NUMERICAL MOVEOUT ESTIMATION FOR MIGRATION VELOCITY ANALYSIS IN SUPER-GATHERS

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ABSTRACT

In migration velocity analysis (MVA), the residual moveout in the image gather is used to correct the velocity field. In this work we propose a numerical approach to describe the moveout in the image gather considering a dipping reflector. The description is valid for neighbouring image gathers, so that several image gathers can be used simultaneously to obtain the parameters. This strategy provides more reliable values for the velocity correction factor. Through some synthetic examples, we validate the strategy of fitting the moveout curves or surfaces numerically.

INTRODUCTION

Migration velocity analysis (MVA) is a seismic processing technique that investigates the migration residual moveout to correct an a priori velocity model. After migration, it is expected that in a common image gather (CIG), the same reflections are imaged to the same depth, so that the events get flattened. However, if the velocity used in migration is wrong, the events have a residual moveout. For a migration velocity lower than the correct one, events curve upward, whereas if the velocity is higher, events curve downward. This moveout can be used to correct the velocity (Sattlegger, 1975).

As in conventional normal moveout (NMO) processing, it is necessary to predict such moveout. Al-Yahya (1989) proposed a formula based on a horizontal reflector embedded in a constant velocity overburden. It allows to detect a velocity correction factor from fitting the theoretical moveout curve to the one observed in the image gather.

Schleicher and Biloti (2007) proposed a generalization of Al-Yahya's formula considering dipping reflectors. During their derivations, they arrived at a 5th-order polynomial that can not be solved analytically. To obtain a solution, they used a Taylor series expansion and further auxiliary approximations. Since Schleicher and Biloti (2007) used a Taylor approximation, theoretically their result is valid only for small dips.

Instead of making approximations, we adopt a numerical approach to solve the 5th-order polynomial and describe the moveout in the image gather. In this way, we get closer to the observed curve. Futhermore, in a similar way as proposed by Klokov et al. (2009), our proposal allows to use several image gathers at the same time. This strategy provides more reliable values for the velocity correction factor. However, while in Klokov et al. (2009) the introduced parameter plays only a mathematical role, our choice for the additional parameter has the advantage of being physically meaningful (the local dip of the reflector).

The main purpose of this work is to validate the strategy of fitting the moveout curves or surfaces in case of super-gathers numerically.

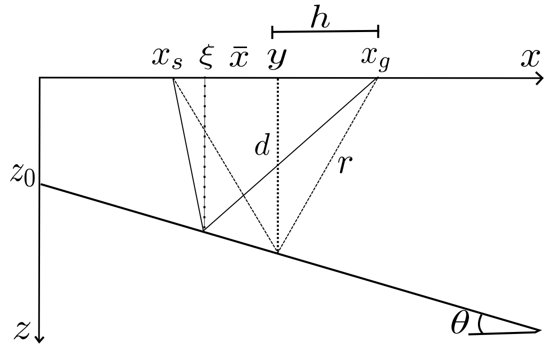


Figure 1: Dipping reflector geometry.

MIGRATION VELOCITY ANALYSIS

For a horizontal reflector embedded in a homogeneous medium, Al-Yahya (1989) derived the following formula to describe the time-migrated event in a CIG:

$$\tau^2_{\text{AY}}(h) = t_0^2 + (\gamma^2 - 1)4h^2/v_m^2, \tag{1}$$

where h is the half-offset, t_0 is the vertical time and $\gamma = v_m/v$ is velocity correction factor, representing the ratio between the migration velocity v_m and the true medium velocity v.

Considering a dipping reflector, Schleicher and Biloti (2007) generalized Al-Yahya's formula 1. In the following, we repeat some theoretical derivations of their work that are important to explain our ideas and where they differ from the previous works. Let us start from the reflection traveltime for a dipping reflector, which is given by

$$t_{\text{ref}} = \frac{2}{v}\frac{r}{\sqrt{1+m^2}} = \frac{2}{v} r\cos\theta. \tag{2}$$

Here, $m = \tan\theta$, with θ denoting the dip angle of the reflector, and $r = \sqrt{d^2 + h^2}$ is the distance between this depth point and the source or receiver, with $d = my + z_0$ being the depth of the reflector vertically under the midpoint coordinate y and z_0 denoting the depth of the reflector at some reference position, where the horizontal x coordinate equals zero (see Figure 1).

