DETERMINATION OF TRAVELTIME PARAMETERS IN VTI MEDIA

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ABSTRACT

For modern long-offset acquisition geometries, a hyperbolic traveltime approximation is no longer sufficient to flatten the CMP gather because of medium inhomogeneity or anisotropy. For transversely isotropic media with a vertical symmetry axis (VTI media), just two traveltime parameters are sufficient for performing all time-related processing. Using an estimate of the NMO velocity from a hyperbolic velocity analysis, one can estimate the anisotropic parameter from a more general traveltime approximation. We extend this two-step procedure using a more accurate nonhyperbolicity term in the traveltime approximation. The used traveltime approximations allow to predict the bias in the NMO velocity estimate, thus providing a means of correcting both the estimated NMO velocity and the resulting anisotropy parameter value. By means of a numerical example, we demonstrate that the estimation of both traveltime parameters is improved considerably.

INTRODUCTION

In the seismic reflection method, knowledge of a high-quality velocity model is indispensible, because it plays a key role in the processing and migration of seismic reflection data. Conventional velocity analysis (Dix, 1955; Yilmaz, 1987) by the common-midpoint (CMP) method fits a hyperbolic traveltime approximation to a seismic reflection event in a CMP section. In this procedure, a single traveltime parameter, usually expressed as the normal-moveout (NMO) velocity, is estimated using a measure of the quality of the fit. However, for modern long-offset acquisition geometries, a hyperbolic traveltime approximation is no longer sufficient to flatten the CMP gather because of medium inhomogeneity or anisotropy (Alkhalifah et al., 1996; Toldi et al., 1999). Many authors proposed alternative ideas of how to extract information about the seismic velocities from the data under these conditions. Most of these ideas resort to the migrated domain for velocity extraction, for example using the focusing properties of seismic diffractions (Harlan et al., 1984; Landa and Keydar, 1998; Fomel et al., 2007; Novais et al., 2008) or horizontalization of the common-image gather (Al-Yahya, 1989; Liu and Bleistein, 1995). Based on the latter approach, Schleicher et al. (2008) used image-wave propagation to determine the subsurface velocity model. Even for anisotropic media, there are several methods to obtain information about the velocity model (Tsvankin and Thomsen, 1994; Al-Dajani and Tsvankin, 1998; Sarkar and Tsvankin, 2004; Behera and Tsvankin, 2007).

A particularly important work in this respect is that of Alkhalifah and Tsvankin (1995). They demonstrated that for transversely isotropic media with a vertical symmetry axis (VTI media), just two traveltime parameters are sufficient for performing all time-related processing such as NMO and dip-moveout (DMO) corrections. The two traveltime parameters are usually expressed as the NMO velocity v_{nmo} and the nonhyperbolicity parameter η , a combination of the well-known weak anisotropy parameters ϵ and δ of Thomsen (1986). Using on these parameters, Alkhalifah and Tsvankin (1995) derived a new traveltime approximation based on continued fractions that describes nonhyperbolic traveltimes for larger offsets.

Alkhalifah (1997) showed that using an estimate of v_{nmo} after a hyperbolic velocity analysis, one can estimate the anisotropic parameter η from the more general traveltime approximation of Alkhalifah and

Tsvankin (1995). He proposes a two-step procedure. The first step uses conventional velocity analysis in the CMP gather up to a short offset to estimate v_{nmo} . In the next step, assuming that the estimative of v_{nmo} is sufficiently accurate, he proposes to use farther offsets to estimate the anisotropy parameter η . As a drawback of his method, he noted the strong sensitivity of the η estimates on the quality of the estimated NMO velocity which, in turn, depends on η .

In this paper, we apply the two-step procedure of Alkhalifah (1997) using the new nonhyperbolic traveltime approximations of Schleicher and Aleixo (2008; see also Aleixo and Schleicher, 2009), based on anelliptic approximations (Fomel, 2004) of the VTI traveltime. These traveltime approximations allow to predict the bias in the NMO velocity estimate, thus providing a means of correcting both the estimated NMO velocity and the resulting η value. In this way, the extraction procedure leads to more reliable estimates of v_{nmo} and η .

