AN INTERPRETATION OF CRS ATTRIBUTES OF TIME-MIGRATED REFLECTIONS

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ABSTRACT

Applied to unmigrated data, the Common-Reflection-Surface (CRS) method is able to produce high-quality stacked sections, as well as useful traveltime parameters for a number of imaging and inversion purposes. One difficulty of the CRS method is the treatment of diffractions, especially when located close to reflections, and also triplications. In these regions, the CRS parameters obtained by coherence-measure procedures become inaccurate. In this context, and also for more general reasons, it might be interesting to investigate the use of the CRS methodology in the time-migrated domain, where diffractions have been collapsed and triplications untangled. The CRS method applied to the time-migrated domain approximates the zero-offset reflection traveltime as a second-order Taylor expansion in image point coordinates, in the vicinity of a given image ray. It is to be remarked the analogy to the conventional CRS application to the unmigrated, poststack domain, in which the zero-offset reflection traveltime is approximated as a second-order Taylor expansion in midpoint coordinates, in the vicinity of a given normal ray. Based on pioneering work of the mid eighties at NORSAR, we use the methodology of surface-to-surface propagator matrices for anisotropic media and obtain expressions that relate reflector dip and curvature to first and second derivatives of the time-migration reflection time with respect to image point coordinates. This provides an interpretation of the CRS coefficients. Besides its intrinsic interest, such quantitative relationships can provide useful constraints for the construction of selected reflectors from interpreted reflection events in the time-migrated domain.

INTRODUCTION

The Common-Reflection-Surface (CRS) stack produces, besides simulated zero-offset sections, also first (slope) and second (curvature) derivatives of zero-offset reflection traveltime with respect to source-receiver midpoint coordinates. As well known, such derivatives are identified as CRS attributes or parameters and are given a physical interpretation, namely: (a) The first derivative is well related to the ray-parameter vector of the normal ray at its emergence point and (b) the second derivative is related to the curvature of the *normal wave* that starts on the normal-incidence point (NIP) of the reflector and is measured also at the normal-ray emergence point.

Time migration, either post-stack or pre-stack, is widespread used in seismic processing to produce initial time-domain images and velocity in a simple and efficient way (Hubral and Krey, 1980; Yilmaz, 2000). Together with its advantages of computational efficiency and robustness with respect to a background velocity model, time migration has the the drawback of producing distorted images, in some cases, even under mild lateral velocity variations (Robein, 2003). An option to overcome the imaging difficulties of time migration is to convert the time-migrated images into depth, which includes, as an obligatory step, the conversion of the time-migrated velocities into a corresponding depth-velocity field.

The theoretical explanation of the time migration procedure has been given in Hubral (1977) by means of the concept of image rays. For an isotropic medium, the image ray starts normally to the measurement

surface and travels down to hit, generally non-normally, the reflector. In this way, it can be seen as a "dual" of a normal ray, which starts, generally non-normally, at the measurement surface and hits normally the reflector. For an anisotropic medium, the duality between the image and normal rays is still valid: in this case, it is not the ray but the slowness vector that is normal to the measurement surface (image ray) and the reflector (normal ray).

Quite recently, Cameron et al. (2006, 2007) unveiled the theoretical relationship between the time-migrated and depth velocity fields and presented algorithms to estimate depth velocities and trace image rays from a given time-migrated velocity field. A modified algorithm for the same purposes has been also presented in Iversen and Tygel (2008). Application of the theory to actual time-to-depth conversion has been presented in Cameron et al. (2008). The above papers show that a depth velocity field and also image rays can be fully constructed from a given time-migrated velocity field. As a consequence, individual time-migrated reflection curves can be readily converted into depth by simply moving time samples along the image ray.

Considering prestack time-migrated data, one can apply the CRS method to obtain, besides a zero-offset data volume, also linear (slope) and quadratic coefficients for zero-offset time-migrated reflections. Such coefficients may be seen as *dual* with respect to their zero-offset unmigrated counterparts. A question that naturally arises is what could be a physical meaning of the time-migrated CRS coefficients. For a given depth velocity model (e.g., a model that has been directly extracted from the time-migrated velocity field) and using the machinery of surface-to-surface propagator matrices (see, e.g., Bortfeld, 1989; Červený, 2001) we provide an interpretation of such coefficients in terms of reflector dip and curvature in depth. Our approach is heavily based on the pioneering work carried out in the eighties by Iversen, Åstebøl and Gjøystdal at NORSAR (see Iversen et al., 1987, 1988). They show that the time-migrated reflection traveltime and its derivative with respect to midpoint determines the reflector dip and, in addition, the second derivatives provides the reflector curvature. Both quantities are evaluated at the point the image ray hits the reflector. Explicit expressions for the reflector dip and curvature are provided in terms of the propagator matrix in ray-centered coordinates of the image ray, assumed to be already derived from the time-migrated velocity field. Here these works are reviewed and suitably modified, to provide the sought-for interpretation of the CRS attributes of time-migrated reflections and to take into account anisotropic media.

