A BRIEF REVIEW OF 3D NIP WAVE TOMOGRAPHY

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ABSTRACT

In this paper, I would like to review the fundamentals of the 3D normal-incidence-point (NIP) wave tomography and to extend my research intention beyond NIP wave tomography. NIP wave tomography is an efficient method to invert for a depth velocity model which is kinematically consistent with the seismic reflection data. However, there still remain some problems to be solved and/or optimized. This includes the efficient handling of the optimization problem itself, but also the inherent second-order limitations of the NIP wave tomography. The goal is to achieve a smooth velocity model sufficiently accurate to allow for accurate depth imaging of reservoir structures.

3D CRS STACK OPERATOR

To do 3D inversion in the way proposed by Duveneck (2004), the starting point will be to prepare the seismic data. The CRS stacking parameters are determined from the seismic prestack data by coherence analysis. A raw prestack dataset is usually very huge and contains various kinds of incoherent and coherent noise. As the CRS stack is looking for coherent events in the prestack data, especially the coherent noise is an issue and should be attenuated as far as possible before the stacking process. The CRS stack method has been initially developed for 2D data (see, e. g., Mann et al., 1999; Jäger et al., 2001). The concepts of this method can be easily extended to the 3D case. The hyperbolic form of the 3D CRS operator can be written as follows:

$$t^2(\mathbf{X}_m, \mathbf{h}) = (t_0 + 2\mathbf{p}\Delta\mathbf{X})^2 + 2t_0(\Delta\mathbf{X}^T\mathbf{N}\Delta\mathbf{X} + \mathbf{h}^T\mathbf{M}\mathbf{h}) \,, \tag{1}$$

where $\mathbf{X}_m$ is the 2D location vector of the source/receiver midpoint on the acquisition surface. $\Delta\mathbf{X} = \mathbf{X}_m - \mathbf{X}_0$ is the midpoint displacement vector with respect to $\mathbf{X}_0$, the location of the zero-offset trace to simulated. Finally, $\mathbf{h}$ is the half-offset vector between source and receiver. This operator is defined by eight independent parameters, here expressed in terms of the kinematic wavefield attributes horizontal slowness vector $\mathbf{p}$ and two matrices of second spatial derivatives, $\mathbf{N}$ and $\mathbf{M}$. Their definition and their relation to geometrical properties can be expressed as follows:

$$\begin{aligned}
2\mathbf{p} &= \left(\frac{\partial t}{\partial x_m}, \frac{\partial t}{\partial y_m}\right)^T = \frac{2}{v_0}(\cos\alpha\sin\theta\sin\alpha\sin\theta) \\
\mathbf{N} &= \begin{pmatrix} \frac{\partial^2 t}{\partial x_m^2} & \frac{\partial^2 t}{\partial x_m \partial y_m} \\ \frac{\partial^2 t}{\partial x_m \partial y_m} & \frac{\partial^2 t}{\partial y_m^2} \end{pmatrix} = \frac{1}{v_0}\mathbf{H}\mathbf{K}_{\mathrm{N}}\mathbf{H}^T \\
\mathbf{M} &= \begin{pmatrix} \frac{\partial^2 t}{\partial h_x^2} & \frac{\partial^2 t}{\partial h_x \partial h_y} \\ \frac{\partial^2 t}{\partial h_y \partial h_x} & \frac{\partial^2 t}{\partial h_y^2} \end{pmatrix} = \frac{1}{v_0}\mathbf{H}\mathbf{K}_{\mathrm{NIP}}\mathbf{H}^T \,,
\end{aligned} \tag{2}$$

where $\mathbf{H}$ describes the transformation from local ray centered Cartesian coordinates to the global Cartesian coordinates system at the normal ray emergence location $\mathbf{x}_m$. v_0 denotes the near-surface velocity. The first derivatives in $\mathbf{p}$ relate to the emergence angle α and the azimuth ϕ of the normal ray. The matrices $\mathbf{M}$ and $\mathbf{N}$ relate to the curvature matrices of two hypothetical wavefronts as observed at $\mathbf{x}_m$: the NIP wave curvature $\mathbf{K}_{\text{NIP}}$ due to a point source at the NIP and the normal wave curvature $\mathbf{K}_{\text{N}}$ (Höcht, 2002).