METHOD

For a homogeneous VTI medium the hyperbolic traveltime approximation is only valid for small offsets, and the velocity coefficient is an NMO velocity that differs from the vertical velocity (Thomsen, 1986). Extending the Taylor series of the traveltime approximation up to fourth order does not extend the validity range significantly (Tsvankin and Thomsen, 1994). However, other types of traveltime approximation can be found that are valid for longer offsets the most famous one being (Tsvankin and Thomsen, 1994; Alkhalifah and Tsvankin, 1995)

$$t^{2}(x) = 1 + x^{2} - \frac{2\eta x^{4}}{1 + (1 + 2\eta)x^{2}}.$$
(1)

Here, we use the normalised half-offset, $x = h/\tau_0 v_{nmo}$, and the normalised traveltime $t(x) = \tau(x)/\tau_0$, where h is half-offset and τ_0 is the zero-offset traveltime. Moreover, the anisotropy parameter is

$$\eta = \frac{\epsilon - \delta}{1 + 2\delta} \tag{2}$$

and the normal-moveout (NMO) velocity

$$V_{nmo} = V_{p0}\sqrt{1+2\delta},\tag{3}$$

where ϵ and δ are Thomsen's (1986) parameters, and V_{p0} is the vertical P-wave velocity.

Alkhalifah (1997) proposed to use a hyperbolic approximation

$$t^2(x) = 1 + x^2 \tag{4}$$

to estimate v_{nmo} by a short-offset conventional velocity analysis. Thereafter, assuming that the estimate of v_{nmo} is sufficiently accurate, the traveltime correction of equation (1) can be used to estimate the anisotropic parameter η . Introducing the notation

$$\Delta t^2 = (1+x^2) - t^2(x) = \frac{2\eta x^4}{1 + (1+2\eta)x^2}$$
(5)

for the traveltime correction of equation (1), η can be obtained at a given normalised half-offset x from

$$\eta = \frac{\Delta t^2 (1+x^2)}{2x^2 (x^2 - \Delta t^2)}.$$
(6)

To measure Δt^2 in the data, Alkhalifah (1997) suggests to apply an NMO correction using v_{nmo} from the first step and then compute $\Delta t^2 = 1 - (t^2(x) - x^2) = 1 - t_{cor}^2$, where t_{cor} corresponds to the moveout traveltime after NMO correction. In other words, Δt^2 measures the residual moveout after a conventional NMO correction. The second quantity needed in equation (6) is the normalised half-offset x. Alkhalifah (1997) showed that the reliability of the estimate increases with increasing offset. Thus, equation (6) should be applied at the farthest offsets available or as a mean over a number of the farthest offsets. Recently, Aleixo and Schleicher (2009) derived a set of new more accurate traveltime approximations in VTI media. These approximations are based on the anelliptic approximation of Fomel (2004) and have the form

$$t^{2}(x) = 1 + \frac{x^{2}}{Q} + B_{i}(\eta) \frac{x^{2}}{1 + x^{2}/Q} , \qquad (7)$$

where $Q = 1 + 2\eta$. For the factor $B_i(\eta)$, they derived five different forms, depending on the actual approximation procedure applied to Fomel's VTI traveltime approximation. These forms are

$$B_1(\eta) = 2\eta/Q, \tag{8}$$

$$B_2(\eta) = 2\eta/(1+\eta)Q,$$
 (9)

$$B_3(\eta) = 2\eta/(1+\eta)^2, \tag{10}$$

$$B_4(\eta) = 2\eta/Q^2, \tag{11}$$

$$B_5(\eta) = 8\eta (1+\eta)/5Q.$$
(12)

Aleixo and Schleicher (2009) demonstrated that while exhibiting a much simpler algebraic form, all these approximations are of similar quality as the original approximation of Fomel (2004).

The aim of this work is to use the traveltime approximations (7) in the two-step procedure of Alkhalifah (1997) to obtain a more accurate estimative for parameter η . The first step of estimating v_{nmo} remains the same as before. The second step needs to be slightly altered due to the different traveltime approximation. To simplify the expressions, we introduce a new traveltime misfit parameter y defined as

$$y = \frac{x^2 - \Delta t^2}{x^2} \,. \tag{13}$$

Here, Δt^2 is determined from the data as described in connection with equation (6). Note that for perfect NMO correction, $\Delta t^2 = 0$ and thus y = 1.

