# SENSITIVITY ANALYSIS OF THE NON-HYPERBOLIC COMMON REFLECTION SURFACE

*M. Tygel, H. Perroud, R. Lopes, and R. Krummenauer*

**email:** *mtygel@gmail.com*
**keywords:** *Multidimensional stacking, non-hyperbolic CRS*

## ABSTRACT

*A new traveltime moveout, referred to as of non-hyperbolic Common-Reflection-Surface (CRS), has been recently proposed by Fomel and Kazinnik, with promising results in accuracy for long offsets and/or curved reflectors. The new moveout, which admits a 2D and a 3D version, depends on the same parameters used in conventional CRS. The aim of the ongoing research is to pave the way for new parameter estimation strategies that exploit the better accuracy of the non-hyperbolic CRS moveout. To that end, we analyze the sensitivity of the CRS parameters in the non-hyperbolic moveout, by means of controlled parameter perturbations. These are carried out in different trace configurations, which go beyond the common-midpoint and zero-offset configurations used in conventional CRS estimation. Still restricted to 2D models, results so far show that the non-hyperbolic CRS moveout has a high potential of becoming a good choice for estimating the CRS parameters.*

## INTRODUCTION

Obtaining the traveltime for reflection events is crucial for methods such as the common-midpoint (CMP) (Mayne, 1962) or its generalizations, such as the multifocus (Berkovitch et al., 2008) and common-reflection-surface (CRS) (Hertweck et al., 2007) methods. In general, approximations of the traveltime are obtained for traces in the vicinity of the normal, zero-offset, ray, and are referred to as (generalized) normal moveouts. When the approximations are designed for the square of the traveltime, one refers to quadratic moveouts. In particular, for the CMP configuration, the quadratic normal traveltime is called *normal moveout (NMO)*. A review on quadratic moveouts can be found in Tygel and Santos (2007). Note, however, that only approximations to the actual traveltime are used, and these are usually only valid for small-offsets. One consequence is that this constrains the number of traces that can be used for stacking, thus limiting the fold for shallow events. Also, large-offset traveltimes are often required in the presence of anisotropy. As a consequence, many moveouts involve anisotropic parameters (see, e.g., Tsvankin, 2005).

In general, the search for traveltime expressions that represent reliable approximations for large offset is part of ongoing research efforts. This task is even more important when one considers that the approximate traveltimes depend on parameters with geophysical significance, such as the velocity, the reflector dip and curvature. Estimating these parameters is crucial for tasks such as stacking, and a good estimate may provide valuable information for interpreters. Here, again, there is an interest in more accurate traveltimes that allow the use of larger offset: with more accurate expressions, and with more traces, the parameters estimates can also be made more accurate.

In the case of Taylor polynomial approximations, the simplest attempt to improve the moveout accuracy to larger offsets is to include fourth-order terms. In the CMP configurations, this has been done by (Al-Chalabi, 1973). For more general configurations, a counterpart fourth-order CRS traveltime was proposed in Oliva et al. (2003). Although such approximations apparently did not give the expected results for long offsets, they nevertheless seem to produce, for short offsets, better parameter estimates if compared to their respective hyperbolic counterparts.

Recently, Fomel and Kazinnik (2011) proposed a new moveout, referred to as the *non-hyperbolic CRS* that, at least for the synthetic examples shown in the paper, provides impressive results for long offsets. The proposed moveout has two main attractive features. First, it depends on the very same parameters that characterize the hyperbolic moveout used in the conventional CRS method. Also, it exactly represents the traveltime response of a hyperbolic reflector in a homogeneous overburden. In this way, the new traveltime is exact for both a dipping planar reflector and a point scatterer (diffraction response). Also interestingly, for the CMP configuration, the non-hyperbolic CRS depends on all CRS parameters, as opposed to the hyperbolic (conventional) CRS, which depends only on the NMO-velocity.

The appealing features of the non-hyperbolic CRS moveout make it a good candidate for replacing its CRS hyperbolic counterpart for parameter estimation and stacking within the CRS method. With this goal in mind, it is valuable, as a preliminary step, to better understand the sensitivity of the various parameters to perturbations. By analyzing how the traveltime is affected by changes in a given parameter, search strategy can be designed to yield more efficient and accurate results. In the present, ongoing investigation, our attention will be focused in the sensitivity analysis of the parameters within the non-hyperbolic CRS moveout, leaving the actual search strategy choices for a future publication. The sensitivity analysis will be done by means of controlled parameter perturbations carried out in selected configurations. These configurations go beyond the CMP and zero-offset (ZO) sections currently used for parameter estimation in conventional CRS. Still restricted to 2D models, the results in this paper, obtained on initial synthetic data show that the non-hyperbolic CRS moveout has a high potential of becoming a good choice for the CRS method.