Furthermore, the matrix $\mathbf{M}$ can be defined in the CMP volume ($\Delta\mathbf{X}_m = \mathbf{0}$), whereas the matrix $\mathbf{N}$ is defined in the ZO volume ($\mathbf{h} = \mathbf{0}$). In practice, we first consider a small aperture in the ZO volume, such that the second order terms can be neglected, meaning that the normal wave matrix $\mathbf{M}$ can be assumed to vanish.

For 2D acquisition, the hyperbolic CRS stack operator in the CMP volume can be reduced by the well-known CMP stack operator (Mayne, 1962) as follows:

$$t^2(h) = t_0^2 + \frac{4h^2}{v_{\text{NMO}}^2}\,, \tag{3}$$

where $v_{\text{NMO}} = v/\cos\alpha$, with α again representing the dip of the reflection event and v the average velocity of the reflector's overburden. Levin (1971) extended this to 3D acquisition. The normal moveout (NMO) velocity here also depends on the source-receiver azimuth angle θ with respect to the normal ray, namely $v_{\text{NMO}} = v/(1 - \sin^2\beta\cos^2\theta)$. For a given azimuth, the relation between the NMO velocity and the CRS attributes is given by

$$v_{\text{NMO}}^{-2} = \frac{t_0}{2}\mathbf{e}_\theta^T\mathbf{M}\mathbf{e}_\theta\,, \tag{4}$$

with the unit vector $\mathbf{e}_\theta$ pointing in the azimuth direction θ.

The parameters of the 3D CRS operator

Subsets of the parameters can be conveniently determined in certain subset of the prestack data: the 3D CRS operator (2) can be decomposed as follows:

$$t^2 = t_0^2 + \mathbf{h}^T\cdot\mathbf{M}\cdot\mathbf{h} \quad \text{for} \quad \Delta\mathbf{X}_m = \mathbf{0}\,, \tag{5}$$

$$t^2 = (t_0^2 + \mathbf{p}\cdot\Delta\mathbf{X}_m)^2 + \Delta\mathbf{X}_m^T\cdot\mathbf{N}\cdot\Delta\mathbf{X}_m \quad \text{for} \quad \mathbf{h} = \mathbf{0}\,. \tag{6}$$

Thus, matrix $\mathbf{M}$ is determined in the CMP volume according to Equation (5), whereas $\mathbf{p}$ and matrix $\mathbf{N}$ can be determined in the ZO volume according to Equation (6). In practice, the problem is further simplified by splitting the the latter problem: within a small ZO aperture, we first determine the horizontal slowness $\mathbf{p}$, followed by a second search in a larger aperture for matrix $\mathbf{N}$.

In practice, the number of relevant and determinable CRS attributes depends on the acquisition geometry and, thus, on the dimension of the prestack data hyper-volume. This ranges from 2D acquisition along a line with a 3D data volume (x_m, h, t) and three CRS attributes via single-azimuth 3D data with a 4D data hyper-volume to the (x_m, y_m, h_x, h_y, t) and eight independent CRS attributes: dip angle α, azimuth angle θ and three independent values in each of the symmetric 2×2 curvatures matrices $\mathbf{K}_{\text{NIP}}$ and $\mathbf{K}_{\text{N}}$, respectively. In principle, all of these eight parameters in 3D CRS operator should be computed simultaneously with a global optimization algorithm. However, this task would require an unacceptable high computational cost. The above described searches in subsets of the data allows to significantly cut this costs, accompanied with some loss of accuracy.

NIP WAVE TOMOGRAPHY

The NIP wave tomography introduced by Duveneck and Hubral (2002) is based on two imaging conditions: one is Snell's law, which is implicitly met by the considered normal rays, the other one is that in a consistent model the hypothetical NIP wave has to focus at zero time and infinite curvature if propagated back into the subsurface. The matrix $\mathbf{M}$ and the horizontal slowness vector $\mathbf{p}$ parameterize the normal ray and the hypothetical NIP wavefront up to second order according to the so-called NIP-wave theorem. The latter states that the (hypothetical) ZO traveltime and the CMP traveltimes actually given in the data coincide up to second order.

For a true diffractor in the subsurface, normal wave and NIP wave coincide, i. e., $\mathbf{M} = \mathbf{N}$. For hypothetical diffractors as used in the NIP wave tomography, an analytic approximation of the diffraction traveltime is obtained by setting $\mathbf{M}$ equal to $\mathbf{N}$ in Equation (1) once all CRS attributes have been determined. This approach has certain advantages:

- We can consider the NIP waves for a discrete number of picked reflection events.
- The picks are independent of each other.
- For convenient and efficient ray tracing, we use a smooth velocity model description.
- There is no need to pick contiguous reflection events in the prestack data hyper-volume.
- Picking is only performed in the stacked ZO volume.