PARAMETERS OF ZERO-OFFSET REFLECTIONS IN UNMIGRATED AND MIGRATED TIME DOMAIN

The unmigrated and migrated time domains are both five-dimensional. For common-offset migration these domains can be defined in terms of the coordinates $(\mathbf{x}, \mathbf{h}, T^X)$ and $(\mathbf{m}, \mathbf{h}, T^M)$. Here, $\mathbf{x}$ defines the common midpoint between source and receiver locations, while $\mathbf{h}$ is half of the source-receiver offset. The vector $\mathbf{m}$ specifies a so-called common-image point in the time-migrated domain, i.e., the location of a common-image gather. The coordinates $(\mathbf{x}, \mathbf{h})$ and $(\mathbf{m}, \mathbf{h})$ are curvilinear and are related to respective measurement surfaces Σ^X and Σ^M specified for the unmigrated and migrated domains. For practical reasons, one will in most cases choose the surfaces Σ^X and Σ^M identical; nevertheless, it is ultimate to distinguish between midpoint coordinates $\mathbf{x}$ and image-point coordinates $\mathbf{m}$ along this common surface.

In the unmigrated domain the common-reflection surface has the form $T^X(\mathbf{x}, \mathbf{h})$. It is common to approximate this surface to second order in $\mathbf{x}$ and $\mathbf{h}$, based on CRS coefficients known at some reference location $(\mathbf{x} = \mathbf{x}_0, \mathbf{h} = \mathbf{0})$. In this respect, probably the most "natural" coefficients one can think of are the traveltime derivatives

$$T_0^X = T^X(\mathbf{x}_0, \mathbf{h} = \mathbf{0}) \,, \tag{1}$$

$$\mathbf{p}_0^{Xx} = \left(\frac{\partial T^X}{\partial x_i}(\mathbf{x}_0, \mathbf{h} = \mathbf{0}) \right) \,, \tag{2}$$

$$\mathbf{M}_0^{Xxx} = \left(\frac{\partial^2 T^X}{\partial x_i \partial x_j}(\mathbf{x}_0, \mathbf{h} = \mathbf{0}) \right) \,, \tag{3}$$

$$\mathbf{M}_0^{Xhh} = \left(\frac{\partial^2 T^X}{\partial h_i \partial h_j}(\mathbf{x}_0, \mathbf{h} = \mathbf{0}) \right) \,. \tag{4}$$

These coefficients can be computed directly from the seismic prestack data, without knowledge of the depth-velocity model. For clarity of notation, subscript zero on the lefthand-side entities is generally skipped. Moreover, in this paper we do not consider traveltime derivatives taken with respect to offset or attributes related to such traveltime derivatives. This allows us to use a more simple notation, $\mathbf{p}^X$ and $\mathbf{M}^X$, for the traveltime slope and curvature entities in equations 2 and 3.

In the migrated domain, the common-reflection surface is given by the function $T^M(\mathbf{m}, \mathbf{h})$, and the traveltime parameters corresponding to those in equations 1-4 are,

$$T_0^M = T^M(\mathbf{m}_0, \mathbf{h} = \mathbf{0}) \,, \tag{5}$$

$$\mathbf{p}_0^{Mm} = \left(\frac{\partial T^M}{\partial m_i}(\mathbf{m}_0, \mathbf{h} = \mathbf{0}) \right) \,, \tag{6}$$

$$\mathbf{M}_0^{Mmm} = \left(\frac{\partial^2 T^M}{\partial m_i \partial m_j}(\mathbf{m}_0, \mathbf{h} = \mathbf{0}) \right) \,, \tag{7}$$

$$\mathbf{M}_0^{Mhh} = \left(\frac{\partial^2 T^M}{\partial h_i \partial h_j}(\mathbf{m}_0, \mathbf{h} = \mathbf{0}) \right) \,. \tag{8}$$

Observe that the matrix $\mathbf{M}_0^{Mhh}$ vanishes if the seismic data have been perfectly time-migrated. The parameters subjected to our interest in this paper are those in equations 5-7, which are generally referred to without subscript zero. For further simplification, we write the traveltime slope and curvature parameters in these equations as $\mathbf{p}^M$ and $\mathbf{M}^M$.