## HYPERBOLIC AND THE NON-HYPERBOLIC CRS

In the following, we briefly review the definitions of the hyperbolic and non-hyperbolic CRS moveouts, on which the sensitivity analysis of the CRS parameters will be based. As to the present stage of our investigations, we also restrict our discussions to the 2D case. It is to be noted that a natural extension of the non-hyperbolic CRS moveout to 3D models has also been given in Fomel and Kazinnik (2011), without any testing. In the following, we consider a single seismic horizontal line, in which source and receiver pairs are specified by midpoint and half-offset coordinates, $(m, h)$, with respect to a reference midpoint, $m_0$, and zero offset, $h = 0$. Adopting the notation as in Fomel and Kazinnik (2011), the 2D hyperbolic CRS traveltime approximation is given by the expression

$$t_{\mathrm{CRS}}(d, h; t_0) = \sqrt{F(d) + b_2 h^2} \tag{1}$$

where $h$ denotes the half-offset, $d = m - m_0$ denotes the midpoint displacement. Moreover,

$$F(d) = (t_0 + a_1 d)^2 + a_2 d^2 \,, \tag{2}$$

with $t_0$ representing the two-way zero-offset traveltime and the set of parameters $\{a_1, a_2, b_2\}$ are the usual CRS parameters

$$a_1 = \frac{2\sin\beta}{v_0} \tag{3}$$

$$a_2 = \frac{2t_0\cos^2\beta}{v_0} K_{\mathrm{N}} \tag{4}$$

$$b_2 = \frac{2t_0\cos^2\beta}{v_0} K_{\mathrm{NIP}} \,. \tag{5}$$

Finally, $v_0$, represents the velocity at the surface, usually assumed known and constant around the central ray, $\beta$ is the emergence angle of the normal ray with respect to the measurement surface at the central point, $K_{\mathrm{NIP}}$ and $K_{\mathrm{N}}$ are the so-called wavefront curvatures of the normal incident point (NIP) wave and the normal (N) wave, respectively, also measured at the central point. For a brief explanation on the above CRS parameters, the reader is referred to (see, e.g., Hertweck et al., 2007).

The non-hyperbolic CRS traveltime of Fomel and Kazinnik (2011) has the form

$$t_{\mathrm{CRS_{FK}}}(d, h; t_0) = \sqrt{\frac{F(d) + c\,h^2 + \sqrt{F(d-h)F(d+h)}}{2}} \,, \tag{6}$$

where $c$ is the composite parameter

$$c = 2b_2^2 + a_1^2 - a_2 \,, \tag{7}$$

which is also a function of the CRS parameters $\{a_1, a_2, b_2\}$.

The differences between the hyperbolic and this non-hyperbolic CRS traveltime can be seen when we consider the dependence on the two coordinates, half-offset $h$ and midpoint displacement $d$, separately. In the zero-offset (ZO) domain, then $h = 0$, and we have that $d$ is the only coordinate and both (1) and (6) reduce to (2). As a consequence, the hyperbolic and non-hyperbolic CRS approximations are identical in a ZO section.

However, in the common-midpoint (CMP) configuration, then, even if $d = 0$, the non-hyperbolic traveltime expression (6) depends on the quantity $c$, which in turn depends on the entire parameter set $\{a_1, a_2, b_2\}$. Moreover, the third term inside the square root at the right-hand-side of (6), depending on $a_1$ and $a_2$, does not vanish. As a consequence, all three parameters have a non-null influence in the traveltime description in the CMP domain and, in principle, should be taken into account in a parameter search strategy. This is in contrast to the hyperbolic CRS (1), which depends only on $b_2$ in the CMP domain.

First results on the accuracy of this new CRS traveltime model have been presented in Fomel and Kazinnik (2011). Here, we concentrate on the role of each of the parameters in the set $\{a_1, a_2, b_2\}$ in the traveltime. More specifically, we will show how changes in each of these parameters affect the traveltime. This will be done for some gather configurations, in an attempt to highlight the particularities of the non-hyperbolic moveout. Although not pursued in this paper, such analysis may pave the way for hopefully more accurate and efficient estimation/search strategies of the CRS parameters.