DATA SPACE AND MODEL SPACE

The determination of the velocity distribution in the subsurface is a crucial inversion problem. In order to solve the inverse problem in the case of NIP wave tomography, we are looking for a velocity model and a distribution of NIPs in the subsurface which is consistent with the CRS attributes, locations, and traveltimes extracted from the CRS stack results: the aim of the NIP wave tomography is to minimize the difference between the data forward-modeled from the NIPs to the acquisition surface and the data-derived CRS attributes. The inversion problem is stated as a minimization of a cost function in a least square sense. The inversion problem is solved by local linearization and iterative updating.

Jacobian Matrix for the model update

The model update can be assessed by the Jacobin matrix which is constructed with the perturbation ratio of the relationship between the data components and the model components. The data components consist of the emergence location $\mathbf{x}_m$ of the normal ray, the one-way ZO traveltime $T = t_0/2$, the horizontal slowness vector $\mathbf{p}$ which is related to the first traveltime derivatives, and the NIP matrix $\mathbf{M}$ related to the second traveltime derivatives for each individual picked reflection event. The data components can be represented as follows:

$$\mathbf{d}^{\text{mea}} = (T_i, \mathbf{x}_{m_i}, \mathbf{p}_i, \mathbf{M}_i^{\text{NIP}}), \qquad i = 1, \ldots, n_{\text{data}} \tag{7}$$

In case the seismic acquisition geometry is limited to a narrow azimuth interval around a given azimuth ϕ as in marine seismic acquisition, only the the component $M^{\text{NIP}}_{\phi,\mathbf{x_m}} = \mathbf{e}_\phi^T \mathbf{M}^{\text{NIP}}_{\mathbf{x}_m} \mathbf{e}_\phi$ occurs in the CRS operator and can, thus, be determined from the data.

The model parameters can be expressed by a 3D location vector of each NIP in the general 3D case and, in order to define the local reflector dip at the NIP, two unit vectors or angles have to be included as model parameters for each NIP. Of course, we also need a parameterization of the velocity model in the inverse process. The model parameters can be expressed as follows:

$$\mathbf{k} = \left((x_i, y_i, z_i, e_{x_i}, e_{y_i})^{\text{NIP}}, v_{jkl} \right) \qquad i = 1 \ldots n_{\text{data}},\ j = 1 \ldots n_x,\ k = 1 \ldots n_y,\ l = 1 \ldots n_z\,, \tag{8}$$

where n_x, n_y, n_z are the number of knots in x, y, and z-direction, respectively. In case of a smooth velocity model, the velocity model can be composed by B-splines (de Boor, 1972):

$$v(x, y, z) = \sum_{j=1}^{n_x} \sum_{k=1}^{n_y} \sum_{l=1}^{n_z} v_{jkl} B_j(x) B_k(y) B_l(-z)\,, \tag{9}$$

where v_{jkl} are the B-spline coefficients and B_j form a B-spline basis according to

$$B^{[m]}_{[x_j,\ldots,x_{j+m+1}]}(x) = \frac{x - x_j}{x_{j+m} - x_j} B^{[m-1]}_{[x_j,\ldots,x_{j+m}]}(x) + \frac{x_{j+m+1} - x}{x_{j+m+1} - x_{j+1}} B^{[m-1]}_{[x_{j+1},\ldots,x_{j+m+1}]}(x)\,, \tag{10}$$

where m denotes the degree. The larger m the more the B-spline order increases. On order of $m = 4$ is suited for 2D and 3D velocity models.

With Equations (7) and (8), the inversion process consist of the minimization of the cost function

$$G(\mathbf{k}) = \frac{1}{2}||\mathbf{d}_{\text{mea}} - \mathbf{d}_{\text{mod}}(\mathbf{k})||^2 = \frac{1}{2}\Delta\mathbf{d}^{\mathbf{T}}(\mathbf{k})\mathbf{C}_D^{-1}\Delta\mathbf{d}(\mathbf{k})\,, \tag{11}$$