The reflection event in the image gather is then desribed by the envelope of the family of all isochrons for points on the traveltime curve given by equation (2). This family is descbribed by

$$t(x, y, h) = \frac{2z}{v_m} = \frac{2b}{v_m}\sqrt{1 - \frac{(x-y)^2}{a^2}}, \tag{3}$$

where x is the horizontal coordinate of the image point, and the half-axes of the ellipse (3) are given by $a = v_m t_{\text{ref}}/2$ and $b = \sqrt{a^2 - h^2}$.

Replacing the reflection traveltime (2), equation (3) can be recast into the form

$$t(x, y, h) = \frac{2}{v_m}\frac{1}{\gamma\sqrt{1+m^2}}\frac{pq}{r}, \tag{4}$$

where $p = \sqrt{\gamma^2 r^2 - (1+m^2)(x-y)^2}$ and $q = \sqrt{\gamma^2 r^2 - (1+m^2)h^2}$.

The envelope condition is

$$\frac{dt}{dy} = \frac{2}{v_m}\frac{1}{\gamma\sqrt{1+m^2}}\frac{1}{pqr^3} f(x, y, h) = 0, \tag{5}$$

where

$$\begin{aligned} f(x, y, h) &= p^2(1+m^2)h^2 m(my + z_0) + \\ &+ q^2 r^2 \left[\gamma^2 m(my + z_0) + (1+m^2)(x-y)\right]. \end{aligned} \tag{6}$$

This condition defines the midpoint y where the reflection associated with the image point x is found in the original data section. After the stationary y is obtained, the event location is calculated substituing y back into (4).

Since f is a polynomial function of degree five in y, $f = 0$ can not be solved analytically, except for the zero-offset case. This motivated Schleicher and Biloti (2007) to use Taylor expansion up to fourth order in m. For doing so, the condition $m \ll 1$ must be fulfilled, which means that their formula is valid for small dips only.

In this work we propose to obtain y by solving equation (5) numerically. This allows to avoid the use of a Taylor expansion. Another advantage of the numerical solution is that we can solve equation (5) for more than a single image gather, extending the work of Schleicher and Biloti (2007), who considered only one image gather at a time.

For a fixed image point $x = \xi$, we have to find a solution y for each h. In general, for an iterative numerical solution a reasonable initial guess for y is necessary. As mentioned above, equation (5) can be solved analytically for $h = 0$. The solution is

$$y_0 = \frac{\gamma^2 m z_0 + (1+m^2)\xi}{(1-\gamma^2)m^2 + 1}. \tag{7}$$

This value is used as the initial value for the smallest half-offset. Employing a continuation strategy, we then use the solution for the previous h as initial guess for the next half-offset. After calculating the numerical solution y, it is substituted into equation (4) to obtain the moveout in the image gather.

The information above can be used to estimate the moveout in a neighbouring image gather. for this purpose, we estimate the vertical time τ_0 upon substitution of equation (7) into formula (4). This yields

$$\tau_0 \equiv t(\xi, y_0, 0) = \frac{2\gamma(m\xi + z_0)}{v_m} \frac{1}{\sqrt{|(1-\gamma^2)m^2 + 1|}}. \tag{8}$$

From the above equation we can obtain $z_0 = v_m \tau_0$ in terms of t_0. In this way, instead of calculating the time in the image gather for each z_0, we can use τ_0.

The true zero-offset vertical time is given by

$$t_0 = \frac{2z}{v} = \frac{2(mx + z_0)\gamma}{v_m}. \tag{9}$$

Combining equations (8) and (9), we can write τ_0 in terms of t_0

$$\tau_0 = \frac{t_0}{\sqrt{|(1-\gamma^2)m^2 + 1|}}.$$

Let us now consider the image gather at $x = \xi$, in the vicinity of another image gather at $\bar{x}$. In other words, $\xi = \bar{x} + \delta x$. The corresponding vertical time $\overline{\tau}_0$ reads

$$\overline{\tau}_0 = \frac{2}{v_m}\gamma(m\bar{x} + z_0)\frac{1}{\sqrt{|(1-\gamma^2)m^2 + 1|}}, \tag{10}$$

and the true vertical time for image gather at $\xi = \bar{x} + \delta x$ is given by

$$\bar{t}_0 = \overline{\tau}_0 + \frac{2}{v_m}\gamma m \delta x \frac{1}{\sqrt{|(1-\gamma^2)m^2 + 1|}}. \tag{11}$$