## SENSITIVITY OF THE NON-HYPERBOLIC CRS TRAVELTIME

In this section we present the sensitivity analysis of the non-hyperbolic moveout (6). To that end, in very basic examples, we perturb the exact values of the parameters $a_1$, $a_2$ and $b_2$, so as to simulate estimation errors that could be found in practical parameter search procedures. The numerical analysis are done for two important configurations, common midpoint and common offset, which best represent the multicoverage characteristic of the moveout. In all the plots that are produced, we adopted a common pattern to analyze the sensitivity of the traveltime with respect to the parameters. For instance, suppose that the sensitivity of $a_1$ is under analysis. In this case, we keep $a_2$ and $b_2$ with the correct values while varying $a_1$ from $-10\,\%$ to $+10\,\%$ of its correct value, in steps of $1\,\%$.

The relevance of this study is that, if the traveltime does not vary much when a certain parameter changes in a certain configuration, we can infer that:

- This parameter cannot be accurately estimated in this configuration. Indeed, as different values of the parameter yield approximately the same traveltime, there is no way to discriminate between the correct and incorrect values of the parameter.
- Moreover, one can keep this parameter fixed at a certain value, and estimate the other parameters. Indeed, as the traveltime has a small dependence on the parameter, keeping it at a fixed value will have a small effect on the estimation of the other parameters.

### Example 1 - Dipping reflector

In this example we analyze the case of a planar dipping reflector in a homogeneous overburden. In this case, both the hyperbolic and the non-hyperbolic CRS models match the exact traveltime. Note that, in this case, the wavefront of the normal wave is infinite, leading to $a_2 = 0$. Therefore, the sensitivity analysis concentrates only on $a_1$ and $b_2$. In this example, we have $t_0 = 0.20984\,\text{s}$ and $v_0 = 2000\,\text{m/s}$. Consequently, the reflector depth is $z \approx 210\,\text{m}$.

We start with the parameter sensitivity under a CMP configuration, as shown in Figure 1. In this scenario we plot the traveltime deviation from the correct traveltime when a controlled disturbance is applied to the exact values of the parameters $a_1$ and $b_2$. Specifically, each curve presented in these graphs represents the time deviation $\Delta t = \tilde{t}_{\text{CRS}_{\text{FK}}} - t_{\text{CRS}_{\text{FK}}}$, where $t_{\text{CRS}_{\text{FK}}}$ denotes the actual traveltime and $\tilde{t}_{\text{CRS}_{\text{FK}}}$ denotes the perturbed traveltime due to a shift in the value of the parameter under consideration.

Figure 1(a) shows the sensitivity of the parameters $a_1$ (blue curves) and $b_2$ (red curves) for a midpoint displacement $d = 0$, that is, the CMP gather based on the reference point. As expected, the variations on $a_1$ have no influence on the traveltime because the moveout (6) reduces to the hyperbolic moveout in the case of $a_2 = 0$ and $d = 0$. Notwithstanding, this behaviour is not shared when we allow for a CMP gather under non-zero midpoint displacements $d$. Figures 1(b) and 1(c) show the sensitivity for $d = 50\,\mathrm{m}$ and $d = 200\,\mathrm{m}$. We note a significant influence of $a_1$, especially for smaller offsets. Note that, since the reflector depth is $z \approx 210\,\mathrm{m}$, our analyses were drawn for relative apertures up to $r \approx 5$, where the relative aperture is defined as the ratio $r$ between half-offset (or midpoint displacement) and depth.

Changing to the CO configuration, we show in Figure 2 the sensitivity of $a_1$ and $b_2$ for the half-offsets $h = 50\,\mathrm{m}$ and $h = 200\,\mathrm{m}$. Figure 2(a) suggests that, for low offsets, $a_1$ is dominant over $b_2$, the later becoming more prominent as the offset increases. Note again that $a_1$ has no effect for $d = 0$.

The results up to now lead to one important conclusion: Due to the multidimensional (offset-midpoint) characteristic of the non-hyperbolic CRS traveltime, and its dependence on at least two parameters in the CMP/CO domain, the influence of each parameter can be amplified using an adequate gather configuration. In other words, we can choose a gather with constant $h$ (or $d$) and some aperture $\Delta d$ (or $\Delta h$) so that the traveltime dependence on $a_1$ or $b_2$ will be dominant.