Manipulating equation (7), we can write

$$y = \frac{Q + x^2 + B_i Q^2}{Q^2 + Q x^2}.$$
(14)

Since y depends on η , we can use equation (14) to extract η from measured values of y. Although more sophisticated methods can be thought of, our first approach was to try and invert equation (14) for a direct equation $\eta(y)$. However, since equation (14) is strongly nonlinear in η , direct inversion is impossible. Therefore, we chose to linearise numerator and denominator separately in η before inverting the expression. For B_1 to B_4 this procedure leads to

$$y = \frac{1+2\eta+x^2+2\eta}{1+4\eta+(1+2\eta)x^2},$$
(15)

which leads to the following extraction formula for η :

$$\eta = \frac{(1+x^2)(1-y)}{4y+2x^2y-4} \,. \tag{16}$$

Correspondingly, we obtain for B_5

$$y = \frac{5 + 10\eta + 5x^2 + 8\eta}{5 + 20\eta + (5 + 10\eta)x^2},$$
(17)

which results in the following expression for η :

$$\eta = \frac{5(1+x^2)(1-y)}{20y+10x^2y-18} \,. \tag{18}$$

Using formulas (16) and (18), we can estimate η from the picked traveltime at any chosen offset x using the value of y extracted from the data according to expression (13).

If the estimate of v_{nmo} is precise, these estimates for η are generally of higher accuracy than the ones obtained with equation (6). However, they suffer from the same sensitivity problems already reported by Alkhalifah (1997). This is a severe drawback, since the estimate of v_{nmo} in the first step is already influenced by anisotropy. However, while traveltime approximation (1) does not predict such a behaviour, approximations (7) do. Rewriting equation (7) as

$$t^{2}(x) = 1 + \frac{x^{2}}{Q} + B_{i} \frac{x^{2} + x^{4}/Q - x^{4}/Q}{1 + x^{2}/Q}$$

$$= 1 + \left(B_{i} + \frac{1}{Q}\right) x^{2} - \frac{B_{i}}{Q} \frac{x^{4}}{1 + x^{2}/Q}, \qquad (19)$$

we see that in this description, the short-offset term is already influenced by the presence of η , resulting in an apparent NMO velocity

$$v_{nmo}^{ap} = v_{nmo} / \sqrt{B_i + 1/Q}$$
 (20)

Only B_1 does not predict a dependence of the NMO velocity on the medium nonhyperbolicity η , since we have $B_1 + 1/Q = 1$. However, all other choices of B_i predict such a dependence. Once η is known, equation (20) permits to calculate the true NMO velocity from the apparent one by

$$v_{nmo} = C_i v_{nmo}^{ap} = \sqrt{B_i + 1/Q} v_{nmo}^{ap}$$
 (21)

Note that for B_1 to B_4 , the correction factor satisfies

$$C_{i} = \frac{v_{nmo}}{v_{nmo}^{ap}} = \sqrt{B_{i} + 1/Q} < 1.$$
(22)

We thus expect apparent NMO velocities with $v_{nmo}^{ap} > v_{nmo}$.

Equation (21) has an important consequence. Once η has been estimated, this equation allows to correct the observed NMO velocity. The corrected value of v_{nmo} can then be used to recalculate y and η according to formulas (14) and (16) or (18). This procedure can then be applied iteratively to correct both the estimates of v_{nmo} and η until both values are consistent.

Procedure

The iterative procedure can be summarised in the following steps:

- (1) Use hyperbolic velocity analysis for the shortest offset to determine a first estimate for the apparent NMO velocity v_{nmo}^{ap} .
- (2) Use this NMO velocity estimate together with equation (16) or equation (18) to obtain an estimate for η from the traveltime misfit at the farthest offsets.
- (3) Use the η estimate to correct the apparent NMO velocity for an improved estimate v_{nmo} according to equation (21).
- (4) While the ratio R between the NMO velocities of subsequent iterations still significantly differs from 1, for instance $|1 R| > \epsilon$, go to step (2) using the new v_{nmo} estimate.

After convergence of this iterative procedure, the estimated parameters are the final estimates for η and v_{nmo} .

NUMERICAL EXAMPLES

In this section we present some numerical examples for the proposed procedure for the η extraction. We consider a single-layer homogeneous VTI medium with $v_{nmo} = 2.5 \ km/s$. Figure 1 shows a synthetic seismogram in such a medium for $\eta = 0.3409$, which represents the Greenhorn shale (Jones and Wang, 1981), with random noise with a signal-to-noise ratio of 10. The objective of the test is to flatten this event.