To evaluate how good a so-determined theoretical curve/surface fits an actual migrated event, we determine the coherence along the trial surface. The parameters that describe the best-fitting curve/surface are obtained by maximizing the coherence measure. The most important parameter for the migration velocity analysis is the velocity correction parameter γ. Once γ is obtained, the velocity field is updated and the data has to be migrated again.

NUMERICAL EXAMPLE

Curve fitting

In this section, we test the fitting of our numerically obtained moveout prediction curve to a migrated event on synthetic test data. We consider three models, each with a single planar reflector with increasing dips of 10, 15 and 20 degrees. In all models, the velocity above the reflector is 2.0 km/s. We have added random noise with a signal-to-noise ratio of 40 to the data. These data were then Kirchhoff time migrated twice, once using a lower migration velocity of 1.5 km/s and once with a higher velocity of 3.0 km/s. Therefore, the theoretical values for the migration velocity correction parameter are $\gamma = 0.75$ and $\gamma = 1.5$, respectively. The theoretical values for the model parameters m and z_0 are given in Table 1.

Dip	m	z_0
10°	0.175	0.875
15°	0.275	1.375
20°	0.375	1.875

Table 1: Theoretical parameter values.

We constructed the theoretical curves using the formulas of Al-Yahya (1989), Schleicher and Biloti (2007) and compared them to those obtained with our numerical approach. The results for the case of the higher migration velocity of 3.0 km/s are shown in Figure 2. Note that for a reflector dip of 10°, Schleicher and Biloti's curve and ours are almost the same. For a dip of 15°, our curve is slightly better than Schleicher and Biloti's, better fitting the event for larger offsets. For a dip of 20°, the improvement of our numerically estimated curve is even more remarkable. For all three dips, Al-Yahya's curve doesn't describe the event very well. As expected, his description gets worse when the reflector dip increases.

The corresponding results for the lower migration velocity of 1.5 km/s are shown in Figure 3. In this case, the deviation of Al-Yahya's curve from the migrated event is much less pronounced. However, the low-velocity result demonstrates the short-comings of Schleicher and Biloti's correction. For large offsets, their curve strongly deviates from the migrated event, even exhibiting a completely wrong curvature. Our numerical solution fits the event much better for all three dipping reflectors.

Parameter estimation

In this section, we test the quality of the extracted model parameters from fitting the trail curvers to the migrated data. Note that in both Al-Yahya's and Schleicher and Biloti's approaches, the parameters are obtained using the data of a single image gather. In our proposal, more than one image gather can be used to find the optimal parameters. However, to compare the three strategies, we use only one image gather.

For this numerical experiment, we use a semblance window of 20 ms and the data migrated with the higher velocity of 3.0 km/s. We apply the following procedure to find the optimal parameter values. For each τ_0, we start by constructing a coarse grid in γ and m, with $\Delta\gamma = 0.25$ and $\Delta m = 0.1$, evaluating the coherence value at each grid point, and find the highest value. The parameter values of γ and m associated to that highest semblance value are the initial guesses for the optmization phase. In this work, we employed the simplex method for unconstrained optmization Nelder and Mead (1965). The results for the three reflectors are shown in Table 2.

We see that the use of Al-Yahya's curve always led to an underestimated velocity correction parameter γ, and that of Schleicher and Biloti always to an overestimated one. Both Schleicher and Biloti's and our curves provided a better precision for γ than for the dip parameter γ. Of course, no value for m is given in Table 2 for Al-Yahya since his curve assumes a horizontal reflector $m = 0$.

For the model with reflector dip 10°, the parameters estimated using Schleicher and Biloti's curve are slightly better than ours. For the models with dips 15° and 20° our procedure estimated the parameters with the smallest error with respect to the theoretical parameters. Moreover, our proposal has the best coherence value, indicating that the fit to the migrated event is better than using the other curves.