### Example 2 - Circular reflector

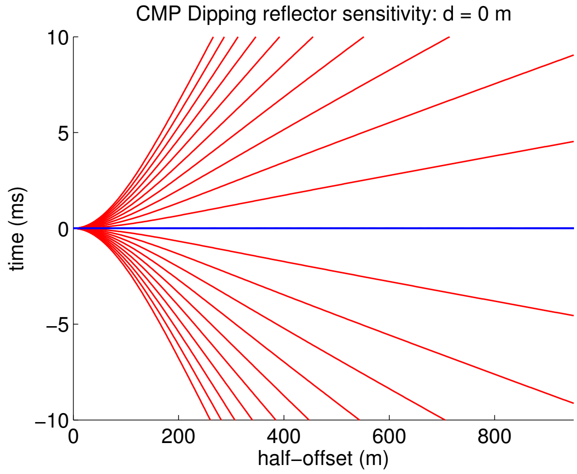
We now move to a second, also very basic, model, in which the planar reflector is replaced by a circular reflector, still within a homogeneous acoustic overburden. The new feature of that model is that, now, the non-hyperbolic moveout is not exact. We analyze the sensitivity of the non-hyperbolic CRS moveout with respect to the complete parameter set $\{a_1, a_2, b_2\}$. It is to be observed that, now, $a_2 \neq 0$. Other than the reflector shape, the model share similarities with the previous example: $t_0 = 0.20984\,\mathrm{s}$ and $v_0 = 2000\,\mathrm{m/s}$, with a reflector depth $z \approx 210\,\mathrm{m}$.

Figure 3 shows the absolute difference between the exact and the non-hyperbolic CRS traveltimes in a CMP gather (with $d = 0$). Besides the hyperbolic CRS of equation (1), the non-hyperbolic CRS of equation (6), we also consider the fourth-order CRS traveltime introduced in Oliva et al. (2003) (we refrain to write down that moveout expression here). Observe that the fourth-order CRS approximation does a good job up to $h \approx 400\,\mathrm{m}$, corresponding to a relative aperture $r \approx 2$, degrading quickly after this threshold. On the other hand, the non-hyperbolic CRS approximation leads to errors close to zero, even for $r > 4$. This is a strong result that shows that, if that moveout is used, very large apertures can be employed in the CMP data gathers, both for parameter search and stacking.

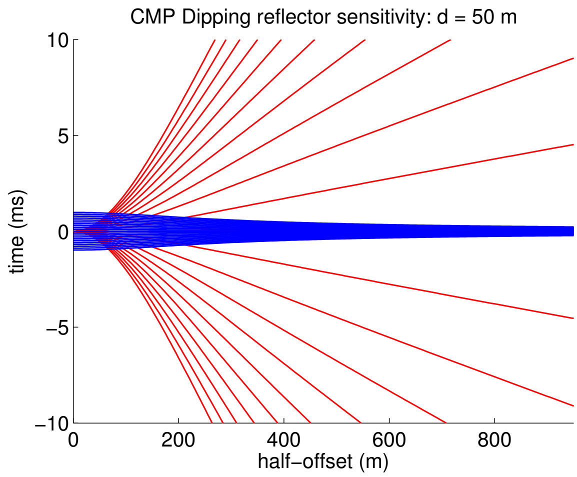
Figure 4 shows the traveltime sensitivity with respect to the parameters $\{a_1, a_2, b_2\}$ in the CMP configuration, for $d = 0\,\mathrm{m}$, $d = 50\,\mathrm{m}$ and $d = 200\,\mathrm{m}$. We note from Figure 4(a) that both $a_1$ (blue curves) and $a_2$ (green curves) have some influence in the traveltime, even when $d = 0$. Note, however, that relatively large deviations ($10\,\%$) in $a_2$ induce little time deviation in the traveltime. Although deviations on $a_1$ induce far more effects if compared to $a_2$, we observe that $b_2$ is the dominant parameter in terms of sensitivity for all the offset range. However, the influence of $a_1$ (and also $a_2$) becomes dominant for low offset values when we allow $d$ to be different from zero, as shown in Figures 4(b) and 4(c) for $d = 50\,\mathrm{m}$ and $d = 200\,\mathrm{m}$, respectively.

