IMPLEMENTATION ASPECTS OF EIGENSTRUCTURE-BASED VELOCITY SPECTRA

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ABSTRACT

In this paper we discuss high-resolution coherence functions for the estimation of stacking shape parameters in seismic signal processing. We focus on the MUltiple SIgnal Classification (MUSIC) algorithm, which uses the eigendecomposition of the seismic data to measure the coherence. MUSIC can outperform the traditional semblance in cases of close or interfering reflections. Our main contribution is to propose several simplifications to the implementation of MUSIC. Namely, we propose an iterative way to obtain the MUSIC coherence function, called power method MUSIC (PM-MUSIC). We also propose a new way to obtain the MUSIC pseudospectrum, based on the eigendecomposition of the spatial covariance matrix of the seismic data. This is in contrast to the algorithms in the literature, which are based on the temporal covariance. We compared temporal and spatial covariance matrices, implemented with PM-MUSIC, in a synthetic example with two reflections corrupted by additive white gaussian noise. The results show that our implementation, although computationally simpler, provides results comparable to the ones in the literature.

INTRODUCTION

In seismic processing, estimation of the parameters that describe the shape of the wavefronts plays an important role. For stacking, the classical estimation problem is to determine the moveout parameters, namely the stacking velocity and zero-offset traveltime. The most used approach calculates a shape parameters spectra, by computing a coherence measure for different values of these parameters. Then, the parameter that leads to the highest coherence is picked (Taner and Koehler, 1969). The standard coherence function is a second-order energy measure, called semblance (Neidell and Taner, 1971). Semblance is calculated for a given window of N_t samples taken from N_r receivers. Each window is centered on the trajectory defined by the moveout parameters being tested, and consists of a few samples before and after the window center. (The samples may need to be interpolated to form the window.) Let $x(l,i)$ be the l-th sample of the window at i-th receiver, Then, semblance is defined as

$$S_c = \frac{\displaystyle\sum_{l=k-N_t/2}^{l=k+N_t/2} \left| \sum_{i=1}^{N_r} x(l,i) \right|^2}{N_r \displaystyle\sum_{l=k-N_t/2}^{l=k+N_t/2} \sum_{i=1}^{N_r} |x(l,i)|^2}. \tag{1}$$

As discussed in Biondi and Kostov (1989) and Kirlin (1992), eigenstructure methods can lead to velocity spectra with higher resolution, when compared to semblance. One of most commonly used high resolution methods is MUltiple SIgnal Classification (MUSIC), introduced by Schmidt (1986). MUSIC is based in some properties of the eigendecomposition of the seismic data. Recently, MUSIC algorithm has been used in Asgedon et al. (2011) for the estimation of the common-reflection surface (CRS) attributes.

The implementation of MUSIC-based velocity spectra is the main focus of this work. In an attempt to reduce its computational complexity, first, we show that the MUSIC coherence measure can be computed based on a single eigenvector, the one associated to the largest eigenvalue[1]. This results in significant computational savings. Further savings can be obtained by noting that the full eigenvalue decomposition is not required, and the largest eigenvector can be computed efficiently with the power method (Golub and Van Loan, 1996).

Finally, we propose a coherence function based on the eigendecomposition of a matrix whose dimension is lower than the one currently used in the literature. Again, this results in computational savings. However, as a byproduct, the use of this lower-dimensional matrix seems to improve the performance of the method when dealing with correlated wavefronts, which is always the case in seismic data (Asgedon et al., 2011).

WINDOWING

As discussed in the introduction, coherence is computed on a window of data centered at some time $\tau_k(i)$, where k corresponds to a given value of the parameters being tested and i corresponds to the receiver[2]. The value of τ_k depends on some parameters to be estimated. In this paper, we assume a hyperbolic moveout, so that τ_k depends on the normal moveout (NMO) velocity v_k and the two-way zero-offset traveltime τ_0. For each τ_0, the windowed data can be written as a matrix $\mathbf{D}(\theta_k)$, with dimension $N_r \times N_t$, where N_t is the number of samples from the window, N_r is the number of receivers (traces) considered and $\theta_k = 1/v_k$. In Figure 1, we illustrate how the window operation is applied in seismic data. Note, however, that the rows of $\mathbf{D}(\theta_k)$ appear as vertical lines on the right of Figure 1.

The semblance velocity spectrum is computed from this windowed data. Each coherence function is calculated for each pair of parameters (v_k, τ_0), and the velocity spectrum is the coherence measure computed at several windows, for several pairs of parameters. The locations of peaks from the coherence function correspond to pairs of parameters from actual reflections arriving at the receivers.

The hyperbolic windowing can also be used for eigenstructure-based coherence calculation. In Appendix A, we explain the theory behind eigenstructure and MUSIC-based velocity spectra computation. As discussed in this appendix, for each τ_0, different values of θ_k result in different windows, and thus in different data matrices. Thus, to each θ_k corresponds a different temporal covariance matrix, and the ones resulting in large values of MUSIC-based coherence function correspond to velocities of reflection events.

Here, we present an interesting interpretation of MUSIC. Indeed, when a window with correct values of τ_0 and v_k is applied, the windowed data matrix will be represented as in Figure 1. In other words, the data contains several repetitions of the reflection, all arriving at the same instant at all the receivers, plus noise terms. In this case, the data can be written as

$$\mathbf{D}(\theta_k) = \mathbf{1}\mathbf{s}^H + \mathbf{N}. \tag{2}$$

where $\mathbf{s}$ is a $N_r \times 1$ vector that contains the samples from the reflected wavelet, $\mathbf{1}$ is a $N_r \times 1$ vector of ones, $\mathbf{N}$ is an $N_r \times N_t$ noise matrix, which may also contain interfering reflections, and the superscript H refers to the transpose conjugate operation. However, if $\mathbf{N}$ is modeled as an additive white Gaussian noise matrix, the estimated temporal sample covariance[3] matrix is given by

$$\hat{\mathbf{R}}(\theta_k) = \frac{1}{N_t}\mathbf{D}(\theta_k)\mathbf{D}^H(\theta_k) \tag{3}$$

$$\approx \frac{\|\mathbf{s}\|^2}{N_t}\mathbf{1}\mathbf{1}^H + \sigma^2\mathbf{I}, \tag{4}$$

where σ^2 is the noise variance and $\mathbf{I}$ is the identity matrix of appropriate dimension. Note that we disregard the cross terms resulting from $\mathbf{D}(\theta_k)\mathbf{D}^H(\theta_k)$, because we assume that the noise is uncorrelated with the signal.

