropy

In order to solve equation (8) in the anisotropic case, the group velocities and their derivatives with respect to the group angle must be known. These quantities are not generally available. A closed-form expression exists only in the case of elliptical anisotropy, i.e. $\varepsilon = \delta$. Therefore, we use the weak anisotropy descriptions introduced by Thomsen (1986) to express the group velocity and its derivative. For polar media with a vertical symmetry axis, we have

$$v_i = v_{i_0} \left(1 + a \sin^2 \vartheta_i + b \sin^4 \vartheta_i\right) \quad , \tag{16}$$

where

- for qP-waves: $v_{i_0} = \alpha$, $a = \delta$, and $b = (\varepsilon - \delta)$,
- for qSV-waves: $v_{i_0} = \beta$, $a = \sigma$, and $b = -\sigma$,
- for SH-waves: $v_{i_0} = \beta$, $a = \gamma$, and $b = 0$,

and α and β are the vertical velocities of P- and S-waves, respectively. For the velocity derivatives, we find

$$\frac{\partial v_i}{\partial \vartheta_i} = 2\, v_{i_0} \sin \vartheta_i \cos \vartheta_i \, \left(a + 2\, b \sin^2 \vartheta_i\right) \quad . \tag{17}$$

For polar media with a tilted symmetry axis and tilt angle ϕ, the angle θ is replaced by $\theta - \phi$.

EXAMPLES

As we can deduce from the previous section, there are two possible sources of contributions to traveltime errors. The first is the new operator itself, and the second is the introduction of the weak anisotropy approximation.

In order to investigate the influence of these contributions separately, we have chosen media with elliptical symmetry, where $\varepsilon = \delta$. For these media, a closed form solution exists for the group velocity and its derivative (see, e.g., Vanelle, 2002). Reference traveltimes were generated using the NORSAR ray tracing package for values of $\varepsilon = \delta$ ranging from 0.0 (i.e. isotropic) to 0.4. Reflector radii were 100 m, 1 km, and 10 km. The top of the reflector was located at x_c =0 and $H - R$ =1 km in each case. The simulated acquisition scheme covered midpoints from 0 to 1 km from x_c and half-offsets up to 1 km.

We have applied the new recursive stacking operator twice, first using the exact velocity and derivative, and then using their weak anisotropy approximation. In both cases, we have chosen the isotropic zero-offset reflection angle θ_0 as starting angle. In the anisotropic case, the zero-offset ray is not perpendicular to the reflector. Therefore, the zero-offset ray angle ϑ_0 and the angle θ_0, which is the phase angle in this case with the phase normal perpendicular to the reflector do not coincide. There is no closed form solution for the determination of ϑ_0. Since the recursive application of the operator converges after two or at most three iterations, we nevertheless used θ_0 as starting angle.

The results after the third iteration are shown in Figure 2. If exact velocities and velocity derivatives are used, the new operator is highly accurate for all reflector curvatures under consideration. If the weak anisotropy approximation is applied, the accuracy decreases with the strength of anisotropy. This result is not surprising. As for the exact case, the accuracy remains independent of the reflector curvature.

CONCLUSIONS AND OUTLOOK

We have introduced a new recursive stacking operator (RSO) for curved subsurface structures in the presence of anisotropy. Simple numerical examples confirm that the operator leads to high accuracy that is independent of the reflector curvature, i.e., it maintains the high accuracy over a wide range of curvatures, from near-plane reflectors to the diffraction limit.

