AVO AND AVA TUNING AND STRETCH EFFECTS

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ABSTRACT

The primary PP, SS or PS reflection response from a thin layer embedded in a homogeneous medium is approximately given by the delayed derivative of the wavelet times the reflection coefficient times a tuning amplitude factor. This results in an amplitude decrease for a common-offset image gather and an amplitude increase for a common-angle image gather. The alignment of a common-image gather to the normal-incidence reflection results in a wavelet stretch for a common-offset image gather and a wavelet squeeze for a common-angle image gather.
For a reflection from a single interface there are the same stretch and squeeze effects, but there is no amplitude effect. The common-angle image gathers have higher resolution than the common-offset gathers, and they can be used directly in an inversion scheme for elastic parameters.

INTRODUCTION

A number of applications in reservoir characterization and production is based on the analysis of the amplitudes of a primary reflection as a function of offset, a technique referred as AVO analysis. After suitable correction for the offset-dependent geometrical spreading, and also disregarding transmission effects, the so-called *true amplitudes* provide a measure of the reflection coefficient as a function of offset. A further transformation of offset into incidence angle provides, then, the reflection coefficient as a function of angle (AVA). A simple offset-to-angle transformation is described in Appendix A. Under the use of available approximations of the angle-dependent reflection coefficient (as a function of velocity and density contrasts or appropriate combinations of these parameters), useful lithology and fluid information can be estimated.

In the case the single target interface is replaced by a composite of one or more thin layers, a similar procedure can be performed once the seismic response of the layer composite is adequately formulated. For a single thin acoustic layer embedded in a homogeneous acoustic background of lower impedance and in the zero-offset situation, Widess (1973) described the behavior of the reflection amplitudes as a function of the layer thickness and the dominant wavelength the wavelet. Considering a zero-phase cosine wavelet, Widess (1973) reports that maximum constructive interference occurs when the layer thickness is one-quarter of the dominant wavelength. When the layer thickness reduces to one-eight of that wavelength (called by Widess the *theoretical threshold of resolution*), the composite response has the shape of the derivative of the incident wavelet. Corresponding results for the case of a thin layer between two different homogeneous media were given in Chung and Lawton (1995). Lin and Phair (1993) extended the results of Widess (1973) to the offset domain, showing also examples of improved images obtained under the proposed tuning corrections. Finally, Bakke and Ursin (1998) provided unified tuning corrections that are valid for general offsets and arbitrary seismic wavelets beyond the dominant-wavelength approximation. These expressions agree with the ones of Widess (1973) and Lin and Phair (1993) in the case of a cosine wavelet. A discussion on the literature on tuning effects in AVO is also provided in Bakke and Ursin (1998).

Significant advantages can be gained when amplitudes (namely, reflection coefficients) are extracted from the data as a function of incidence angle (AVA) instead of offset (AVO). Ray-based imaging in the angle domain tend to be more reliable as they suffer less from multi-pathing and other artifacts, that are

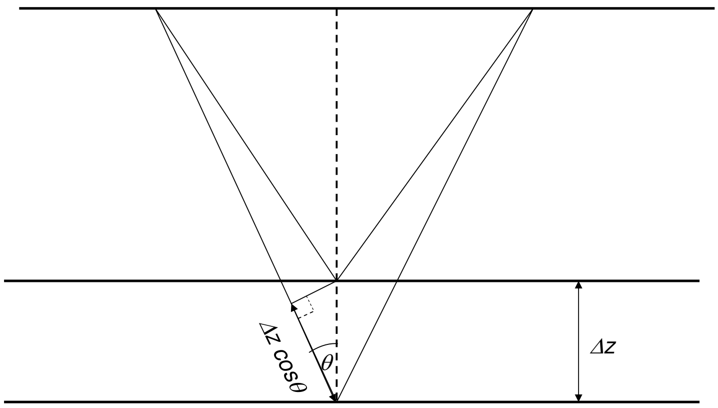

Figure 1: Geometry of a common-offset image gather.

present in the offset domain (Brandsberg-Dahl et al., 2003). Migration directly into angle and depth coordinates with 3-D corrections for geometrical spreading results in gathers which can be used directly for AVA analysis and inversion for elastic parameters. As shown, for example in Ursin et al. (2005a,b), angle-domain common-image gathers can be used very effectively for imaging and tomography applications. In this way, there is a motivation to extend present results on tuning amplitude and wavelet stretch effects from offset (AVO) to angle domain (AVA).