PARABOLIC REFLECTION TRAVELTIME IN THE UNMIGRATED DOMAIN

In the following, we find it convenient to consider *parabolic* approximations of traveltime instead of the usual *hyperbolic* approximations. There is no lack of generality in doing so, and moreover, the obtained results are the same for the two approximations. We assume that, for a target reflection in the zero-offset volume, the reflection traveltime in the vicinity of a reference common midpoint $\mathbf{x}_0$, is approximated by the Taylor parabolic polynomial

$$T^X(\mathbf{x}) = T^X(\mathbf{x}_0) + \Delta\mathbf{x}^T \mathbf{p}^X + \frac{1}{2} \Delta\mathbf{x}^T \mathbf{M}^X \Delta\mathbf{x} \,, \tag{9}$$

where $\mathbf{x} = \mathbf{x}_0 + \Delta\mathbf{x}$ and $\mathbf{p}^X$ and $\mathbf{M}^X$ are the first-derivative vector and second-derivative matrix of traveltime with respect to midpoint, both evaluated at $\mathbf{x}_0$. In order to draw attention to the wave-theoretical interpretation of $\mathbf{p}^X$ and $\mathbf{M}^X$, let us for a moment assume that the medium along the measurement surface Σ^X is isotropic and homogeneous. The linear (2D-vector) coefficient, $\mathbf{p}^X$, can then be written as (see, e.g., Spinner, 2007, equation 3.8)

$$\mathbf{p}^X = \frac{\partial T^X}{\partial \mathbf{x}} = \left(\frac{\partial T^X}{\partial x_i} \right) = \frac{2 \sin\theta}{v_0} (\cos\phi, \sin\phi)^T \,, \tag{10}$$

where θ is the emergence angle of the normal ray with respect to the normal to the surface Σ^X, ϕ is the angle between the x_1 axis and the projection of the ray onto Σ^X, and v_0 is the isotropic medium velocity at $\mathbf{x}_0$. In other words, $\mathbf{p}^X$ corresponds to the ray parameter vector of the normal ray emerging at the point $\mathbf{x}_0$. The quadratic (2×2-matrix) coefficient, $\mathbf{M}^X$, has the expression (see, e.g., Spinner, 2007, equation 3.10)

$$\mathbf{M}^X = \frac{\partial^2 T^X}{\partial \mathbf{x}^2} = \left(\frac{\partial^2 T^X}{\partial x_i \partial x_j} \right) = \frac{2}{v_0} \mathbf{H} \mathbf{K}_N \mathbf{H}^T \,, \tag{11}$$

where $\mathbf{K}_N$ is the wavefront curvature matrix of the normal wave (or N-wave; see, e.g., Hubral, 1983) and $\mathbf{H}$ is a matrix describing the transformation

$$\Delta\mathbf{x} = \mathbf{H}\mathbf{q} \,, \tag{12}$$

which relates the $\mathbf{x}$-coordinates and the vector $\mathbf{q}$ of the first two wavefront-orthonormal coordinates of the normal ray at $\mathbf{x}_0$. It is our aim to provide a similar interpretation of the coefficients of a parabolic approximation of reflections in the time-migrated domain.

PARABOLIC REFLECTION TRAVELTIME IN TIME-MIGRATED DOMAIN

As previously indicated, we use the coordinate vectors $\mathbf{m}$, to locate the zero-offset traces in the time-migrated domain. We consider the Taylor parabolic approximation of the time-migrated reflection traveltime of a target reflector, $T^M(\mathbf{m})$, at trace location, $\mathbf{m} = \mathbf{m}_0 + \Delta\mathbf{m}$, in the vicinity of a reference trace $\mathbf{m}_0$. It is given by

$$T^M(\mathbf{m}) = T^M(\mathbf{m}_0) + \Delta\mathbf{m}^T\mathbf{p}^M + \frac{1}{2}\Delta\mathbf{m}^T\mathbf{M}^M\Delta\mathbf{m}\,, \tag{13}$$

in which the linear and quadratic coefficients, $\mathbf{p}^M$ and $\mathbf{M}^M$ are given by

$$\mathbf{p}^M = \frac{\partial T^M}{\partial \mathbf{m}} = \left(\frac{\partial T^M}{\partial m_i}\right) \quad \text{and} \quad \mathbf{M}^M = \frac{\partial^2 T^M}{\partial \mathbf{m}^2} = \left(\frac{\partial^2 T^M}{\partial m_i\,\partial m_j}\right)\,, \tag{14}$$

all derivatives being evaluated at $\mathbf{m} = \mathbf{m}_0$. The above expression can be interpreted as twice the traveltime along the *image ray* from the initial point with coordinate $\mathbf{m}$ to the point where it hits the reflector. The time $T^M(\mathbf{m}_0)$ is twice the traveltime along the central image ray from the reference point to the *image incident point (IIP)*, namely the point where the (central) image ray hits the reflector.