[1]In order to simplify our explanations, we will commit an abuse of notation and refer to the eigenvector associate to the largest eigenvalue as the largest eigenvector.

[2]For notational convenience, we will drop the explicit reference on the receiver, and use only τ_k to refer to the center of the window.

[3]In this work, we will refer to the estimated sample covariance matrix as covariance matrix.

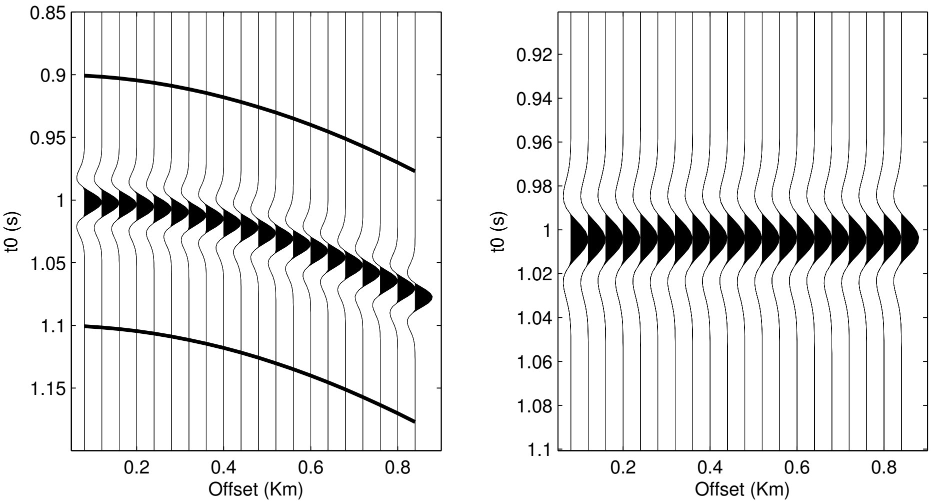


Figure 1: Non windowed data, with superimposed window (left); and windowed data (right).

Clearly, the largest eigenvector of $\hat{\mathbf{R}}(\theta_k)$ is $\mathbf{1}$. Thus, one may interpret MUSIC is an attempt to answer the question: "Is $\mathbf{1}$ the largest eigenvector or $\hat{\mathbf{R}}(\theta_k)$?" If this answer is positive, then we may assume that $\hat{\mathbf{R}}(\theta_k)$ was formed from a window that contains a reflection. The MUSIC *pseudospectrum* is a coherency measure that presents large values when $\mathbf{1}$ is close to the largest eigenvector of $\hat{\mathbf{R}}(\theta_k)$:

$$P_{MU}(\theta_k) = \frac{\mathbf{1}^H\mathbf{1}}{\mathbf{1}^H\mathbf{P}_n(\theta_k)\mathbf{1}}. \tag{5}$$

As discussed in Appendix A, $\mathbf{P}_n(\theta_k)$ in (5) is the projection matrix on the so-called noise subspace, consisting of the subspace spanned by the eigenvectors associated to the smallest eigenvalues of $\mathbf{R}(\theta_k)$.

Some advantages of the windowing operation have been discussed in Kirlin (1992). Perhaps the main advantage, from the point of view of this work, is that, when searching for events with zero delay, the methods apply to both broadband and narrowband signals.

SIGNAL AND NOISE SUBSPACES

To begin the simplification of MUSIC, let $\mathbf{R}(\theta_k) = \mathbf{V}(\theta_k)\mathbf{\Lambda}(\theta_k)\mathbf{V}^H(\theta_k)$ be the eigendecomposition of the covariance matrix. Now, $\mathbf{V}(\theta_k)$ is unitary, so that

$$\mathbf{1}^H\mathbf{V}(\theta_k)\mathbf{V}^H(\theta_k)\mathbf{1} = \mathbf{1}^H\mathbf{1} = N_r. \tag{6}$$

On the other hand, as described in Appendix A, the eigenvectors of the covariance matrix may be separated into those associated to the signal and noise subspaces: $\mathbf{V}(\theta_k) = [\mathbf{V}_s(\theta_k)\,\mathbf{V}_n(\theta_k)]$. Thus

$$\mathbf{1}^H\mathbf{V}(\theta_k)\mathbf{V}^H(\theta_k)\mathbf{1} = \mathbf{1}^H\mathbf{V}_s(\theta_k)\mathbf{V}_s^H(\theta_k)\mathbf{1} + \mathbf{1}^H\mathbf{V}_n(\theta_k)\mathbf{V}_n^H(\theta_k)\mathbf{1}. \tag{7}$$

Now, recall that $\mathbf{P}_n(\theta_k) = \mathbf{V}_n(\theta_k)\mathbf{V}_n^H(\theta_k)$ is the projection matrix onto the noise subspace. If we let $\mathbf{P}_s(\theta_k) = \mathbf{V}_s(\theta_k)\mathbf{V}_s^H(\theta_k)$ be the projection matrix onto the signal subspace, then (7) yields

$$\mathbf{1}^H\mathbf{P}_n(\theta_k)\mathbf{1} + \mathbf{1}^H\mathbf{P}_s(\theta_k)\mathbf{1} = N_r. \tag{8}$$

We can, therefore, modify the MUSIC equation (5), in order to use the signal subspace projection instead of the noise one:

$$P_{MU}(\theta_k) = \frac{N_r}{N_r - \mathbf{1}^H\mathbf{P}_s(\theta_k)\mathbf{1}}. \tag{9}$$

The benefit of using the signal subspace projection is that its dimension is usually smaller than that of the noise subspace. In consequence, computing $\mathbf{1}^H\mathbf{P}_s(\theta_k)\mathbf{1}$ is also simpler than computing $\mathbf{1}^H\mathbf{P}_n(\theta_k)\mathbf{1}$. In fact, in the sequel we discuss this issue in more detail, and show how projecting onto the signal subspace may allow us to compute only the largest eigenvector of the covariance matrix, which can be done efficiently with the power method.