Here we show that simple modifications in the AVO expressions of Bakke and Ursin (1998) provide corresponding expressions for AVA. The thin-layer tuning effect results in reduced amplitude in a common-offset image gather and increased amplitude in a common-angle gather. The alignment of traces to the zero-offset or normal-incidence trace results in a wavelet stretch for a common-offset gather and a wavelet squeeze for a common-angle gather.

The obtained results are in accordance with the behavior of common-image gathers from the Jotun data example presented by Sollid and Ursin (2003), which show that common-angle image gathers have higher resolution than common-offset image gathers.

TUNING EFFECTS FOR A COMMON-OFFSET GATHER

We consider a thin layer imbedded in a homogeneous background. We assume isotropic elastic media both in the thin layer and the background. In principle, both media can be transversely isotropic (VTI). In this situation, the reflection coefficients at the top and bottom layer interfaces satisfy (Stovas and Ursin, 2003)

$$R_t(\theta) = -R_b(\theta) = R(\theta). \tag{1}$$

where t denotes “top", and b denotes “bottom". We assume straight rays for the bottom reflector, and also that the reflection angles at the top and bottom reflectors are the same, denoted by θ. Both these assumptions are justified by the thin-layer condition. In the following, we shall use, besides $R(\theta)$, also the notation $R(x)$, when offset, x, instead of angle, $\theta = \theta(x)$, is being considered.

Referring to Figure 1, we consider PP- or SS-primary reflections from top and bottom layer interfaces for a source-receiver pair at a given fixed offset, x. We assume that their amplitudes have been corrected for offset-dependent geometrical spreading (Ursin, 1990) and also that transmission losses and intra-bed multiples can be neglected. In the zero-order ray approximation, the top and bottom layer primaries, $p_t(x,t)$ and $p_b(x,t)$, where x is offset and t is time, can be written

$$p_t(x,t) = R(x)s(t - T(x)) \tag{2}$$

and

$$p_b(x,t) = -R(x)s(t - T(x) - \Delta T(x)), \tag{3}$$

where $T(x)$ is the traveltime to the top interface, $\Delta T(x)$ the traveltime across the layer and $s(t)$ denotes the source wavelet. The thin layer assumption implies that $\Delta T(x)$ is small, as compared with $T(x)$. Following Bakke and Ursin (1998), the composite response, $p_c(x,t)$, of the two primaries can be conveniently approximated as

$$\begin{aligned} p_c(x,t) &= p_t(x,t) + p_b(x,t) = R(x)[s(t - T(x)) - s(t - T(x) - \Delta T(x)] \\ &\approx R(x)\,\Delta T(x) s'(t - \Delta T(x)), \end{aligned} \tag{4}$$

where $s'(t)$ is the derivative of $s(t)$.

In accordance with Figure 1, the traveltime difference for reflection primaries at top and bottom interfaces (also called *differential traveltime*), $\Delta T(x)$, is given by (Lin and Phair, 1993; Bakke and Ursin, 1998)

$$\Delta T(x) = \frac{2\Delta z}{v} \cos\theta(x) = \Delta T(0)\cos\theta(x), \tag{5}$$

in which Δz denotes the layer thickness, $\Delta T(0)$ is the vertical traveltime and v is the velocity within the layer. Substitution into equation (4) yields the approximation

$$p_c(x,t) = [R(x)\Delta T(0)\cos\theta(x)]\ s'(t - T(x)). \tag{6}$$

We see that the impact of tuning on the single-layer situation can be as described by Bakke and Ursin (1998):

(a) The tuning amplitude is $R(x)\Delta(x) = R(x)\Delta T(0)\cos\theta(x)$ instead of simply $R(x)$ for a single interface;

(b) The tuning wavelet is $s'(t)$ instead of simply $s(t)$ for a single interface.

The tuning behavior of amplitude has the important implication for AVO, namely that under tuning conditions, there is *a decrease in amplitude* with offset as compared with the single interface situation.