It is our aim to characterize the geometric attributes, namely dip and curvature of the reflector at the point IIP in depth, as functions of the coefficients $\mathbf{p}^M$ and $\mathbf{M}^M$.

PROPAGATOR MATRIX OF THE CENTRAL IMAGE RAY

It is convenient to formulate our problem using the theoretical framework of ray-propagator matrices (see, e.g., Červený, 2001; Iversen, 2006). In this way, we introduce the 4×4 surface-to-surface propagator matrix of the central (downgoing) image ray

$$\boldsymbol{\mathcal{T}} = \begin{pmatrix} \boldsymbol{\mathcal{A}} & \boldsymbol{\mathcal{B}} \\ \boldsymbol{\mathcal{C}} & \boldsymbol{\mathcal{D}} \end{pmatrix}\,, \tag{15}$$

which connects the (known) measurement surface Σ^M, called *anterior* surface, to the (unknown) target reflector, Σ^Z, called *posterior* surface.

Points on the reflector will be specified by 2D orthogonal curvilinear coordinates, $\mathbf{z} = (z_1, z_2)^T$. The initial and final points of the central image ray are specified by the coordinates $\mathbf{m}_0$ and $\mathbf{z}_0$, respectively. With the above considerations, the (one-way) traveltime of a paraxial image ray that joins the point $\mathbf{m} = \mathbf{m}_0 + \Delta\mathbf{m}$ at Σ^M to the point $\mathbf{z} = \mathbf{z}_0 + \Delta\mathbf{z}$ at Σ^Z can be expressed as

$$T(\mathbf{z}, \mathbf{m}) = T(\mathbf{z}_0, \mathbf{m}_0) + \Delta\mathbf{z}^T\mathbf{p}^Z(\mathbf{z}_0) - \Delta\mathbf{m}^T\boldsymbol{\mathcal{B}}^{-1}\Delta\mathbf{z} + \frac{1}{2}\Delta\mathbf{m}^T\boldsymbol{\mathcal{B}}^{-1}\boldsymbol{\mathcal{A}}\Delta\mathbf{m} + \frac{1}{2}\Delta\mathbf{z}^T\boldsymbol{\mathcal{D}}\boldsymbol{\mathcal{B}}^{-1}\Delta\mathbf{z}\,. \tag{16}$$

In the above expression, $\mathbf{p}^Z(\mathbf{z}_0)$ is the projection of the slowness vector of the central ray on the tangent plane to the posterior surface at its end point. Observe that we have used the fact that for an image ray the slowness projection, $\mathbf{p}^M(\mathbf{m}_0)$, on the tangent plane to the anterior surface at its initial point vanishes.

The coordinates $\mathbf{m}$ and $\mathbf{z}$, which refer to the initial and endpoints of a specified paraxial image ray, are connected. To find that connection, we recall the properties of the propagator matrix to write

$$\Delta\mathbf{z} = \boldsymbol{\mathcal{A}}\Delta\mathbf{m} + \boldsymbol{\mathcal{B}}\Delta\mathbf{p}^M = \boldsymbol{\mathcal{A}}\Delta\mathbf{m}\,. \tag{17}$$

Note that $\Delta\mathbf{p}^M = \mathbf{p}^M(\mathbf{m}) - \mathbf{p}^M(\mathbf{m}_0) = \mathbf{0}$, since both the central and paraxial rays are image rays, so that $\mathbf{p}^M(\mathbf{m}) = \mathbf{p}^M(\mathbf{m}_0) = \mathbf{0}$. Now, we express $\Delta\mathbf{z}$ as a second-order expansion of $\Delta\mathbf{m}$,

$$\Delta z_k = \frac{\partial z_k}{\partial m_j}\Delta m_j + \frac{1}{2}\frac{\partial^2 z_k}{\partial m_i \partial m_j}\Delta m_i \Delta m_j\,. \tag{18}$$

We recognize that the first derivative in equation 18 equals the matrix element $\mathcal{A}_{kj}$, while the second derivative equals $\partial\mathcal{A}_{kj}/\partial m_i$. It should be noted that the latter term cannot be computed from standard paraxial ray theory, which is based on first-order expansions of position and slowness. We insert equation