Signal Subspace Dimension

If the windowed data contains more than one wavefront, the signal subspace dimension will be greater than one. As discussed in Kirlin (1992), if the number of wavefronts in the windowed data is greater than one, the vector $\mathbf{1}$ will never be an eigenvector of $\hat{\mathbf{R}}(\theta_k)$. On the other hand, assume that the wavefronts have similar but different parameters. Then, only one wavefront will be flattened by the window based on its parameters, while the other reflections will still appear in the windows, but as slightly incoherent interference. In consequence, we may assume that the dimension of the signal subspace is one, so that $\mathbf{V}_s(\theta_k)$ consists only of the largest eigenvalue of the covariance matrix.

Note that, if the wavefronts are similar, as might happen when there is a multiple, even eigenstructure methods may not resolve the two different events. Eigenstructure-based velocity spectra methods have higher resolution than methods like semblance, which are based on energy measures, but we must know, or estimate, as in Biondi and Kostov (1989), the number of wavefronts. If we overestimate or underestimate the signal subspace dimension, MUSIC coherence method will fail to locate the spots from the right parameters. However, the assumption of a single event in each window seems reasonable, as indicated by simulations.

POWER METHOD

One of the most known methods to calculate the eigenvalues and eigenvectors of a matrix is the power method (Golub and Van Loan, 1996). It starts with an initial vector that is iteratively updated so that, after convergence, it becomes proportional to the largest eigenvectors of the matrix.

As mentioned before, if we use a coherence measure based on the signal subspace, as in equation (9), we only need to estimate the largest eigenvector $\mathbf{v}$ of the covariance matrix. As we suspect that this eigenvector should be $\mathbf{v} = \mathbf{1}$, we initialize the power method with the vector $\mathbf{v}^{(0)} = \mathbf{1}$. For the n-th iteration, the estimated eigenvector will be:

$$\mathbf{v}^{(n)} = \frac{\hat{\mathbf{R}}(\theta_k)\mathbf{v}^{(n-1)}}{||\hat{\mathbf{R}}(\theta_k)\mathbf{v}^{(n-1)}||}. \tag{10}$$

The stop criterion for the power method is based on the difference between consecutive estimates of the eigenvector, $\mathbf{v}^{(n)}$. We say that $\mathbf{v}^{(n)}$ is the desired eigenvector if

$$||\mathbf{v}^{(n)} - \mathbf{v}^{(n-1)}|| < \xi, \tag{11}$$

where ξ is a threshold value that controls the desired precision of the algorithm. In general, the number of iterations n is smaller when the window, for the parameters τ_0 and v_k, fits well the wavefront reflection. This is because, in this case, the largest eigenvector will be close to $\mathbf{1}$, the initial value of the power method. For the right parameters, usually only one iteration is enough for convergence.

The MUSIC *pseudospectrum*, combined with the power method (PM-MUSIC) can be written as:

$$P_{MU}(\theta_k) = \frac{N_r}{N_r - \mathbf{1}^H\hat{\mathbf{v}}\hat{\mathbf{v}}^H\mathbf{1}} = \frac{N_r}{N_r - |\mathbf{1}^H\hat{\mathbf{v}}|^2}, \tag{12}$$

where $\hat{\mathbf{v}}$ is the eigenvector computed by the power method.

PM-MUSIC has lower complexity than the conventional MUSIC from equation (5), as the latter requires a full eigendecomposition. Its complexity, however, is larger than that of semblance. This is illustrated in Table 1, where we show the number of calculations needed for the semblance and PM-MUSIC algorithms. In this table, n represents the number of iterations of the power method. In general we have

$N_r > N_t$ and the calculations in semblance and PM-MUSIC algorithms are dominated by N_r. Disregarding the N_t portion in the algorithms and considering only the number of multiplications from Table 1, we can see that, asymptotically, PM-MUSIC has order $O(N_r^2)$, while semblance has order $O(N_r)$. Note that MUSIC has a complexity of order $O(N_r^3)$, as it needs to compute all N_r eigenvalues and eigenvectors from $\hat{\mathbf{R}}(\theta_k)$.

Table 1: Number of Operations for Semblance and PM-MUSIC Algorithms

Algorithm	Additions	Multiplications
Semblance	$2[(N_r-1)+N_t]$	N_rN_t
PM-MUSIC (temporal)	$n[N_r(N_r-1)+2(N_r-1)+N_r]+(N_r-1)$	$n(N_r^2+2N_r)+N_r$
PM-MUSIC (spatial)	$N_t(N_r-1)+n[N_t(N_t-1)+2(N_t-1)+N_t]+2(N_t-1)$	$N_rN_t+n(N_t^2+2N_t)+2N_t^2$

The extra complexity in PM-MUSIC is the price to pay for its higher resolution. There is also the possibility of reducing N_r by the use of partial stacking, as discussed in Key and Smithson (1990).

THE SPATIAL COVARIANCE MATRIX

The spatial covariance matrix of the data in a window with parameter θ_k can be computed as

$$\mathbf{r}(\theta_k) = E\{\mathbf{D}^H(\theta_k)\mathbf{D}(\theta_k)\}, \tag{13}$$

where the dimension of $\mathbf{r}(\theta_k)$ is $N_t \times N_t$. Recall that N_t is the number of samples in the window, which is usually smaller than the number of receivers, N_r. Thus, the dimension of $\mathbf{r}(\theta_k)$ is usually smaller than $\mathbf{R}(\theta_k)$. As before, the spatial covariance matrix can be estimated as

$$\hat{\mathbf{r}}(\theta_k) = \frac{1}{N_r}\mathbf{D}^H(\theta_k)\mathbf{D}(\theta_k). \tag{14}$$

Now, the temporal correlation matrix is estimated as $\hat{\mathbf{R}}(\theta_k) = \frac{1}{N_r}\mathbf{D}(\theta_k)\mathbf{D}^H(\theta_k)$. Thus, it can be easily shown that both $\hat{\mathbf{r}}(\theta_k)$ and $\hat{\mathbf{R}}(\theta_k)$ have the same non-zero eigenvalues, and the corresponding eigenvectors of $\hat{\mathbf{r}}(\theta_k)$, $\mathbf{u}$, are related to the corresponding eigenvectors of $\hat{\mathbf{R}}(\theta_k)$, $\mathbf{v}$, as $\mathbf{u} = \mathbf{D}^H(\theta_k)\mathbf{v}$. Thus, instead of testing whether the all-ones vector, $\mathbf{1}$, is the largest eigenvector of $\hat{\mathbf{R}}(\theta_k)$, as is done in the usual MUSIC, we may test whether $\mathbf{D}^H(\theta_k)\mathbf{1}$, is the largest eigenvector of $\hat{\mathbf{r}}(\theta_k)$. The advantage is that, in the second case, we have to compute the eigenvectors of a smaller matrix.