Figure 5 shows the sensitivity results for the CO configuration with half-offsets $h = 50\,\mathrm{m}$ and $h = 200\,\mathrm{m}$, respectively. From Figure 5(a), we conclude that, for small midpoint apertures, $b_2$ continues to dominate. However, as the midpoint aperture becomes larger, the traveltime sensitivity to $a_1$ and $a_2$ becomes more relevant, with a significant emphasis on $a_1$, especially for smaller midpoint apertures. Increasing the offset to $h = 200\,\mathrm{m}$, Figure 5 still shows a predominance of $b_2$, but not as much as in the CMP configuration.

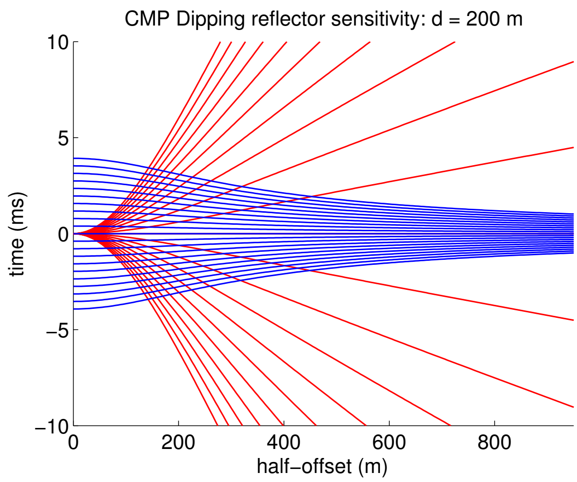
The results presented in this example confirm, as generally accepted in practice, that the most relevant parameters are, in fact, $a_1$ and $b_2$. Nevertheless, depending on the gather configuration, say CMP/CO, we can choose a constant $h$ (or $d$) and some aperture $\Delta d$ (or $\Delta h$), so that $a_1$ or $b_2$ will be dominant. Based on these conclusions, we can devise possibilities for efficient search strategies to estimate the set of parameters $\{a_1, a_2, b_2\}$. This is briefly discussed below.




(a)

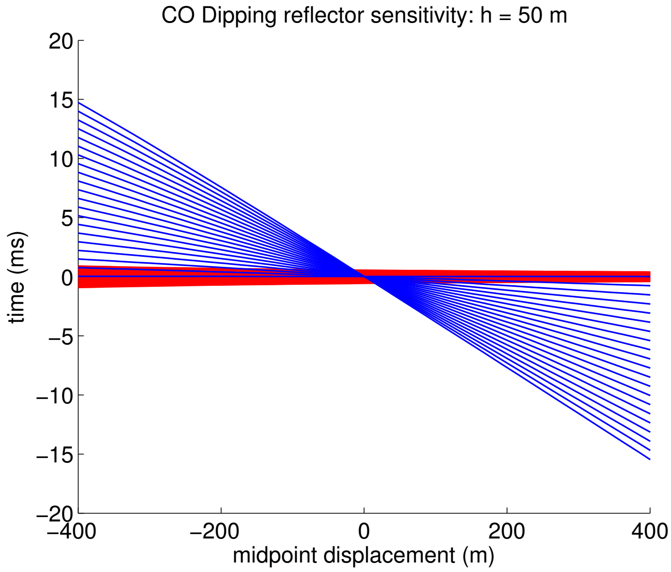



(b)

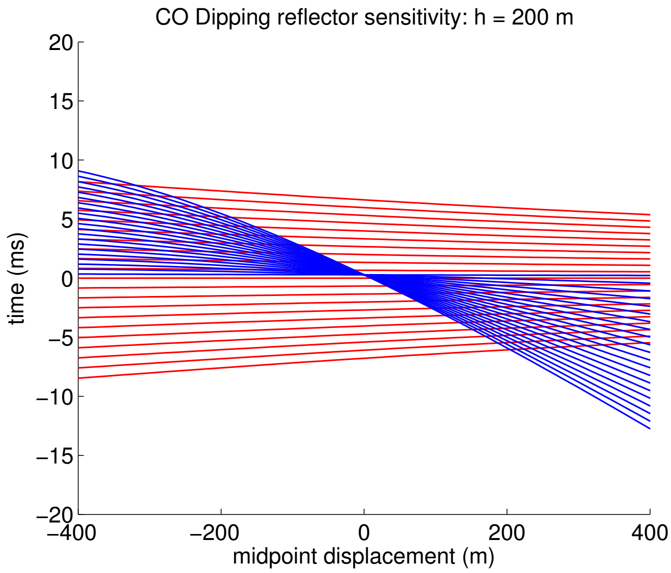



(c)

**Figure 1:** Sensitivity of the non-hyperbolic CRS moveout with respect to the parameters $a_1$ and $b_2$ in a CMP configuration for a planar dipping reflector. Blue curves refer to $a_1$, and red curves refer to $b_2$.