18 into equation 16, while neglecting terms of order three and higher in the resulting traveltime expansion, and we use the property

$$\boldsymbol{\mathcal{A}}^T\boldsymbol{\mathcal{D}} - \boldsymbol{\mathcal{C}}^T\boldsymbol{\mathcal{B}} = \mathbf{I} \tag{19}$$

inherent in the formulation of surface-to-surface ray propagator matrices. The result is

$$T(\mathbf{z},\mathbf{m}) = T(\mathbf{z}_0,\mathbf{m}_0) + \Delta\mathbf{m}^T\boldsymbol{\mathcal{A}}^T\mathbf{p}^Z + \frac{1}{2}\Delta\mathbf{m}^T\left(\boldsymbol{\mathcal{E}} + \boldsymbol{\mathcal{C}}^T\boldsymbol{\mathcal{A}}\right)\Delta\mathbf{m}\,, \tag{20}$$

where the elements of matrix $\boldsymbol{\mathcal{E}}$ are given by

$$\mathcal{E}_{ij} = \frac{\partial^2 z_k}{\partial m_i \partial m_j} p_k^Z = \frac{\partial \mathcal{A}_{kj}}{\partial m_i} p_k^Z\,. \tag{21}$$

We now require that the observed two-way traveltime parameters are consistent with the one-way traveltime parameters simulated by the field of image rays. Comparison of equations 13 and 20 yields

$$T^M(\mathbf{m}_0) = 2T(\mathbf{z}_0,\mathbf{m}_0)\,, \tag{22}$$

$$\mathbf{p}^M = 2\boldsymbol{\mathcal{A}}^T\mathbf{p}^Z\,, \tag{23}$$

$$\mathbf{M}^M = 2\left(\boldsymbol{\mathcal{E}} + \boldsymbol{\mathcal{C}}^T\boldsymbol{\mathcal{A}}\right)\,. \tag{24}$$

WAVEFRONT-ORTHONORMAL LOCAL COORDINATE SYSTEM

As indicated above, we assume that a depth-velocity model is known. As a consequence, the central image ray can be traced into depth, so that the point where it hits the reflector can also be assumed as known. That point, called IIP and specified by the curvilinear coordinate, z_0, is obtained by following the central image ray trajectory until the time $T = T^M/2$ is consumed. Moreover, the slowness vector is normal to the wavefront at the IIP, namely,

$$\hat{\mathbf{p}} = \frac{1}{c}\hat{\mathbf{n}}\,, \tag{25}$$

where c denotes phase velocity.

Following (Červený, 2001), it is helpful to introduce a local wavefront-orthonormal coordinate system (y_1, y_2, y_3) such that the third axis (y_3) is normal to the wavefront. The other two axes can be freely specified so that a right-hand Cartesian system is obtained. Quantities belonging to the wavefront-orthonormal coordinate system will be denoted with a superscript Y. Considering wavefront-orthonormal coordinates, the slowness and ray-velocity vectors are written

$$\hat{\mathbf{p}}^Y = \begin{pmatrix} \mathbf{0} \\ 1/c \end{pmatrix}\,, \qquad \hat{\mathbf{v}}^Y = \begin{pmatrix} \mathbf{v}^Y \\ c \end{pmatrix}\,. \tag{26}$$

Moreover, the vector $\mathbf{v}^Y$ is zero if the medium is isotropic at the actual ray/interface intersection point.

As shown in (Iversen, 2006), the wavefront coordinate system allows for the computation of the submatrix systems $(\mathbf{Q}_E, \mathbf{P}_E)$ and $(\mathbf{Q}_D, \mathbf{P}_D)$, which correspond to hypothetical wavefront solutions initialized, respectively, as an exploding reflector (E) and a point diffractor (D) at the anterior surface. The two solutions form the 4×4 matrix

$$\boldsymbol{\Phi} = \begin{pmatrix} \mathbf{Q}_E & \mathbf{Q}_D \\ \mathbf{P}_E & \mathbf{P}_D \end{pmatrix}\,, \tag{27}$$

which constitutes the surface-to-surface propagator matrix of the central image ray with respect to the given measurement (anterior) surface and the wavefront surface that corresponds to the central image ray at the IIP. We remark that the above-defined propagator matrix is an inherent property of the given central image ray and measurement surface, defined independently of the interface. To account for that interface, we need to introduce an additional local coordinate system. The new coordinate system enables the construction of a so-called *projection matrix*, $\mathbf{Y}$, which embodies the properties of the interface and provides the link between the propagator matrices $\boldsymbol{\mathcal{T}}$ and $\boldsymbol{\Phi}$ in the form

$$\boldsymbol{\mathcal{T}} = \mathbf{Y}\boldsymbol{\Phi}\,. \tag{28}$$

The explicit expression of $\mathbf{Y}$ will be given in the next section.