As an alternative interpretation, consider, as before, the case of a window with correct parameters, resulting in a data matrix that can be approximated as

$$\mathbf{D}(\theta_k) = \mathbf{1}\mathbf{s}^H + \mathbf{N}. \tag{15}$$

where $\mathbf{s}$ is a vector that contains the samples from the reflected wavelet. In this case, the estimated correlation matrix is given by

$$\begin{aligned} \hat{\mathbf{r}}(\theta_k) &= \frac{1}{N_t}\mathbf{D}^H(\theta_k)\mathbf{D}(\theta_k) & (16) \\ &\approx \mathbf{s}\mathbf{s}^H + \sigma^2\mathbf{I}. & (17) \end{aligned}$$

Clearly, the largest eigenvector of $\hat{\mathbf{r}}$ is $\mathbf{s}$. Unfortunately, the exact wavelet corresponding to a reflection is not know. However, if the window contains a reflection, $\mathbf{1}$ is the largest eigenvector of $\hat{\mathbf{R}}(\theta_k)$, so that $\mathbf{D}^H(\theta_k)\mathbf{1}$ is the largest eigenvector of $\hat{\mathbf{r}}(\theta_k)$. Thus, we may interpret $\mathbf{D}^H(\theta_k)\mathbf{1}$ as an estimate of the wavelet $\mathbf{s}$.

Now, for each window formed from the parameters τ_0 and v_k, we will test, using an eigestructure-based coherence measure, if $\hat{\mathbf{s}} = \mathbf{D}^H(\theta_k)\mathbf{1}$ is an eigenvector from matrix $\hat{\mathbf{r}}(\theta_k)$. The resulting MUSIC *pseudospectrum*, obtained by $\hat{\mathbf{r}}(\theta_k)$, can be written as

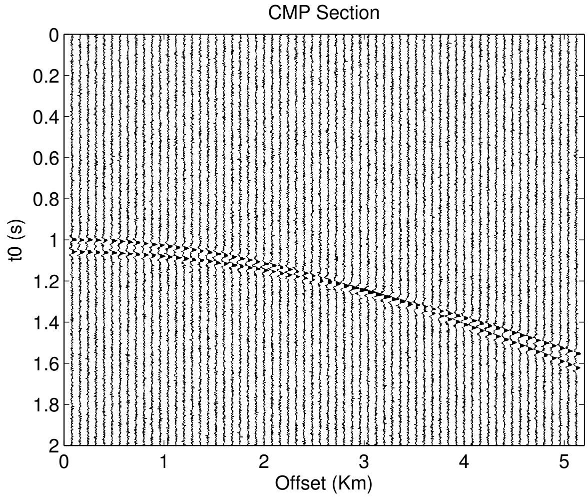


Figure 2: CMP section used in simulations.

$$P_{MU}(\theta_k) = \frac{\hat{\mathbf{s}}^H \hat{\mathbf{s}}}{\hat{\mathbf{s}}^H \hat{\mathbf{s}} - \hat{\mathbf{s}}^H \mathbf{P}_{\hat{r}_s}(\theta_k) \hat{\mathbf{s}}}. \tag{18}$$

where $\mathbf{P}_{\hat{r}_s}(\theta_k)$ is the projection matrix onto the signal subspace of $\hat{\mathbf{r}}(\theta_k)$.

As before, we may assume that the signal subspace of $\hat{\mathbf{r}}(\theta_k)$ is one-dimensional, so that only the largest eigenvalue of $\hat{\mathbf{r}}(\theta_k)$ needs to be computed. This can be done efficiently with the power method. The resulting complexity is also illustrated in Table 1. By doing the same assumptions we done for the other algorithms, we can see that PM-MUSIC, obtained from the spatial covariance matrix, has order $O(N_r)$, which is comparable to semblance.

NUMERICAL EXAMPLES

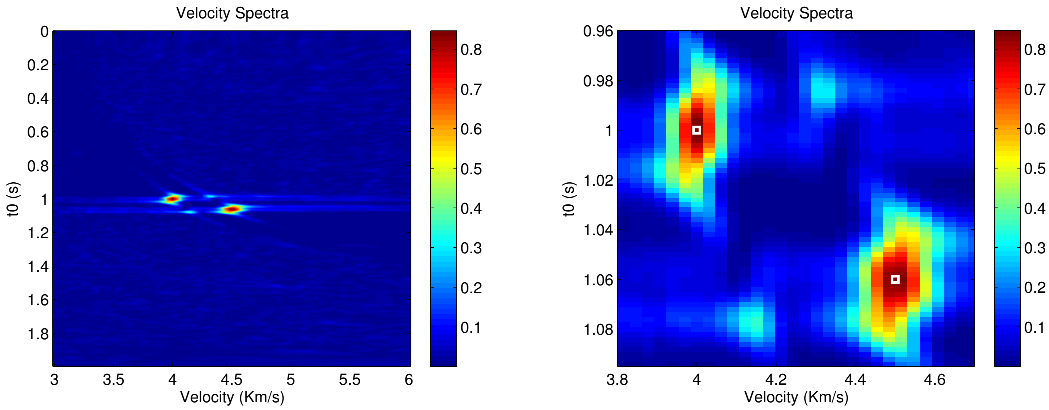
In this section we compare the use of temporal and spatial covariance matrices to obtain high resolution velocity spectra. We will use the semblance coherence function as a benchmark. We also show high resolution velocity spectra obtained by the power method, for both temporal and spatial covariance matrices.