Figure 1: Synthetic seismogram for horizontal reflector at 1 km depth below a homogeneous VTI layer with $\eta = 0.3409$ and $v_{nmo} = 2.5 \ km/s$. Signal-to-noise ratio is 3.

However, to put our numerical tests on a broader basis, we carried out the extraction procedure for a set of models with a single homogeneous VTI layer with the same parameters of the Greenhorn Shale except for a varying η . The value of η varies from 0.01 to 0.5, which covers the range of η observed in practice.

Our first experiment was to extract η using equations (16) and (18) under the assumption that v_{nmo} is known exactly. Of course, for this constant-velocity layer, $v_{nmo} = v = 2.5$ km/s. We have tested the extraction for 50 different values of η between $\eta = 0.01$ and $\eta = 0.5$. Figure 2 shows the results of the η extraction and Figure 3 compares the relative error of the estimates for η with formulas (6) (Tsvankin and Thomsen, 1994), (16), and (18). We see that the extraction using formula (16) is the most accurate one, with a relative error below 1% for the complete range of η . Formula (18) is slightly less accurate, with a error of about 0.25% even for the smallest η and with a maximum error of about 1.25%. The results of the extractions using B_2 to B_4 are not shown here. The fall in the range between B_1 and B_5 . The error of Tsvankin and Thomsen's formula has an error close to zero at the smallest η , but increases much faster with η than the two B_i estimates, reaching a maximum of 3% at $\eta = 0.5$.

The improved values of η in this extraction are a consequence of the better approximation that traveltimes (7) achieve as compared to equation (1). However, this kind of comparison suffers from an important lack of practicality. In practice, the exact value of the NMO velocity is not known a priori. It is necessary to estimate η using the value for v_{nmo} as obtained from a short-offset conventional velocity analysis. Thus, we repeated the above experiment using the estimated apparent NMO velocities. We carried out a conventional hyperbolic velocity analysis in an offset range of $0.1 \le x \le 0.5$. Figure 4 shows the estimated apparent NMO velocity as a function of η . We observe a visible dependence of the estimated NMO velocity on η , varying between 2.5 km/s for very small η and 2.7 km/s for $\eta = 0.5$.

We then used these estimated values of v_{nmo} in the estimation of η . The incorrect estimation of v_{nmo} strongly deteriorates the quality of the η estimates. Figures 5 and 6 show the extracted values and the relative error of the η estimates obtained with formulas (6) (Tsvankin and Thomsen, 1994), (16) (B_1), and



Figure 2: Extraction of η with formulas (6) (Tsvankin and Thomsen, 1994), (16) (i.e., B_1), and (18) (i.e., B_5) using the exact NMO velocity.



Figure 3: Relative error of η extraction with formulas (6) (Tsvankin and Thomsen, 1994), (16) (i.e., B_1), and (18) (i.e., B_5) using the exact NMO velocity.



Figure 4: Apparent NMO velocity, estimated from a short-offset conventional hyperbolic velocity analysis, as a function of η .

(18) (B_5) , using the estimated NMO velocities. We see that the error of the three estimates is of comparable size, reaching about 30%, much larger than the velocity error of less than 10%. This confirms the sensitivity problems of this technique reported by Alkhalifah (1997). We conclude that the error in the estimate of v_{nmo} affects the η estimates much stronger than the choice of the traveltime approximation.

Thus, it turns out that the most important feature of the new traveltime approximations (7) is not their slightly improved quality over equation (1), but their prediction of an η dependence of the apparent NMO velocity. This feature allows for a correction of the NMO velocity value as obtained in a short-offset hyperbolic velocity analysis. Figure 7 shows the values of the apparent NMO velocity v_{nmo}^{ap} as a function of η as predicted using the four values of B_i (i = 1, 2, 3, 4) that describe such an η -dependence. Also shown is the observed trend of Figure 4 (black curve). We observe that the different approximations predict the apparent NMO velocity with different quality. The best approximation for η below 0.1 is the one using B_5 . Up to about 0.2, the B_4 approximation remains closest. The approximation that most closely resembles the observed curve over the full range of tested values of η is the one using B_2 . The B_3 approximation underestimates the η dependence of the NMO velocity.