TUNING EFFECTS FOR A COMMON-ANGLE GATHER

We now refer to Figure 2, which depicts the common-angle gather situation, as encountered in common-angle migration (see, e.g., Sollid and Ursin, 2003). We again assume straight rays for the reflection from the bottom layer, and now the differential traveltime, $\Delta T(\theta)$, is given by

$$\Delta T(\theta) = \frac{2\Delta z}{v\cos\theta} = \Delta T(0)\sec\theta\ . \tag{7}$$

Substituting into equation (4) we obtain (compare with the expression (6) for the case of a common-offset gather)

$$p_c(\theta,t) = [R(\theta)\Delta T(0)\sec\theta]\ s'(t - T(\theta)). \tag{8}$$

Equation (8) has the following important consequence for AVA purposes: For AVA analysis derived from common-angle gathers, there is *an increase in amplitude* with incident angle as compared with the single interface situation.

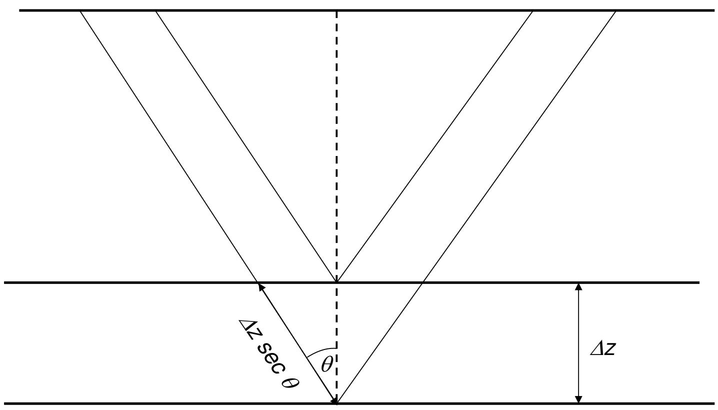

Figure 2: Geometry for a common-angle image gather.

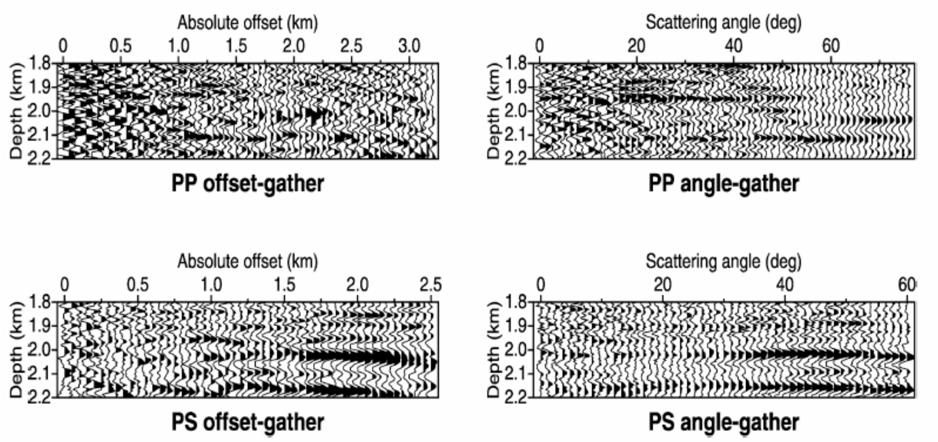

Figure 3: Comparison between common-offset Kirchhoff migration and scattering-angle migration. The Top Heimdal event is at about 2.1 km depth. This is Figure 14 of Sollid and Ursin (2003).

WAVELET STRETCH EFFECTS

We consider stretch effects that are observed in $p_c(x,t)$ (common-offset gathers) and $p_c(\theta,t)$ (common-angle gathers) due to traveltime corrections. For common-offset gathers, the stretch factor is

$$S(x) = \frac{\Delta T_{new}(x)}{\Delta T_{old}(x)} = \frac{\Delta T(0)}{\Delta T(x)} = \frac{1}{\cos\theta(x)} > 1, \tag{9}$$

which means a wavelet stretch increase. For common-angle gathers, the stretch factor is

$$S(\theta) = \frac{\Delta T_{new}(\theta)}{\Delta T_{old}(\theta)} = \frac{\Delta T(0)}{\Delta T(\theta)} = \cos\theta < 1 \ , \tag{10}$$

which means a wavelet squeeze.