In the simulations, we used a simple synthetic model with two reflections, generated by equation (20) in Appendix A. The first one has a zero offset traveltime of 1 s and a velocity of 4000 m/s; the second one has a zero offset traveltime of 1.06 s and a velocity of 4500 m/s. Both reflections are modeled by a zero-phase Ricker wavelet, with a dominant frequency of 25 Hz and are fully correlated. The CMP section contains 64 receivers. The offset of the first one is 80 m and the distance between them is also 80m. The sample period is 2 ms and white Gaussian noise was added to the data, in order to get a signal to noise ratio (SNR) of 15 dB. The CMP section can be viewed in Figure 2.

Covariance Matrices Comparison

Figures 3, 4 and 5 show velocity spectra calculated with semblance from equation (1) and MUSIC from equations (5) and (18), using temporal and spatial covariance matrices, respectively. The figures show both the full spectra and a zoom into the part of the spectra close to the actual values of the parameters. The window size used was 19 samples and velocities were tested from 3000 m/s to 6000 m/s, in increments of 10 m/s. We have assumed that the signal subspace has rank one, *i.e.*, it is formed by a single signal. For the temporal covariance matrix, we performed spatial smoothing, described in the appendix, using 47 subarrays, each consisting of 18 receivers. We have also used forward-backward averaging, also described in the appendix, in the spatially smoothed temporal covariance matrices to increase its rank.

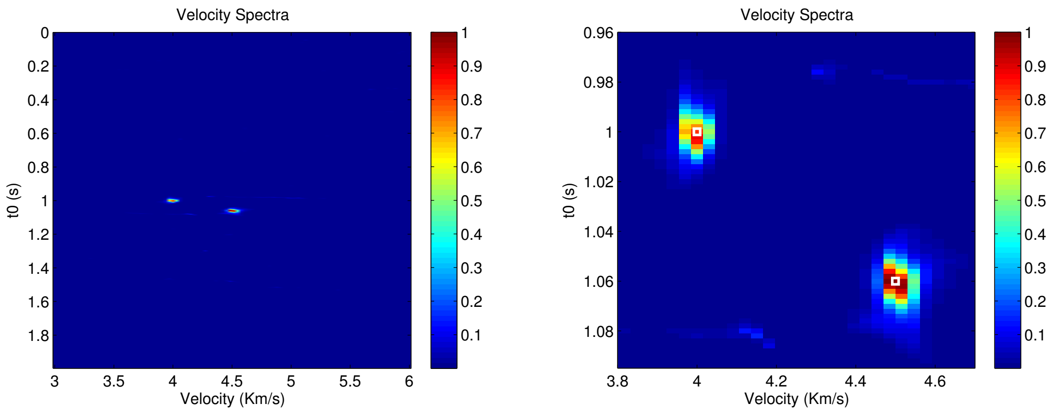
The white squares in the velocity spectra indicate the true location of the parameters, and MUSIC velocity spectra in figures 4 and 5, are normalized with respect to the largest coherence value of the cor-



(a) Semblance velocity spectra.

(b) Semblance velocity spectra: closer view.

Figure 3: Semblance velocity spectra.



(a) Temporal covariance MUSIC velocity spectra.

(b) Temporal covariance MUSIC velocity spectra: closer view.

Figure 4: Temporal covariance MUSIC velocity spectra.

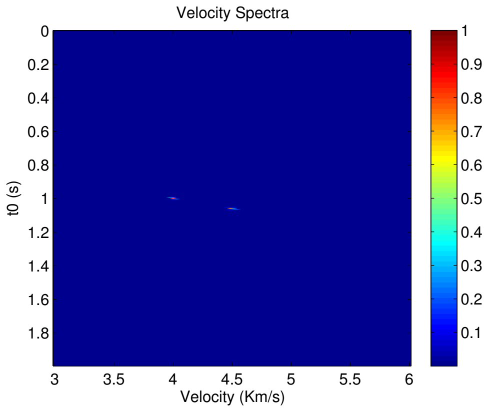
responding spectrum. The results in the figures clearly show that both MUSIC algorithms outperform semblance in terms of resolution, resulting in more precise velocity estimates. They also indicate that MUSIC with spatial correlation may present even better resolution than temporal correlation, despite their lower complexity.

Power Method

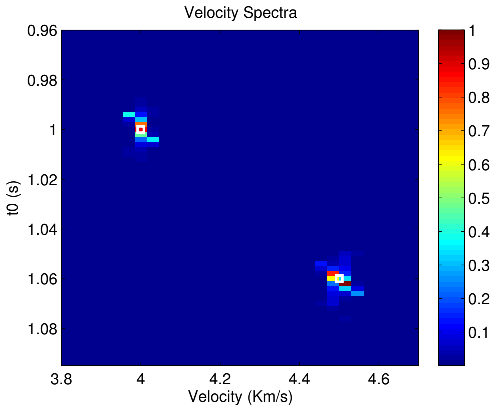
Figures 6 and 7 show, for the same CMP section of the previous example, velocity spectra calculated with PM-MUSIC for temporal and spatial covariance matrices, normalized with respect to the largest coherence value of the corresponding spectrum. Clearly, the use of the power method has no impact on the results, when compared to figures 4 and 5.

In Figure 8, we show the histogram of the number of iterations needed for PM-MUSIC convergence, for both temporal and spatial covariance matrices example. We used $\xi = 0.3$ in the simulations. As seen in figure 8(a), for the temporal covariance matrix, in 84.71% of the cases the power method converged in a single iteration. For the spatial covariance matrix, figure 8(b) shows that PM-MUSIC converged in one iteration in 76.53% of the cases. Both figures illustrate that the power method converges quickly, hardly ever requiring more than three iterations.