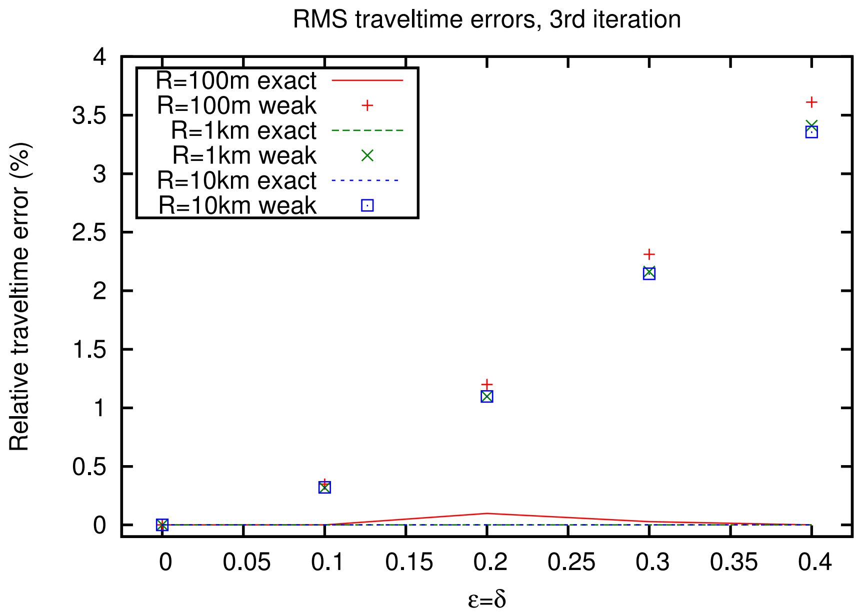

Figure 2: Accuracy of the new recursive stacking operator after three iterations in the presence of elliptical anisotropy. Solid lines indicate that the new operator performs with high accuracy for all reflector radii if exact velocities and derivatives are chosen. If these are expressed by their weakly anisotropic counterpart, the accuracy degrades with increasing strength of anisotropy.

Future work includes the application to more complex models. Schwarz et al. (2011a) have shown that the corresponding isotropic RSO performs well in the presence of heterogeneity, leading not only to highly accurate traveltimes, but also to better stack results and higher resolution than with, e.g., the CRS method, in particular in regions with diffractions.

Another important aspect of our future work is to establish relationships between the model parameters describing the reflector in the homogeneous medium to the CRS parameters in the heterogeneous case. For isotropic media, this has been achieved by Schwarz et al. (2011b). For anisotropic media, we have to deal with the additional difficulty that an anisotropic CRS formulation does not exist. Therefore, this task will include the derivation of an anisotropic CRS equivalent operator.

ACKNOWLEDGEMENTS

We thank the Applied Seismics group in Hamburg for continuous discussion, in particular Benjamin Schwarz and Sergius Dell. The NORSAR 3D software was used to generate reference traveltimes. This work was kindly supported by the sponsors of the Wave Inversion Technology (WIT) Consortium.

REFERENCES

de Bazelaire, E. (1988). Normal moveout revisited – inhomogeneous media and curved interfaces. *Geophysics*, 52:143–157.

Landa, E., Keydar, S., and Moser, T. J. (2010). Multifocusing revisited – inhomogeneous media and curved interfaces. *Geophysical Prospecting*, 1–14:doi: 10.1111/j.1365–2478.2010.00865.x.

Müller, T. (1999). *The Common Reflection Surface stack method – seismic imaging without explicit knowledge of the velocity model*. PhD thesis, University of Karlsruhe.

Schwarz, B., Vanelle, C., and Gajewski, D. (2011a). Application of RSO in heterogeneous media. *15th Annual WIT report*.

Schwarz, B., Vanelle, C., and Gajewski, D. (2011b). The relationship between RSO and CRS parameters. *15th Annual WIT report*.

Thomsen, L. (1986). Weak elastic anisotropy. *Geophysics*, 51:1954–1966.

Vanelle, C. (2002). A tutorial on elliptical anisotropy. *6th Annual WIT report*, pages 267–275.

Vanelle, C., Kashtan, B., Dell, S., and Gajewski, D. (2010). A new stacking operator for curved subsurface structures. *14th Annual WIT report*, pages 247–253.