Using the estimate of η , we can correct the estimate for v_{nmo} according to equation (21). This in turn gives a new estimate for η . We continue this process iteratively until both values are consistent, i.e., until the ratio R between the values of the NMO velocity in two subsequent iterations differs from one by less than $\epsilon = 10^{-4}$. In principle, and depending on the actual value of η , this should be possible with all four possible choices of B_i for which the correction factor is different from one. Because of the fact that B_2 best predicts the bias in the estimation of v_{nmo} for the full range of tested η (see Figure 4), we use only this approximation for the correction procedure.

Figure 8 shows the final corrected values of v_{nmo} after this iterative procedure. We see that the final estimates for v_{nmo} are improved quite considerably as compared to those of Figure 4. Of course, because of the strongly nonlinear behaviour of the apparent NMO velocity, complete correction is impossible. The deviation from the true value of 2.5 km/s is largest at an η of about 0.2, with a relative error of about 3%.



Figure 5: Estimated values of η when using the estimated v_{nmo} from a short-offset hyperbolic velocity analysis. Also shown are the true η values (dashed line).



Figure 6: Error of the estimated values of η when using the estimated v_{nmo} from a short-offset hyperbolic velocity analysis.



Figure 7: Prediction of apparent v_{nmo} according to equation (19) using four different choices of B_i . Also shown is the observed trend of Figure 4 (black curve).

Correspondingly, Figures 9 and 10 show the extracted values and the relative error of the resulting final η estimate after the iterative procedure. As we can see in these figures, the extracted values of η have been considerably improved and the resulting error of the η estimates has been reduced significantly in comparison to Figure 6, except in the range of very small values of η . The errors are below 20% for all η values above 0.03 and below 10% almost everywhere except in the range around $\eta = 0.2$ where the apparent NMO velocity has the most nonlinear behaviour. Around $\eta = 0.4$, the error becomes close to zero. This confirms that the iterative procedure to correct v_{nmo} and η based on the traveltime approximations of Aleixo and Schleicher (2009) helps to reduce the sensitivity of the η extraction procedure of Alkhalifah (1997).

Finally, let us see how the extracted traveltime parameters act when trying to apply a nonhyperbolic NMO correction. Figure 11 depicts the synthetic data section of Figure 1 after nonhyperbolic NMO correction using the described iterative procedure for the extraction of v_{nmo} and η . The extracted values for these data where $v_{nmo} = 2.5162$ km/s and $\eta = 0.3285$. We see that the nonhyperbolic reflection event has been nicely flattened. Moreover, the extracted values of v_{nmo} and η are quite close to the true values, with acceptable errors of 0.5% and -3.6%, respectively.

CONCLUSIONS

In this work, we have refined the technique of Alkhalifah (1997) to compute the anisotropic parameter η . The technique consists of a conventional velocity analysis for short offsets plus a calculation of η based on the nonhyperbolicity term, assuming that an accurate value for the NMO velocity has been obtained. In our refined version, we have replaced the nonhyperbolicity term from the traveltime approximation derived by Tsvankin and Thomsen (1994) by those of the more accurate ones of Aleixo and Schleicher (2009).

We have seen that the new traveltime approximations make the η extraction more precise if the NMO velocity is known exactly. However, the general problem of the technique is its high sensitivity to errors in



Figure 8: Estimates of v_{nmo} after correction according to equation (19) with B_2 of equation (9).

the estimate of the NMO velocity (Alkhalifah, 1997). This generally leads to large errors of the η estimates. The traveltime approximations of Aleixo and Schleicher (2009) pave the way to overcome this problem. They allow to predict the bias in the NMO velocity estimate in dependence on η , thus providing a means of correcting both the estimated NMO velocity and the resulting η value in an iterative procedure. By means of a numerical example, we have demonstrated the improvement in the estimation of v_{nmo} and η that can be achieved in this way.

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Figure 9: Final estimated values of η after iterative correction of v_{nmo} and η . Also shown are the true η values (dashed line).



Figure 10: Relative error of the final estimated values of η after iterative correction of v_{nmo} and η .



Figure 11: Section of Figure 1 after nonhyperbolic NMO-correction with final estimated values for v_{nmo} and η .

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