These stretch effects will influence the wavelet for a simple reflection, and also the composite wavelet from a thin layer. In Appendix B it is shown that the PS-reflected wave shows a similar effect. The wavelet stretch effects are clearly seen in the Jotum data example from Sollid and Ursin (2003), shown in Figure 3. It is evident that the wavelets in the common-offset image gathers to the left, are much broader than the ones for the common-angle image gathers to the right, both for the PP and PS reflections.

CONCLUSIONS

The composite reflection from a thin layer can be approximated by the reflection coefficient times the derivative of the wavelet together with an amplitude change. For a common-offset image gather, there is an amplitude decrease and a wavelet stretch. For a common-angle image gather, there is an amplitude increase and a wavelet squeeze.

For a simple reflection, the stretch and squeeze effects are the same, but there is no amplitude change. These effects have been observed on pre-stack depth migrated data, and they explain why the common-angle image gathers have better resolution then the common-offset image gathers.

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APPENDIX A

OFFSET TO ANGLE TRANSFORMATIONS IN 1-D MEDIA

For the sake of completeness, we reproduce here the main expressions that approximate the reflection angle, $\theta = \theta(x)$, as a function of offset in 1D media. In this situation, we start with the observation that the horizontal slowness,

$$p = \frac{dT}{dx} = \frac{\sin\theta(z)}{v(z)} \tag{A-1}$$

is constant for a given ray and $v(z)$ and $\theta(z)$ are the velocity and ray angle at depth z, respectively. For a fixed reflector, we consider the hyperbolic traveltime approximation

$$T(x) = \sqrt{T(0)^2 + \frac{x^2}{v_{rms}^2}}, \tag{A-2}$$

where $T(0)$ is the vertical traveltime, x is offset and v_{rms} is the rms-velocity, corresponding to that reflector. From the above two equations, it follows that

$$\frac{dT}{dx} = \frac{x}{T(x)v_{rms}^2} \ . \tag{A-3}$$

from which the reflection angle from the target reflector at the given offset, $\theta = \theta(x)$, satisfies

$$\sin\theta(x) = \frac{v\,x}{T(x)v_{rms}^2}, \tag{A-4}$$

where v is the interval velocity at the reflecting layer. As a consequence, we also find that

$$\cos\theta(x) = \sqrt{1 - \frac{v^2x^2}{T(x)^2v_{rms}^2}} = \frac{T(0)}{T(x)}\sqrt{1 + (v_{rms}^2 - v^2)\left(\frac{x}{T(0)\,v_{rms}^2}\right)^2}. \tag{A-5}$$

APPENDIX B

TUNING AND STRETCH EFFECTS FOR A CONVERTED WAVE

For a converted PS-wave, the normal-incidence traveltime differential is

$$\Delta T(0) = \Delta T_P(0) + \Delta T_S(0) = \frac{\Delta z}{v_P} + \frac{\Delta z}{v_S} \ , \tag{B-1}$$

where v_P and v_S are the P- and S-wave velocities in the thin layer, respectively. For a common-offset gather, the thin-layer tuning effect is derived as for a non-converted wave, but now with (see Figure 1)

$$\Delta T(x) = \Delta T_P(x) + \Delta T_S(x) = \Delta T_P(0)\cos\theta + \Delta T_S(0)\cos\chi \ , \tag{B-2}$$

where χ is the ray angle for the reflected S-wave. It can be computed from Snell's law

$$\frac{\sin\theta}{v_P} = \frac{\sin\chi}{v_S} \ . \tag{B-3}$$

It is easily seen that $\Delta T(x) < \Delta T(0)$, which results in a wavelet stretch, and an amplitude decrease for a thin layer reflection.

For a common-angle gather, the differential traveltime is (see Figure 2)

$$\Delta T(x) = \frac{\Delta T_P(0)}{\cos\theta} + \frac{\Delta T_S(0)}{\cos\chi} \ . \tag{B-4}$$

We see that $\Delta T(x) > \Delta T(0)$, so that for a common-angle gather there is a wavelet squeeze. The thin-layer tuning effect results in an increase in reflection amplitude.