**(a)**




**(b)**

**Figure 2:** Sensitivity of the non-hyperbolic CRS moveout with respect to the parameters $a_1$ and $b_2$ in a CO configuration for a planar dipping reflector. Blue curves refer to $a_1$, and red curves refer to $b_2$.

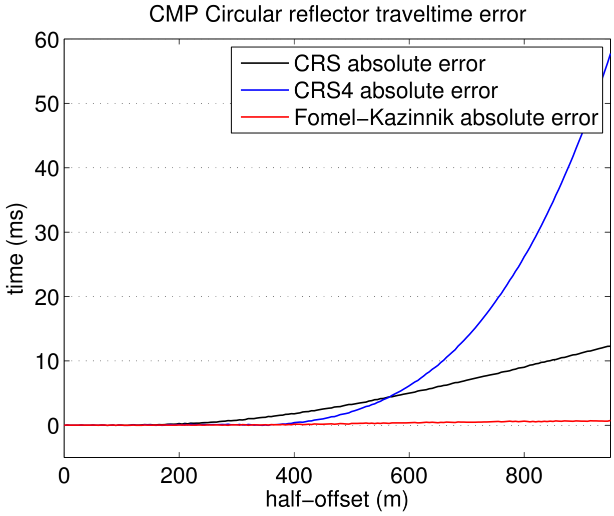



**Figure 3:** Comparison of the absolute time deviation from the exact traveltime for a circular reflector within a homogeneous acoustic overburden under in a CMP configuration: CRS stands for the conventional (hyperbolic) CRS, CRS4 stands for the fourth-order approximation and Fomel-Kazinnik stands for the non-hyperbolic CRS.

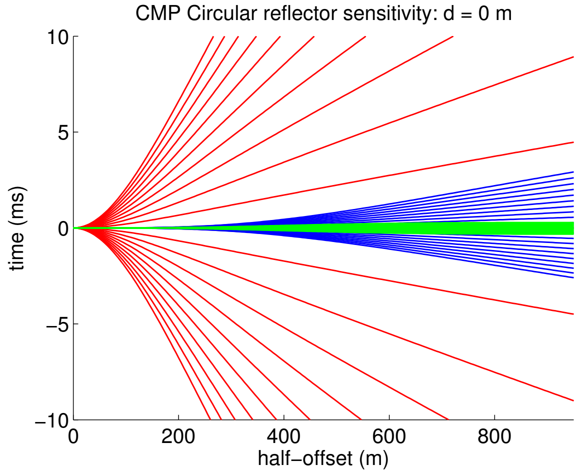
## Brief discussion on search strategies

The usual approach to implement the hyperbolic CRS stack is to divide the estimation of the parameters $\{a_1, a_2, b_2\}$ into CMP and ZO configurations. In the CMP configuration, the conventional, hyperbolic, CRS traveltime of equation (1) reduces to the well-known normal moveout (NMO) and, thus, only a single parameter, $b_2$, is estimated. Using the estimation of $b_2$, a ZO section is obtained through stacking, based on the hyperbolic moveout traveltime. In a second stage, the estimation of $a_1$ and $a_2$ is performed on the stacked section. This is done using the fact that, in the ZO stacked section, we have $h = 0$, so the hyperbolic CRS taveltime (1) does not depend on $b_2$. Then, with the complete set of estimated parameters, a final stacking is performed using the hyperbolic CRS traveltime in (1).