In Figure 9, we show images of the number of iterations performed by PM-MUSIC, for each point of the velocity spectra coherency functions. It is possible to see that when we are close to the true shape

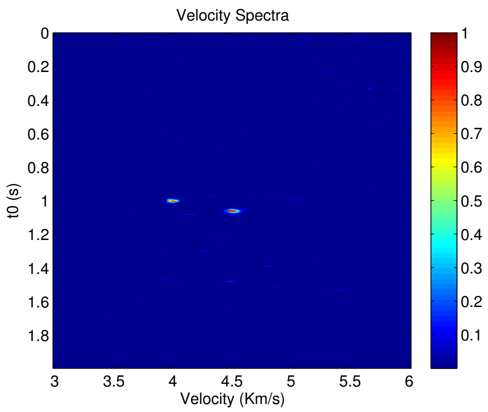


(a) Spatial covariance MUSIC velocity spectra.

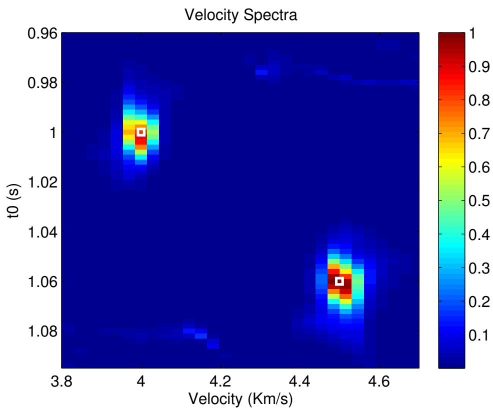


(b) Spatial covariance MUSIC velocity spectra: closer view.

Figure 5: Spatial covariance MUSIC velocity spectra.

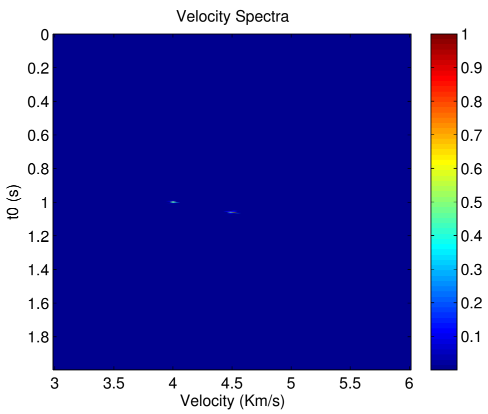


(a) Temporal covariance PM-MUSIC velocity spectra.

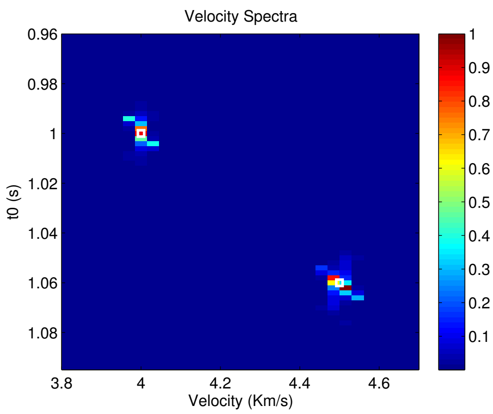


(b) Temporal covariance PM-MUSIC velocity spectra: closer view.

Figure 6: Temporal covariance PM-MUSIC velocity spectra.

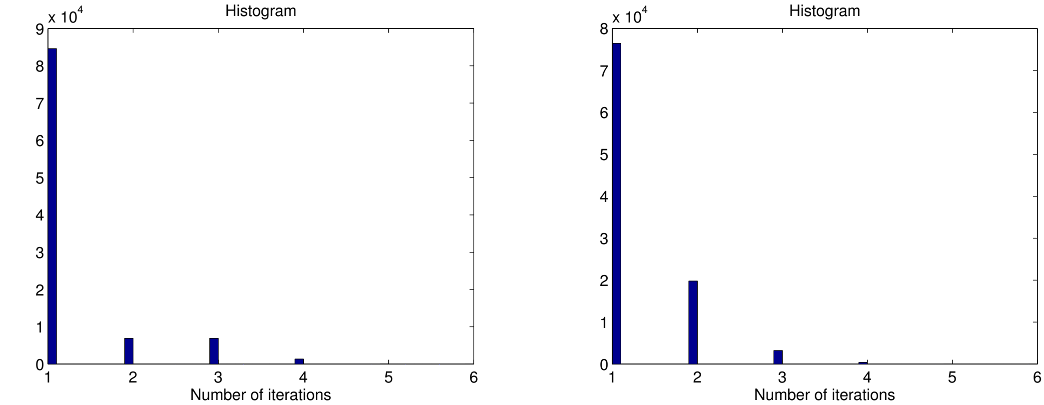


(a) Spatial covariance PM-MUSIC velocity spectra.



(b) Spatial covariance PM-MUSIC velocity spectra: closer view.

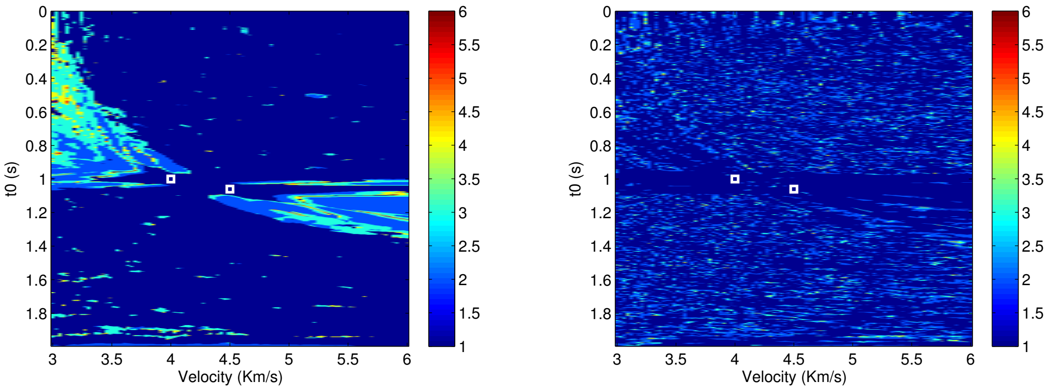
Figure 7: Spatial covariance PM-MUSIC velocity spectra.



(a) PM-MUSIC for temporal covariance matrix.

(b) PM-MUSIC for spatial covariance matrix.

