# TRAVELTIME APPROXIMATIONS IN VTI MEDIA

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## ABSTRACT

*As exploration targets have gotten deeper, cable lengths have increased accordingly, making the conventional two term hyperbolic traveltime approximation produce increasingly erroneous traveltimes. To overcome this problem, many traveltime formulas were proposed in the literature that provide approximations of different quality. In this paper, we give an overview over a number of those approximations and compare their quality. Moreover, we propose some new traveltime approximations based on the approximations found in the literature. The main advantage of our approximations is that some of them are have rather simple analytic expressions that makes them easy to use, while achieving the same quality as the better of the established formulas.*

## INTRODUCTION

Traveltime approximations play a key hole in the processing of reflection data. They are used in, for example, migration (Alkhalifah and Larner, 1994; Vestrum et al., 1999; Mukherjee et al., 2001), moveout correction and velocity analysis (Tsvankin and Thomsen, 1994; Alkhalifah and Tsvankin, 1995; Fomel, 2003) as well as remigration (Fomel, 1994; Hubral et al., 1996; Schleicher and Aleixo, 2007) .

The standard hyperbolic approximation (Dix, 1955) of the P-wave reflection traveltime commonly used in seismic data precessing is exact for a homogeneous isotropic medium and a planar reflector. It remains a good approximation for short offsets in layered media with not too strong lateral variations. However, as exploration targets have gotten deeper, cable lengths have increased accordingly. Increased offsets have made the conventional two term hyperbolic equation produce increasingly erroneous traveltimes. To overcome this problem, it is important to include a nonhyperbolic correction to the reflection moveout to guarantee an accurate determination of the model parameters.

Many attempts have been made over the years to provide higher-order reflection moveout equations that provide good approximations for higher offsets. Working with a layered earth model, Bolshix (1956) obtained a sixth-order equation that approximates traveltime. Later, Taner and Koehler (1969) provided a high-order approximation for traveltimes based on a exact Taylor-series expansion of the traveltime. May and Straley (1979) used orthogonal polynomials to derive a high-order traveltime approximation. These approximations based on polynomials, Taylor series or orthogonal polynomials are rather inaccurate for larger offsets. Therefore other approximations are necessary.

To improve accuracy, various authors proposed a shifted-hyperbola approximation (Malovichko, 1978; Claerbout, 1987; Sword, 1987; de Bazelaire, 1988; Castle, 1994). This equation describes a hyperbola that is symmetric about the $t$-axis and has asymptotes that intersect the time axis $x = 0$ at a time $t = \tau_s$ that is different from the zero-offset traveltime $\tau_0$. The shifted hyperbola proposed by Claerbout (1987) contains a free parameter, called $a$, that can be used to find the best fitting traveltime approximation. The shifted hyperbola's parameter can be related to the anisotropy parameter $\eta$ (Siliqi and Bousquié, 2000; Ursin and Stovas, 2006) generating a VTI approximation for traveltime.

However, for a homogeneous transversely isotropic medium with a vertical symmetry axis (a VTI medium) the hyperbolic approximation is only valid for small offsets, and the velocity coefficient is an

NMO velocity that differs from the vertical velocity (Thomsen, 1986). Tsvankin and Thomsen (1994) generalize the results of Hake et al. (1984) and give a forth-order approximation, but this equation rapidly loses accuracy with increasing offset. Alternatively, they proposed a continued-fraction approximation, that is valid for long offsets (Tsvankin and Thomsen, 1994; Alkhalifah and Tsvankin, 1995). Based on the approximation of Tsvankin and Thomsen (1994), Douma and Calvert (2006) proposed new approximations for the traveltime function based on the Padé approximation or rational interpolation. Stovas and Ursin (2004), using another methodology, derive a different continued-fraction approximation for the traveltime function, but this approximation is only valid for short and intermediate offsets. Fowler et al. (2006) provide a methodology using orthogonal parameters to describe the traveltime approximation. They study the orthogonal approximations to Tsvankin and Thomsen's equation and to the shifted hyperbola.

Zhang and Uren (2001) observed that the ray velocity in general transversely isotropic (TI) media can be approximated by a simple equation. Based on this equation, they provide a traveltime approximation for P-waves in homogeneous TI media. Additionally, they found a equation for a single horizontal reflector overlain by transversely isotropic media with a vertical symmetrical axis (VTI medium).

Fomel (2004) provides a anelliptic approximations for qP velocities in VTI media generalizing the anelliptic approximation result of Muir and Dellinger (1985). Fomel's (2004) approximation is a long offset approximation for traveltime function in VTI media.

A tutorial on traveltime approximations in VTI media, summarizing the most practical of the above formulas, can be found in Fowler (2003). In this paper, we give an overview over a collection of traveltime approximations found in the literature and compare their quality. Moreover, we propose some new traveltime approximations based on the approximations found in the literature. The main advantage of our approximations is that some of them are have rather simple analytic expressions that makes them easy to use, while achieving the same quality as the better of the established formulas.

## TRAVELTIME APPROXIMATIONS

In this section we present a collection of traveltime approximations from the literature. The first approximation is the standard hyperbolic traveltime (Dix, 1955),

$$t^2(x) = 1 + x^2, \tag{1}$$

Here and in everywhere in this paper, we use the normalized half-offset,

$$x = \frac{h}{\tau_0 v_{nmo}}, \tag{2}$$

and the normalized traveltime

$$t(x) = \frac{\tau(x)}{\tau_0}. \tag{3}$$

The first attempts to improve on equation (1) were higher-order approximations based on Taylor series and orthogonal polynomials (Taner and Koehler, 1969; May and Straley, 1979). However, those approximations do not reach to much father offsets than equation (1).