As mentioned before, in our procedure more than one image gather can be used at once to estimate the parameters. To test this feature, we used the image gathers immediately to the left and right of the central

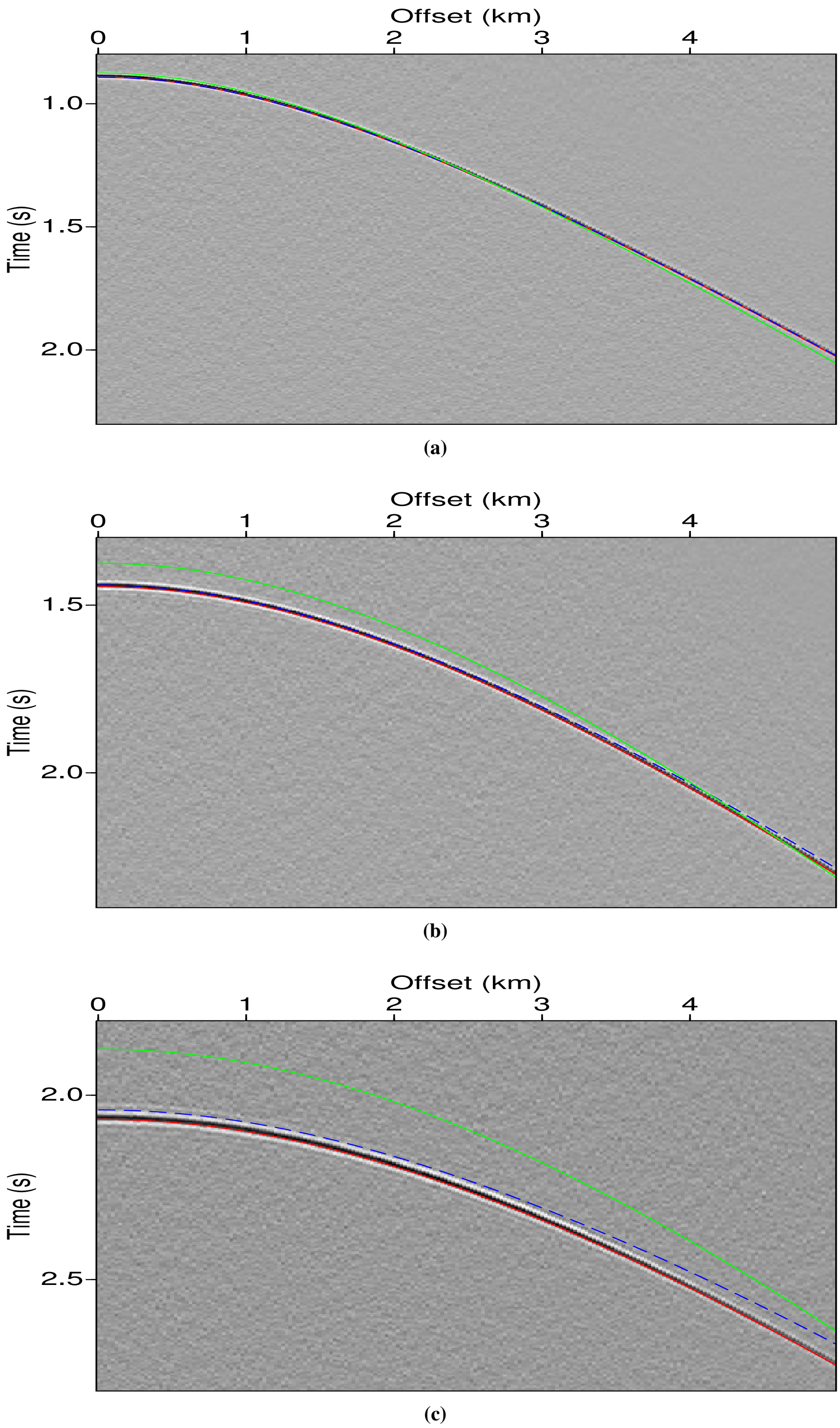
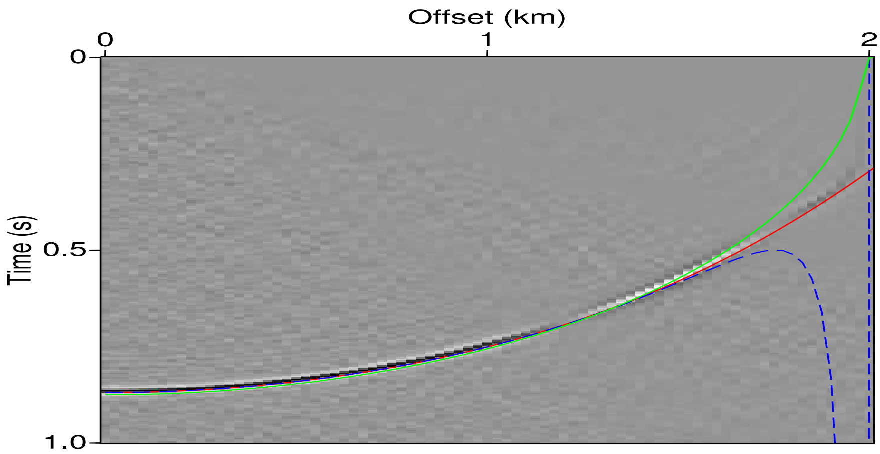
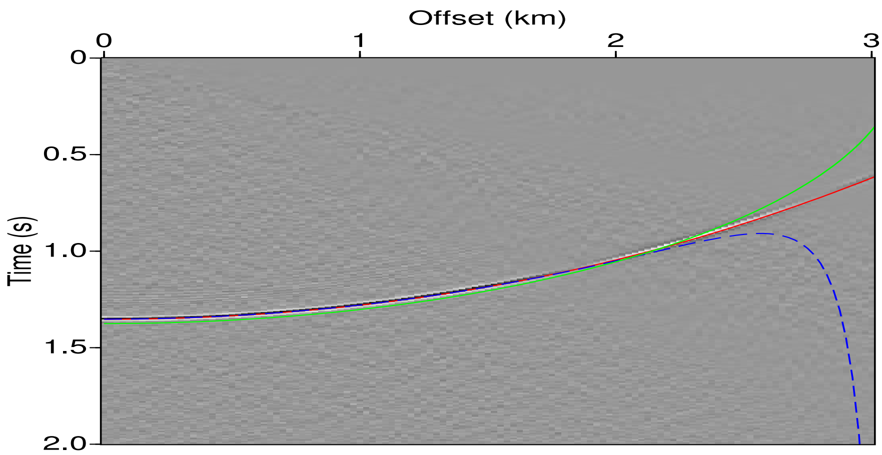