Figure 8: Histogram of number of iterations for convergence of PM-MUSIC applied on temporal and spatial covariance matrices.



(a) PM-MUSIC for temporal covariance matrix.

(b) PM-MUSIC for spatial covariance matrix.

Figure 9: Images with the number of iterations required for convergence of PM-MUSIC.

parameters, the number of iterations is small.

CONCLUSIONS

In this paper we discussed the stacking parameters estimation in seismic signal processing. The focus of our discussion was on the high-resolution method known as MUSIC, which has a better resolution when compared to the classical semblance. In the literature, MUSIC is traditionally based on the eigendecomposition of the temporal covariance matrix, calculated from seismic data. We presented an iterative method called PM-MUSIC, to perform the eigendecomposition and compute the coherence. When compared to MUSIC, PM-MUSIC algorithm presents reduced complexity, due to the fact that it is based on the the iterative estimation of only the eigenvector associated with the largest eigenvalue from the temporal covariance matrix. This is in contrast to MUSIC, which requires the estimation of all the other remaining eigenvectors. We also presented a new way to perform eigenstructure-based velocity spectra calculation, based on eigendecomposition of the spatial covariance matrix.

We have compared, with simulations of a simple synthetic model, high-resolution methods and found that eigenstructure algorithms, based on the spatial covariance matrix, are better than the ones based on the temporal covariance matrix, when dealing with correlated signals. We observed on simulations that, unlike MUSIC with temporal matrices, we do not need to use spatial smoothing of forward-backward averaging to make the signals uncorrelated, when using the spatial covariance matrix.

We also showed, for this example, the impact of the use of the power method to compute the eigenvectors, for both temporal and spatial covariance matrices. The results confirm that the number of iterations needed for its convergence is small and the accuracy of the velocity spectra is not affected by its use.

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REFERENCES

Asgedon, E. G., Gelius, L. G., and Tygel, M. (2011). Higher resolution determination of zero-offset common-reflexion-surface stack parameters. *International Journal of Geophysics*, 2011(10.1155/2011/819831):http://dx.doi.org/10.1155/2011/819831.

Biondi, B. and Kostov, C. (1989). High-resolution velocity spectra using eigenstructures methods. *Geophysics*, 54(7):832–842.

Dix, C. H. (1955). Seismic velocities from surfaces measurements. *Geophysics*, 20(1):68–86.

Golub, B. H. and Van Loan, C. F. (1996). *Matrix Computations*. The Johns Hopkins University Press, Baltimore, 3rd edition.

Key, S. C. and Smithson, S. D. (1990). New approach to seismic-reflection event detection and velocity determination. *Geophysics*, 55(8):1057–1069.

Kirlin, R. L. (1992). The relationship between semblance and eigenstructures velocity estimators. *Geophysics*, 57(8):1027–1033.

Neidell, N. and Taner, M. (1971). Semblance and other coherency measures for multichannel data. *Geophysics*, 36(3):482–497.

Sacchi, M. (1998). A bootstrap procedure for high-resolution velocity analysis. *Geophysics*, 63(5):1716–1725.

Schmidt, R. O. (1986). Multiple emitter location and signal parameter estimation. *IEEE Trans. on Antennas and Propagation*, 34(3):276–280.

Taner, M. and Koehler, F. (1969). Velocity spectra digital computer derivation and applications of velocity functions. *Geophysics*, 34(6):859–881.

Willians, R. T., Prasad, S., Mahalanabis, A. K., and Sibul, L. H. (1988). An improved spatial smoothing technique for bearing estimation in a multipath environment. *IEEE Transactions on Acoustics, Speech and Signal Processing*, 36(4):425–432.

APPENDIX A

EIGENSTRUCTURE-BASED VELOCITY SPECTRA

The seismic data, recorded at the array of N_r receivers can be modeled as a combination of N_s reflections from the wavefront source. At the time instant t and at the i-th receiver, the recorded data can be described as

$$d_i(t) = \sum_{k=1}^{N_s} s_k(t - \tau_k(x_i, v_k)) + n_i(t) \tag{19}$$

where $s_k(t)$ is the k-th observed reflection from the source $s(t)$, $n_i(t)$ is additive noise, uncorrelated with the source reflections, and $\tau_k(x_i, v_k)$ is the time difference, referred to as delays, between the wavefronts arrivals at i-th receiver and the two-way zero-offset traveltime τ_0. Although not explicit from the notation, each reflection is related to different values of τ_0.

When the data is reordered in common-midpoint (CMP) gathers, the delays are approximated by the hyperbolic moveout equation of Dix (1955)

$$\tau_k(x_i, v_k) = \sqrt{\tau_0^2 + \frac{x_i^2}{v_k^2}} - \tau_0 \tag{20}$$

where x_i is the offset between the source and the i-th receiver, and v_k is the stacking velocity of the k-th reflection.

If we assume that the source reflections $s_k(t)$ are narrow-band, with center frequency ω, the delays can be approximated as complex exponentials (Asgedon et al., 2011), so the data can be expressed as

$$d_i(t) = \sum_{k=1}^{N_s} s_k(t) e^{j\omega\tau_k(x_i, v_k)} + n_i(t) \tag{21}$$

In order to simplify data manipulations, equation (21) can be written in matrix notation, considering N_T data time samples

$$\mathbf{D} = \mathbf{A}(\mathbf{\Theta})\mathbf{S} + \mathbf{N} \tag{22}$$

where $\mathbf{D}$ and $\mathbf{N}$ are $N_r \times N_T$ data and noise matrices, $\mathbf{S}$ is an $N_s \times N_T$ source matrix. Note here that data matrix $\mathbf{D}$ consists in a large rectangular window formed by N_T samples, usually much greater than the hyperbolic window formed by N_t samples. $\mathbf{A}(\mathbf{\Theta})$ is an $N_r \times N_s$ matrix formed by the $\mathbf{a}_k(\theta_k)$ steering vectors. These $N_r \times 1$ vectors are formed by the wavefront arrival delays at each receiver

$$\mathbf{a}_k(\theta_k) = \begin{bmatrix} e^{j\omega\tau_k(x_1, \theta_k)} \\ e^{j\omega\tau_k(x_2, \theta_k)} \\ \vdots \\ e^{j\omega\tau_k(x_{N_r}, \theta_k)} \end{bmatrix} \tag{23}$$

where $\theta_k = 1/v_k$.