where $\mathbf{d}_{\text{mod}} = (T_i, \mathbf{x}_{m_i}, \mathbf{p}_i, \mathbf{M}_i^{\text{NIP}})_{\text{mod}}$ represents the forward-model variables for the model specified by Equation (8). $\mathbf{C}_D$ is a weight matrix for the difference between the different data points. The elements of $\mathbf{C}_D$ regarded as a data covariance matrix (Tarantola, 1987) are the weights for t and $\mathbf{x}_m$ due to the the uncertainties of 3D CRS stacking operator parameters. The weight matrix is diagonal matrix, i. e., $(C_D)_{ii} = \sigma_i^2$, owing to the fact that the individual data vectors are not correlated with each other: the meaning of the fact that $\mathbf{C}_D$ is diagonal is that data errors are uncorrelated. In the NIP wave tomography, there are four different data types: traveltime, first traveltime derivatives, second traveltime derivatives, and lateral coordinates with the corresponding weight factors $\sigma_T, \sigma_{\mathbf{M}}, \sigma_{\mathbf{p}}$ and $\sigma_{\mathbf{x}_m}$.

The involved forward model is nonlinear. Thus, the inverse problem which can be minimized according to Equation (11) is nonlinear as well. The direct minimization by means of a global nonlinear optimization method is prohibitively costly. Therefore, the inverse process is addressed in an iterative manner using local linearization: the modeling operator $\mathbf{d}_{\text{mod}}(\mathbf{k})$ is approximated by

$$\mathbf{d}_{mod}(\mathbf{k}_n + \Delta\mathbf{k}) \approx \mathbf{d}_{\text{mea}}(\mathbf{k}_n) + \mathbf{J}\Delta(\mathbf{k})\,, \tag{12}$$

where $\mathbf{J}_{ij} = \partial J_i/\partial m_j$. In the inversion process, this Jacobin matrix or tomographic matrix hast to be determined. It can can be expressed in terms of the partial derivatives of the forward-modeled parameters with respect to the NIP and velocity model parameters as follows:

$$\mathbf{J} = \begin{pmatrix}
\frac{1}{\sigma_T}\left[\frac{\partial T}{\partial x_i^{\text{NIP}}}\right] & \frac{1}{\sigma_T}\left[\frac{\partial T}{\partial y_i^{\text{NIP}}}\right] & \frac{1}{\sigma_T}\left[\frac{\partial T}{\partial z_i^{\text{NIP}}}\right] & \frac{1}{\sigma_T}\left[\frac{\partial T}{\partial e_x^{\text{NIP}}}\right] & \frac{1}{\sigma_T}\left[\frac{\partial T}{\partial e_y^{\text{NIP}}}\right] & \frac{1}{\sigma_T}\left[\frac{\partial T}{\partial v}\right] \\
\frac{1}{\sigma_M}\left[\frac{\partial M}{\partial x_i^{\text{NIP}}}\right] & \frac{1}{\sigma_M}\left[\frac{\partial M}{\partial y_i^{\text{NIP}}}\right] & \frac{1}{\sigma_M}\left[\frac{\partial M}{\partial z_i^{\text{NIP}}}\right] & \frac{1}{\sigma_M}\left[\frac{\partial M}{\partial e_x^{\text{NIP}}}\right] & \frac{1}{\sigma_M}\left[\frac{\partial M}{\partial e_y^{\text{NIP}}}\right] & \frac{1}{\sigma_M}\left[\frac{\partial M}{\partial v}\right] \\
\frac{1}{\sigma_{p_x}}\left[\frac{\partial p_x}{\partial x_i^{\text{NIP}}}\right] & \frac{1}{\sigma_{p_x}}\left[\frac{\partial p_x}{\partial y_i^{\text{NIP}}}\right] & \frac{1}{\sigma_{p_x}}\left[\frac{\partial p_x}{\partial z_i^{\text{NIP}}}\right] & \frac{1}{\sigma_{p_x}}\left[\frac{\partial p_x}{\partial e_x^{\text{NIP}}}\right] & \frac{1}{\sigma_{p_x}}\left[\frac{\partial p_x}{\partial e_y^{\text{NIP}}}\right] & \frac{1}{\sigma_{p_x}}\left[\frac{\partial p_x}{\partial v}\right] \\
\frac{1}{\sigma_{p_y}}\left[\frac{\partial p_y}{\partial x_i^{\text{NIP}}}\right] & \frac{1}{\sigma_{p_y}}\left[\frac{\partial p_y}{\partial y_i^{\text{NIP}}}\right] & \frac{1}{\sigma_{p_y}}\left[\frac{\partial p_y}{\partial z_i^{\text{NIP}}}\right] & \frac{1}{\sigma_{p_y}}\left[\frac{\partial p_y}{\partial e_x^{\text{NIP}}}\right] & \frac{1}{\sigma_{p_y}}\left[\frac{\partial p_y}{\partial e_y^{\text{NIP}}}\right] & \frac{1}{\sigma_{p_y}}\left[\frac{\partial p_y}{\partial v}\right] \\
\frac{1}{\sigma_{\mathbf{x}_x}}\left[\frac{\partial \mathbf{x}_x}{\partial x_i^{\text{NIP}}}\right] & \frac{1}{\sigma_{\mathbf{x}_x}}\left[\frac{\partial \mathbf{x}_x}{\partial y_i^{\text{NIP}}}\right] & \frac{1}{\sigma_{\mathbf{x}_x}}\left[\frac{\partial \mathbf{x}_x}{\partial z_i^{\text{NIP}}}\right] & \frac{1}{\sigma_{\mathbf{x}_x}}\left[\frac{\partial \mathbf{x}_x}{\partial e_x^{\text{NIP}}}\right] & \frac{1}{\sigma_{\mathbf{x}_x}}\left[\frac{\partial \mathbf{x}_x}{\partial e_y^{\text{NIP}}}\right] & \frac{1}{\sigma_{\mathbf{x}_x}}\left[\frac{\partial \mathbf{x}_x}{\partial v}\right] \\
\frac{1}{\sigma_{\mathbf{x}_y}}\left[\frac{\partial \mathbf{x}_y}{\partial x_i^{\text{NIP}}}\right] & \frac{1}{\sigma_{\mathbf{x}_y}}\left[\frac{\partial \mathbf{x}_y}{\partial y_i^{\text{NIP}}}\right] & \frac{1}{\sigma_{\mathbf{x}_y}}\left[\frac{\partial \mathbf{x}_y}{\partial z_i^{\text{NIP}}}\right] & \frac{1}{\sigma_{\mathbf{x}_y}}\left[\frac{\partial \mathbf{x}_y}{\partial e_x^{\text{NIP}}}\right] & \frac{1}{\sigma_{\mathbf{x}_y}}\left[\frac{\partial \mathbf{x}_y}{\partial e_y^{\text{NIP}}}\right] & \frac{1}{\sigma_{\mathbf{x}_y}}\left[\frac{\partial \mathbf{x}_y}{\partial v}\right]
\end{pmatrix}$$