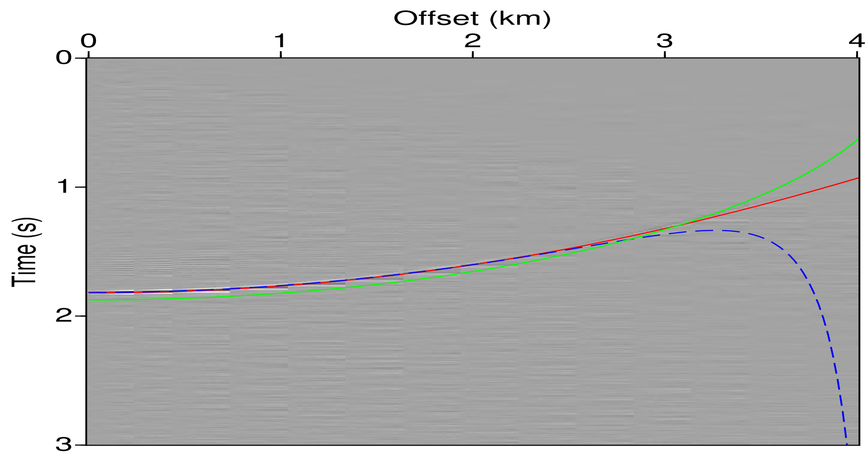
Figure 2: Image gather of the data migrated with an incorrect velocity of 3.0 km/s for 10° (a), 15° (b) , 20° (c) dipping reflector. The green, dashed blue and red lines indicate, respectively, Al-Yahya's, Schleicher and Biloti's and our predicted curve.