INTERFACE LOCAL COORDINATE SYSTEM

In accordance with the previous observation, the determination of the sought-for reflector dip and curvature at the point IIP will be obtained with the help of an additional local 3D-Cartesian system, also having its origin at the IIP. This is the *interface-orthonormal coordinate* system, defined such that the third axis is perpendicular to the interface. The remaining axes, are arbitrarily defined so that the system is also right-hand oriented.

We observe that our results will be the same if we consider an orthogonal curvilinear coordinate system for the interface or a continuum of local Cartesian interface-orthonormal systems, with axes tangential to those of the curvilinear coordinate system at every point along the interface. Therefore, to avoid unnecessarily complications of notations in the following, the local interface coordinates are written simply as $\hat{\mathbf{z}} = (\mathbf{z}, z_3)$.

Following Červený (2001) the transformation from wavefront-orthonormal to interface-orthonormal coordinates is described to the first order by the relation

$$\hat{\mathbf{z}} = \hat{\mathbf{G}}\hat{\mathbf{y}} \; . \tag{29}$$

Being a coordinate transformation between orthonormal Cartesian coordinate systems, matrix $\hat{\mathbf{G}}$ is an orthonormal matrix. As such, it satisfies the relations

$$\hat{\mathbf{G}}^{-1} = \hat{\mathbf{G}}^{T} \; , \qquad \hat{\mathbf{G}}^{-1}\hat{\mathbf{G}} = \hat{\mathbf{I}} \; , \qquad \hat{\mathbf{G}}\hat{\mathbf{G}}^{-1} = \hat{\mathbf{I}} \; . \tag{30}$$

Matrix $\hat{\mathbf{G}}$ has as columns the unit vectors of the wavefront-orthonormal coordinate system expressed with respect to the interface coordinate system. The third column of matrix $\hat{\mathbf{G}}$ is therefore a unit vector normal to the wavefront, for which we use the notation $\hat{\mathbf{n}}^{Z}$. Here, superscript (z) signifies that the vector belongs to the interface coordinate system. It is convenient to express vector $\hat{\mathbf{n}}^{Z}$ as a 3×1 column matrix

$$\hat{\mathbf{n}}^{Z} = \begin{pmatrix} \mathbf{n}^{Z} \\ n_3^{Z} \end{pmatrix} \; , \tag{31}$$

where $\mathbf{n}^{Z} = (n_1^{Z}, n_2^{Z})^{T}$ is the two-component vector (2×1 column matrix) of the vector $\hat{\mathbf{n}}$. In the same way, the orthogonality of matrix $\hat{\mathbf{G}}$ (equation 30), implies that the third line of that matrix is given by the transpose of the unit vector normal to the interface

$$\hat{\boldsymbol{\nu}}^{Y} = \begin{pmatrix} \boldsymbol{\nu}^{Y} \\ \nu_3^{Y} \end{pmatrix} \; , \tag{32}$$

where $\boldsymbol{\nu}^{Z} = (\nu_1^{Z}, \nu_2^{Z})^{T}$ is the two-component vector (2×1 column matrix) of the vector $\hat{\boldsymbol{\nu}}$. Note especially that

$$\nu_3^{Y} = n_3^{Z} = G_{33} \; . \tag{33}$$

In accordance with Červený (2001), we let $\mathbf{G}$ denote the 2×2 upper left sub-matrix of matrix $\hat{\mathbf{G}}$. Moreover, we adopt the form of matrix $\hat{\mathbf{G}}$ used by Iversen (2005),

$$\hat{\mathbf{G}} = \begin{pmatrix} \mathbf{G} & \mathbf{n}^{Z} \\ \boldsymbol{\nu}^{Y^{T}} & n_3^{Z} \end{pmatrix} \; . \tag{34}$$

Here, $\boldsymbol{\nu}^{Y}$ is a two-component vector yet to be determined. It belongs to the wavefront-orthonormal coordinate system, as indicated by the superscript (y), and constitutes the projection of the unit interface normal into the tangent plane to the wavefront. Using equation 30 we find that

$$\mathbf{G}^{T}\mathbf{G} + \boldsymbol{\nu}^{Y}\boldsymbol{\nu}^{Y^{T}} = \mathbf{I} \; , \qquad \mathbf{G}\mathbf{G}^{T} + \mathbf{n}^{Z}\mathbf{n}^{Z^{T}} = \mathbf{I} \; , \tag{35}$$

as well as two equivalent results for the vector $\boldsymbol{\nu}^{Y}$,

$$\boldsymbol{\nu}^{Y} = -\mathbf{G}^{T}\frac{\mathbf{n}^{Z}}{n_3^{Z}} \; , \qquad \boldsymbol{\nu}^{Y} = -n_3^{Z}\mathbf{G}^{-1}\mathbf{n}^{Z} \; . \tag{36}$$