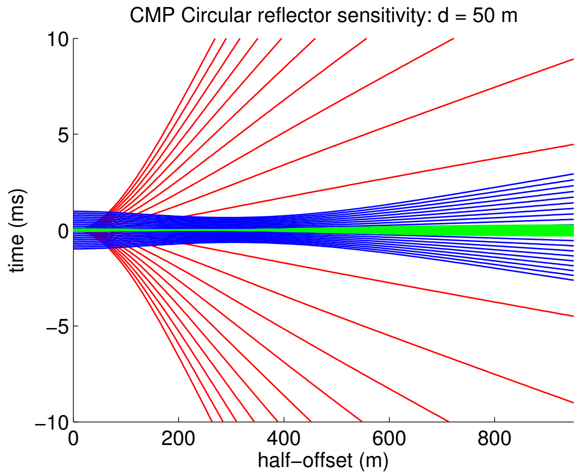
This seems to be a wise strategy because in the CMP and ZO domains, offset and midpoint, are decoupled parameters in the hyperbolic CRS moveout. While the CMP component (offset domain) is described only by $b_2$, the ZO component (midpoint domain) is described only by $a_1$ and $a_2$. This is, however, not the case for the non-hyperbolic CRS model, as can be seen from equation (6) and also from the sensitivity results presented previously.

Under the light of the results so far obtained, we believe that different search strategies can be devised to efficiently exploit the potential of the non-hyperbolic CRS traveltime. The main evidences that corroborate this discussion were shown in Figure 4 and Figure 5.

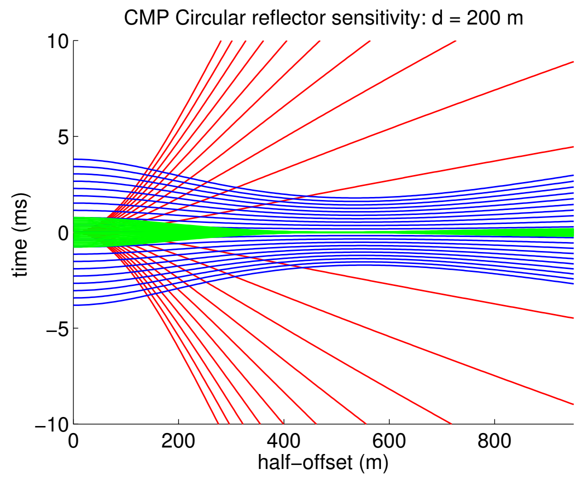
For instance, Figure 4(a) justifies a search for $b_2$ parameter alone as traditionally done in the CMP gather (namely using the hyperbolic CRS on a zero-midpoint CMP), since in this situation, $a_1$ and $a_2$ have almost no influence on the traveltime. In other words, a search for $b_2$ as done in conventional velocity analysis is very justified. While this is true in the traditional CMP (zero-displacent) configuration, Figures 4(b) and 4(c) indicate that the search for $a_1$ and $a_2$ can be also an alternative within a CMP configuration based on non-zero midpoint displacements. Additionally, Figure 5(a) and Figure 5(b) also justify the search for $a_1$ and $a_2$ in a CO configuration, with very emphatic relevance of these parameters over $b_2$. In summary, although derived for very simple models, our results suggest that the relevance of the parameters, especially in the non-hyperbolic CRS moveout, can be amplified if appropriate configuration, CMP with various midpoint displacements and CO with various half-offsets, is employed. Efficient search strategies should consider, thus, this multidimensional characteristic of the non-hyperbolic CRS traveltime to improve the estimation accuracy of the CRS parameters.