(a)



(b)



(c)

Figure 3: Image gather of the data migrated with an incorrect velocity of 1.5 km/s for 10° (a), 15° (b) , 20° (c) dipping reflector. The green, dashed blue and red lines indicate, respectively, Al-Yahya's, Schleicher and Biloti's and our predicted curve.

Dip	Proposal	γ	m	z_0	Coherence
10°	Al-Yahya	1.4762	-	0.8860	0.8281
	Schleicher & Biloti	1.5072	0.1872	0.8801	0.8976
	Sakamori & Biloti	1.4878	0.1429	0.8682	0.9016
15°	Al-Yahya	1.4816	-	1.4944	0.6278
	Schleicher & Biloti	1.5063	0.2573	1.3915	0.9554
	Sakamori & Biloti	1.4970	0.2706	1.3720	0.9597
20°	Al-Yahya	1.4793	-	2.1132	0.6942
	Schleicher & Biloti	1.8061	0.6175	1.1698	0.6112
	Sakakmori & Biloti	1.5026	0.4080	1.8357	0.9478

Table 2: Results for the three proposals. The blue numbers highlights the best agreement with the theoretical parameters.

gather.

As a first step, we analyse the behaviour of the objective function when increasing the number of image gathers used. For this purpose, we evaluate the coherence measure at a grid of parameters γ and m for the correct values of z_0 presented in Table 1. For all values of γ and m, we construct our surface and evaluate the coherence of the samples over it. Figure 4 shows the surface behaviour for the model with reflector dip 20°, where $z_0 = 1.875$, for an increasing number of image gathers. Analysing the semblance values, we observe that for a single image gather, the objective function is a long-stretched valley, but as the number of image gathers increases, the semblance gets more concentrated around the correct values of m and γ (white cross).

Next, we compare the estimation of the model parameters using different numbers of image gathers. In this experiment, we added a stronger random noise with a signal-to-noise ratio of 3 to the data. The results are compiled in Table 3.

Dip	CIGs	γ error [%]	m error [%]	z_0 error [%]	Coherence
10°	1	0.06	1.14	0.01	0.2609
	3	0.29	3.94	0.46	0.2863
	5	0.25	2.17	0.57	0.2855
	7	0.31	1.94	0.96	0.2802
	9	0.13	0.00	0.34	0.2750
	11	0.21	1.26	1.59	0.2286
15°	1	0.07	3.35	0.23	0.2969
	3	42.71	64.58	88.67	0.0133
	5	0.03	0.69	0.10	0.3117
	7	0.03	0.73	0.10	0.3123
	9	0.15	0.33	0.47	0.3159
	11	0.15	0.40	0.48	0.3153
20°	1	0.45	23.49	7.06	0.3576
	3	0.04	0.37	0.47	0.3687
	5	0.04	0.29	0.45	0.3618
	7	0.05	0.35	0.30	0.3599
	9	0.88	11.89	5.73	0.0311
	11	6.27	7.92	5.87	0.0177

Table 3: Error values for the parameters estimation using super-gathers.

For the model of dip 10°, the best agreement with the theoretical parameters is achieved using a single image gather. However, the coherence value is greater using 5 image gathers, indicating that further improvement is possible.

For the 15° dipping reflector, the best γ and z_0 values are estimated using 5 or 7 image gathers. the best