The eigenstructure-based methods for velocity spectra calculation are based on the temporal covariance matrix of the data matrix. Assuming that the sources are zero-mean random process and the noise is uncorrelated with the source, this covariance matrix can be computed as

$$\mathbf{R} = E\{\mathbf{D}\mathbf{D}^H\} = \mathbf{A}(\mathbf{\Theta})E\{\mathbf{S}\mathbf{S}^H\}\mathbf{A}^H(\mathbf{\Theta}) + E\{\mathbf{N}\mathbf{N}^H\} = \mathbf{A}(\mathbf{\Theta})\mathbf{R}_s\mathbf{A}^H(\mathbf{\Theta}) + \sigma_n^2\mathbf{I} \tag{24}$$

where $E\{\}$ is the expectation operator, the superscript H denotes matrix hermitian, $\mathbf{R}_s$ is the source temporal covariance matrix, σ_n^2 is the noise variance and $\mathbf{I}$ is the identity matrix. Note that $\mathbf{R}$ is an $N_r \times N_r$ matrix.

One of the most popular eigenstructure-based algorithms is MUSIC, which takes into account the eigendecomposition of $\mathbf{R}$. Let $\mathbf{V}_s\mathbf{\Lambda}_s\mathbf{V}_s^H$ be the eigendecomposition of $\mathbf{A}(\mathbf{\Theta})\mathbf{R}_s\mathbf{A}^H(\mathbf{\Theta})$. Assuming that $\mathbf{R}_s$ is nonsingular, with no pair of sources fully correlated, then $\mathbf{\Lambda}_s$ has N_s non-zero eigenvalues and $\mathbf{V}_s$ span the same subspace as $\mathbf{A}(\mathbf{\Theta})$, which is called the signal subspace. In this case, the eigendecomposition of $\mathbf{R}$ can be written as (Kirlin, 1992)

$$\mathbf{R} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^H = \mathbf{V}_s\mathbf{\Lambda}_s\mathbf{V}_s^H + \mathbf{V}_n\mathbf{\Lambda}_n\mathbf{V}_n^H \tag{25}$$

where $\mathbf{\Lambda} = \text{diag}(\lambda_1, \ldots, \lambda_{N_r})$ is a diagonal matrix that contains the eigenvalues of $\mathbf{R}$. We assume that $\lambda_1 \geq \ldots \geq \lambda_{N_r}$. Since $\mathbf{\Lambda}_s$ has N_s non-zero eigenvalues, we see that the smallest eigenvalues of $\mathbf{R}$ are all equal to σ^2. The matrix $\mathbf{V}_n$ spans a space, called the noise subspace, that orthogonal to the image of $\mathbf{A}(\Theta)$. Finally, note that that $\mathbf{V}$ is an unitary matrix that can be decomposed as $\mathbf{V} = [\mathbf{V}_s\, \mathbf{V}_n]$.

Since the noise subspace is orthogonal to the steering vectors that constitute the signal, we may search all the possible steering vectors and measure how orthogonal they are to $\mathbf{V}_n$. Then, we may estimate the parameters $\mathbf{\Theta}$ as those that yield the steering vectors most orthogonal to the noise subspace. This gives rise to the MUSIC *pseudospectrum*, given by

$$P_{MU}(\theta_k) = \frac{\mathbf{a}_k^H(\theta_k)\mathbf{a}_k(\theta_k)}{\mathbf{a}_k^H(\theta_k)\mathbf{P}_n\mathbf{a}_k(\theta_k)} \tag{26}$$

where $\mathbf{a}_k(\theta_k)$ is a candidate steering vector and $\mathbf{P}_n$ is the projection matrix onto the noise subspace, given by $\mathbf{P}_n = \mathbf{V}_n\mathbf{V}_n^H$. When the candidate steering vector and noise the subspaces are orthogonal, the denominator in (26) tends to zero. Thus, large values of $P_{MU}(\theta_k)$ will generally correspond to the actual steering vectors.

The subspace decomposition of the temporal covariance matrix has been widely used in the literature for the calculation of high resolution velocity spectra coherence functions (Biondi and Kostov, 1989; Key and Smithson, 1990; Sacchi, 1998; Asgedon et al., 2011).

In practice, the temporal covariance matrix must be estimated from the data. The usual approach is to use the sample covariance matrix (also called the correlation matrix) as an approximation to the covariance matrix. For that, we subtract the mean from the data and calculate

$$\hat{\mathbf{R}} = \frac{1}{N_t}\mathbf{D}\mathbf{D}^H \tag{27}$$

where $\hat{\mathbf{R}}$ is the correlation matrix.

Correlated Signals

Seismic signals are strongly correlated. If two correlated wavefronts are being analyzed at the same window, the source temporal covariance matrix $\mathbf{R}_s$ will be rank deficient, which will result in a mix between signal and noise subspaces. The eigenstructure methods will then fail to resolve the correlated events in the coherence spectra.

To deal with correlated signals without losing resolution, a method called spatial smoothing in the temporal covariance matrix is used (Biondi and Kostov, 1989). Spatial smoothing consists in dividing the original array of N_r receivers into K overlapping subarrays of $(N_r - K + 1)$ receivers. A temporal covariance matrix $\hat{\mathbf{R}}^k$ can be computed for each subarray. The spatial smoothed temporal covariance matrix, $\hat{\mathbf{R}}^K$, will be the average from the K covariance matrices from each subarray

$$\hat{\mathbf{R}}^K = \frac{1}{K}\sum_{k=1}^{K}\hat{\mathbf{R}}^k \tag{28}$$

When dealing with correlated signals, we can apply into the spatial smoothed temporal covariance matrices an operation to increase its rank, called forward-backward (FB) averaging and defined in Willians et al. (1988).

Spatial smoothing has two main drawbacks. First, it reduces the effective number of sensors in the array from N_r to $(N_r - K + 1)$, reducing the resolution of the eigenstructures methods. It also increases the computational complexity to determine the estimated temporal covariance matrix.