With the help of the matrix $\hat{\mathbf{G}}$, we can obtain an explicit expression of the projection matrix, $\mathbf{Y}$, of equation 28. We have

$$\mathbf{Y} = \begin{pmatrix} (\mathbf{G} - \mathbf{A}^{an})^{-T} & \mathbf{0} \\ \left(\mathbf{E} - p_3^Z \mathbf{D}\right)(\mathbf{G} - \mathbf{A}^{an})^{-T} & (\mathbf{G} - \mathbf{A}^{an}) \end{pmatrix} . \tag{37}$$

Here, $\mathbf{D}$ is the *curvature matrix* of the posterior surface. More specifically, in the interface-orthonormal coordinate system the equation of the reflector is

$$z_3 = -\frac{1}{2}\mathbf{z}^T \mathbf{D} \mathbf{z} \, . \tag{38}$$

The 2×2 matrix $\mathbf{E}$ is given by

$$E_{ij} = \frac{1}{c}\left[G_{i3} G_{jm} \eta_m^Y + G_{j3} G_{ik} \eta_k^Y + G_{i3} G_{j3} (\eta_3^Y - \frac{1}{c} \eta_l^Y v_l{}^Y) \right] , \tag{39}$$

and

$$\mathbf{A}^{an} \equiv \mathbf{p}^Z \mathbf{v}^{Y\,T} \tag{40}$$

is the 2×2 *anisotropy matrix*. Both matrices $\mathbf{E}$ and $\mathbf{A}^{an}$ are introduced and described in great detail by Červený (2001). The entities $v_i{}^Y$ and $\eta_i{}^Y$, $i = 1, 2, 3$, are components of the ray-velocity vector $\hat{\mathbf{v}}^Y$ and the slowness-derivative vector $\hat{\boldsymbol{\eta}}^Y = d\hat{\mathbf{p}}^Y/dT$, specified in wavefront-orthonormal coordinates. For isotropic media, we have $\mathbf{A}^{an} = \mathbf{0}$. It is to be observed that in many situations, one can also consider that $\mathbf{E} = \mathbf{0}$. For example, this is the case if the medium is locally homogeneous. Matrix $\mathbf{E}$ is also zero if the slowness vector is normal to the interface.

INTERPRETATION OF CRS COEFFICIENTS OF TIME-MIGRATED REFLECTIONS

Interpretation of the CRS coefficients $\mathbf{p}^M$ and $\mathbf{M}^M$ of the time-migrated traveltime of equation 13, will be given in terms of the of reflector dip and curvature determined by them. Accuracy of results will depend, of course, on the quality of CRS parameter estimation and the given or obtained depth velocity field.

Expressions of the reflector dip and curvature can be readily derived once the propagator matrix, $\boldsymbol{\mathcal{T}}$, is obtained. For that matter, we insert equation 37 into equation 28 to find, after some algebra,

$$\boldsymbol{\mathcal{A}} = (\mathbf{G} - \mathbf{A}^{an})^{-T} \mathbf{Q}_E \, , \tag{41}$$

$$\boldsymbol{\mathcal{C}} = \left(\mathbf{E} - p_3^Z \mathbf{D}\right) \boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{A}}^{-T} \mathbf{Q}_E{}^T \mathbf{P}_E \, , \tag{42}$$

and similarly

$$\boldsymbol{\mathcal{B}} = (\mathbf{G} - \mathbf{A}^{an})^{-T} \mathbf{Q}_D \, , \tag{43}$$

$$\boldsymbol{\mathcal{D}} = \left(\mathbf{E} - p_3^Z \mathbf{D}\right) \boldsymbol{\mathcal{B}} + \boldsymbol{\mathcal{B}}^{-T} \mathbf{Q}_D{}^T \mathbf{P}_D \, . \tag{44}$$

Interpretation of $\mathbf{p}^M$: Estimation of reflector dip

A consequence of equations 33 and 36 is that the projection of the slowness vector into the tangent plane of the interface can be expressed as

$$\mathbf{p}^Z = -\frac{1}{c}\mathbf{G}\mathbf{f} \, , \tag{45}$$

where

$$\mathbf{f} \equiv \frac{\boldsymbol{\nu}^Y}{\nu_3^Y} \, . \tag{46}$$

Vector $\mathbf{f}$ is of particular importance in the following, since it provides all the information required to estimate the normal vector to the reflector. Matrix $\mathbf{G}$, however, needs not be known for this purpose, which will be shown below.