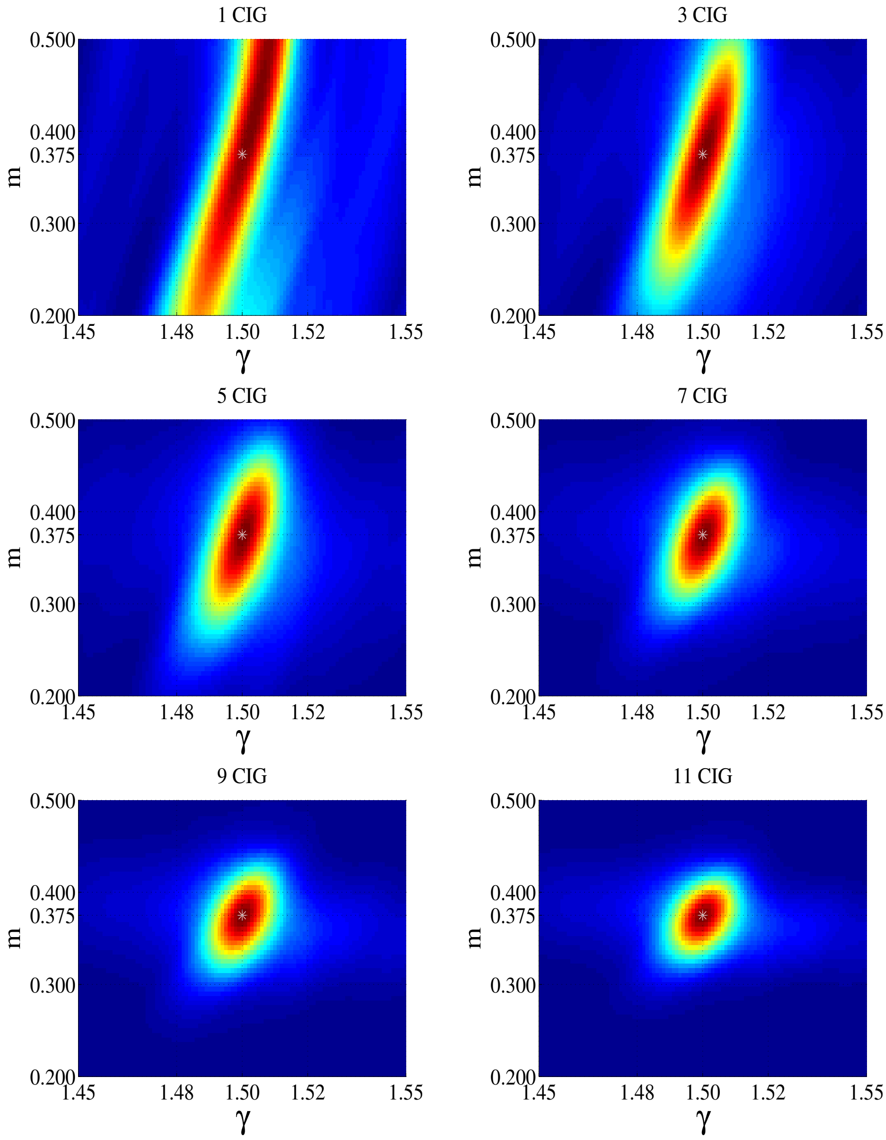


Figure 4: Semblance value for the model of dip 20° for several image gathers, fixing $z_0 = 1.875$. The white cross marks the location of the theoretical parameter values.

m values are obtained from using 9 image gathers, which also produced the highest coherence. Note that for 3 image gathers, the parameters are very poorly estimated, and the coherence value is very low. This unexpected result indicates that the optimization never found the migrated event in the data. As indicated by Figure 4 the objective function cannot be blamed for this behaviour. Therefore, to avoid such situations, the initial values of the optimization method need to be improved. When we tried another initial guess, the method converged and the estimated parameter quality is good.

The improvement in the quality of the estimated parameter values with increasing number of image gathers can be clearly observed in the case of the 20° dipping reflector. Here, the best γ was estimated using 3 or 5 image gathers, the best m using 5 image gathers, and the best z_0 using 7 image gathers. The highest coherence was found for 3 image gathers. Again, for 9 and 11 image gathers we have some problems with the convergence of the optimization method.

These preliminary numerical experiments indicate that the use of several image gathers can be helpful to improve the model-parameter estimation in coherence-based migration velocity analysis. Apparently, the use of about 5 image gathers is sufficient to help the objective function focus at very accurate parameter values. Further tests will be necessary to chose the best optimization method and appropriate initial guesses.

CONCLUSIONS

The residual moveout in a common-image gather after migration with an incorrect velocity is governed by a fifth-order polinomial. In this work, we have proposed to solve this polinomial numerically to describe the moveout in a coherence-based migration velocity analysis. We have validated our numerical expression for the moveout curve and showed that it fits the migrated event better than previously derived approximations.

A major advantage of our numerical description over the approximations available in the literature is that it allows to extend the residual-moveout analysis to neighbouring image gathers. In this way, more information can be used to determine estimates for the velocity model parameters.

Our numerical experiments demonstrate that the use of a few neighbouring image gathers can stabilize and improve the parameter extraction. Further research is necessary to improve the inherent optimization procedure.

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