**(a)**

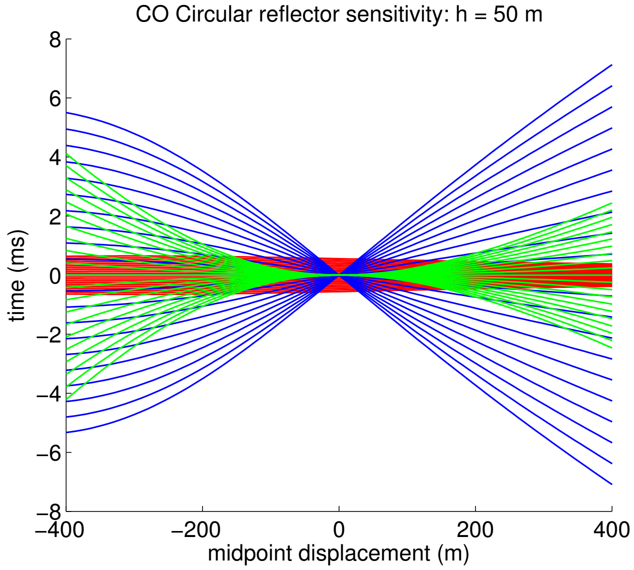



**(b)**

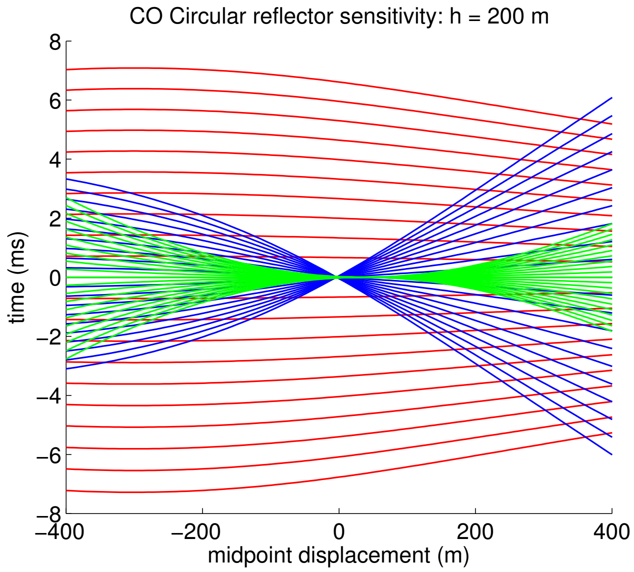



**(c)**

**Figure 4:** Sensitivity of the non-hyperbolic CRS traveltime model with respect to the parameters $a_1$, $a_2$ and $b_2$ in a CMP configuration for a circular reflector within a homogeneous acoustic overburden. Blue curves refer to $a_1$, green curves refer to $a_2$, and red curves refer to $b_2$.




**(a)**




**(b)**

**Figure 5:** Sensitivity of the non-hyperbolic CRS traveltime model with respect to the parameters $a_1$, $a_2$ and $b_2$ in a CO configuration for a circular reflector. Blue curves refer to $a_1$ parameter, green curves refer to $a_2$ parameter and red curves refer to $b_2$ parameter.

## CONCLUSIONS

This paper addressed the sensitivity of CRS parameters in the framework of a recently proposed non-hyperbolic CRS traveltime. The study was carried out in two simple 2D cases of a single reflector (with a planar dipping and a circular shape) overlain by a homogeneous acoustic overburden. We analyzed the sensitivity of the parameters within CMP (away from the reference midpoint) and CO. Under these configurations the midpoint and half-offset are uncoupled, so that our analysis goes beyond the usual ones associated with conventional CRS estimation. Under the use of the non-hyperbolic CRS moveout, the results show that the relevance or influence of each CRS parameter can be amplified if such more general CMP and CO gathers are considered. This means that the efficiency and accuracy of the estimations may be improved. We hope that the directions suggested by the present first study may inspire new and more powerful methods for CRS parameter estimations, which is the main challenge faced today by the CRS method.

## ACKNOWLEDGMENTS

We acknowledge support of the National Council of Scientific and Technological Development (CNPq-Brazil), the Research Foundation of State of São Paulo (FAPESP-Brazil) and from the sponsors of the *Wave Inversion Technology (WIT) Consortium*.

## REFERENCES

Al-Chalabi, M. (1973). Series approximation in velocity and traveltime computations. *Geophys. Prospect.*, 21:783–795.

Berkovitch, A., Belfer, I., and Landa, E. (2008). Multifocusing as a method of improving subsurface imaging. *The Leading Edge*, 27(2):250–256.

Fomel, S. and Kazinnik, R. (2011). Non-hyperbolic common reflection surface. *Geophys. Prosp.: In print*.

Hertweck, T., Schleicher, J., and Mann, J. (2007). Data stacking beyond cmp. *The Leading Edge*, 26:818–827.

Mayne, W. H. (1962). Horizontal data stacking technique. *Supplements to Geophysics*, 27:927–938.

Oliva, P. C., Tygel, M., Hubral, P., and Schleicher, J. (2003). A fourth-order crs moveout for reflection and diffraction events. *Journal of Seismic Exploration*, 12:197–219.

Tsvankin, I. (2005). *Seismic signatures and analysis of reflection data in anisotropic media*. Elsevier Science, second edition.

Tygel, M. and Santos, L. T. (2007). Quadratic normal moveouts of symmetric reflections in elastic media: a quick tutorial. *Studia Geophysica et Geodaetica*, 51:185–206.