The image-ray field corresponds to an exploding reflector initial condition at the measurement surface in the time-migration domain. We can therefore take $\mathbf{Q}_E$ as the 2×2 geometric spreading matrix belonging to the image-ray wavefront and apply a slightly restated form of equation 41,

$$\boldsymbol{\mathcal{A}}^{-T} = (\mathbf{G} - \mathbf{A}^{an})\, {\mathbf{Q}_E}^{-T} \; . \tag{47}$$

Using equation 45 and the definition of the anisotropy matrix $\mathbf{A}^{an}$ in equation 40, we find that

$$\boldsymbol{\mathcal{A}}^{-T} = \mathbf{G} \left(\mathbf{I} + \mathbf{f} \frac{\mathbf{v}^Y}{v_3^Y} \right) {\mathbf{Q}_E}^{-T} \; . \tag{48}$$

We apply equations 45 and 48 in equation 23, which yields

$$\mathbf{f} = -\frac{c}{2} \left(1 + \frac{1}{2} {\mathbf{v}^Y}^T {\mathbf{Q}_E}^{-T} \mathbf{p}^M \right) {\mathbf{Q}_E}^{-T} \mathbf{p}^M \; , \tag{49}$$

which applies to anisotropic conditions at the point where the image ray hits the reflector. For isotropic conditions, equation 49 reduces to the simple result

$$\mathbf{f} = -\frac{c}{2} {\mathbf{Q}_E}^{-T} \mathbf{p}^M \; . \tag{50}$$

Knowing vector $\mathbf{f}$, we can compute the reflector normal as

$$\hat{\boldsymbol{\nu}}^Y = \frac{1}{\pm\sqrt{1 + \mathbf{f}^T \mathbf{f}}} \begin{pmatrix} \mathbf{f} \\ 1 \end{pmatrix} \; , \tag{51}$$

where a convention for the vector direction must be specified.

Interpretation of $\mathbf{M}^M$: Estimation of reflector curvature

Having estimated the reflector normal vector by equation 51, one can construct matrix $\hat{\mathbf{G}}$ in equation 34 according to any preferred convention. Note especially that it is not necessary to require the local interface coordinate system to be aligned with the plane of incidence. Using matrix $\hat{\mathbf{G}}$ we can compute the anisotropy matrix $\mathbf{A}^{an}$, the projected geometric spreading matrix $\boldsymbol{\mathcal{A}}$, and the inhomogeneity matrix $\mathbf{E}$. On these grounds we can consider all the latter matrices to be known. Now we want to express the curvature matrix of the reflector, $\mathbf{D}$, in terms of the known matrices. For that, we use equation 42 in combination with the condition given in equation 24, which yields the formula

$$\mathbf{D} = \frac{1}{p_3^Z} \left[\boldsymbol{\mathcal{A}}^{-T} \left({\mathbf{Q}_E}^T \mathbf{P}_E + \boldsymbol{\mathcal{E}} - \frac{1}{2} \mathbf{M}^M \right) \boldsymbol{\mathcal{A}}^{-1} + \mathbf{E} \right] \; . \tag{52}$$

Equation 52 is exact. However, since matrix $\boldsymbol{\mathcal{E}}$ can not be computed from conventional dynamic ray tracing along a single image ray, it is tempting to assume that its effect is negligible. In particular, we observe that matrix $\boldsymbol{\mathcal{E}}$ is zero if the slowness vector of the image ray is normal to the reflector. The effect of matrix $\boldsymbol{\mathcal{E}}$ will clearly be quite small as long as the angle between the slowness vector and the reflector normal is also small.

CONCLUSIONS

Application of the CRS method in the time-migrated domain provides, besides a refined time-migrated volume, also linear and quadratic CRS coefficients. These coefficients can be interpreted in terms of the reflector dip and curvature in depth. For a given anisotropic depth-velocity model, the CRS coefficients (or time-migrated CRS parameters) determine the dip and curvature of the reflector at the point the reference image ray hits the reflector. Recent literature has shown how an isotropic depth-velocity model can be constructed directly from the time-migrated velocity field, which is naturally obtained in the pre-stack time-migration process. Potential application of the obtained results can be, for example, the construction of selected reflectors in depth to help setting constraints for velocity-model building. We look forward to seeing further research in this direction.

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