where v denotes the B-spline coefficients. In the 3D case, $v = v_{ijk}$ with i, j, k varying of the number of knots for x, y, and z. The Jacobian matrix is iteratively updated after calculating the misfit by means of LSQR method.

Future directions of research

By the given source points and receiver points from the dataset and the inverted normal rays and NIPs, the scattering angle vector $\mathbf{\Phi}$ and the illumination angle vector $\mathbf{\Psi}$ can be defined instead of half-offset and midpoint coordinates so that the Jacobian Matrix would be minimized in terms of these angles (Klüver, 2007). The 3D CRS operator can be reformulated with respect to these angles as follows:

$$t^2(\mathbf{\Psi}_m, \mathbf{\Phi}) = (t_0 + 2\mathbf{p}\Delta\mathbf{\Psi})^2 + 2t_0(\Delta\mathbf{\Psi}^T\mathbf{N}\Delta\mathbf{\Psi} + \mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi})\,, \tag{13}$$

where $\mathbf{\Psi}_m$ is the illumination angle and $\Delta\mathbf{\Psi} = \mathbf{\Psi}_m - \mathbf{\Psi}_0$, $\mathbf{\Psi}_0$ is the illumination angle relative to the actual reflector dip $\mathbf{\Psi}_0$. $\mathbf{\Phi}$ is the scattering angle vector:

$$\mathbf{\Phi} = \frac{\mathbf{A}_S - \mathbf{A}_G}{2}\,, \tag{14}$$

$$\mathbf{\Psi} = \frac{\mathbf{A}_S + \mathbf{A}_G}{2}\,, \tag{15}$$

where $\mathbf{A}_S$ is the angle between the normal on the reflector and the ray to the source and $\mathbf{A}_G$ is the angle between this normal and the ray to the receiver. According to the change of these angles, the attributes and the traveltime may be represented in the angle domain. This describes a beam which might be named NIP beam. I would like to investigate the traveltime with respect to these angles. The 3D CRS operator can be utilized as the variable of a Gaussian beam which is also used in tomographic methods.

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