The first approximation that extends to farther offsets is the so-called shifted hyperbola. It was proposed and studied by a number of authors (Malovichko, 1978; Claerbout, 1987; Sword, 1987; de Bazelaire, 1988; Castle, 1994). It has the general form

$$t(x) = 1 + \frac{1}{S}\left[\sqrt{1 + S\,x^2} - 1\right]. \tag{4}$$

Malovichko (1978) considered a layered medium and expressed the parameter $S$ as that $S = \mu_4/\mu_2^2$, where $\mu_j$ is the $j$th velocity momentum, given by

$$\mu_j = \sum_{k=1}^{N} \Delta\tau_k\, V_k^j \Bigg/ \sum_{k=1}^{N} \Delta\tau_k\ , \tag{5}$$

where $V_k$ is the interval velocity of the $k$th layer and $\Delta\tau_k$ is the vertical traveltime in the $k$th layer. Claerbout (1987) suggests to use a free parameter $a = 1/(1-S)$ in the shifted hyperbola to fit the approximation to the observed traveltime. He gives no interpretation of $a$ in terms of medium parameters. In the shifted hyperbola of Castle (1994), $S$ is no longer a constant but is allowed to vary with offset, i.e., $S = S(x)$. For VTI media, Siliqi and Bousquié (2000) and Ursin and Stovas (2006) expressed the parameter $S$ as $S = 8\eta + 1$.

To further improve accuracy for large offsets in VTI media, Tsvankin and Thomsen (1994) proposed to use the continued-fraction approximation

$$t^2(x) = 1 + x^2 - \frac{2\eta x^4}{1 + (1 + 2\eta)x^2}. \tag{6}$$

Based on the approximation of Tsvankin and Thomsen (1994), Douma and Calvert (2006) proposed new approximations for traveltime function based on Padé approximation or rational interpolation.

Stovas and Ursin (2004), using another methodology, derive a different continued-fraction approximation for traveltime function. In our notation, it reads

$$t^2(x) = 1 + x^2 - \frac{G\,x^4}{1 + (1 + 4G)x^2}, \tag{7}$$

where $G$ is a parameter that depends of the anisotropic parameters $\epsilon$ and $\delta$. It has the form

$$G = \frac{2(\epsilon - \delta)}{(1 + 2\delta)^2}\left[1 + \frac{2\gamma_0^2\delta}{\gamma_0^2 - 1}\right], \tag{8}$$

where $\gamma_0$ is vertical P-wave velocity over vertical S-wave velocity.

Zhang and Uren (2001) observed that ray velocity in TI media can be approximated with a simple equation. Based on this observation, they provide a traveltime approximation for P-waves in homogeneous TI media,

$$t^2(x) = \frac{1}{2}\left[1 + x^2/Q + \sqrt{(1 + x^2/Q)^2 + 4A\,x^2/Q}\right], \tag{9}$$

They give no rule for how the anisotropy parameter $A$ depends on the actual medium parameters.

Generalizing the anelliptic approximation result of Muir and Dellinger (1985), Fomel (2004) found the traveltime approximation

$$t^2(x) = \frac{3 + 4\eta}{4(1 + \eta)}\,t_h^2(x) + \frac{1}{4(1 + \eta)}\sqrt{t_h^4(x) + 16\eta(1 + \eta)x^2/Q}\,, \tag{10}$$

where,

$$t_h^2(x) = 1 + x^2/Q \tag{11}$$

with $Q = 1 + 2\eta$. Note that $t_h^2(x)$ is the hyperbolic part of equation (10), however using the horizontal velocity $v_h = v_{nmo}\sqrt{1 + 2\eta}$ rather than the NMO velocity.

## NEW TRAVELTIME APPROXIMATIONS

In this section, we study a few additional traveltime approximations. Most of them are obtained by further approximation of one of the above formulas, mainly the ones of Zhang and Uren (2001) and Fomel (2004). Others are the result of adaptations that are based on the numerical experiments.

Using the Padé approximation (Baker and Graves-Morris, 1981) in Fomel's traveltime equation (10) we obtain the following expressions. The Padé $[2m, 2n]$ approximation has the form

$$t^2(x) = \sum_{i=0}^{m} a_{2i}x^{2i} \Bigg/ \sum_{j=0}^{n} b_{2j}x^{2j}\;. \tag{12}$$

For a VTI medium, the coefficients for the Padé $[2, 2]$ approximation are

$$a_0 = 1;\; a_2 = (1 + 2\eta);\; b_0 = 1, b_2 = 2\eta\;. \tag{13}$$

The coefficients for the Padé $[4,2]$ approximation for the same equation are

$$a_0 = 1+2\eta,\ a_2 = 2(\eta+1)(4\eta+1),\ a_4 = 4\eta^2+6\eta+1;\ b_0 = 1+2\eta,\ b_2 = 8\eta(\eta+1)+1. \tag{14}$$

Finally, the Padé $[4,4]$ approximation for equation (10) has the coefficients

$$\begin{aligned} a_0 &= 1+8\eta(\eta+1)^2,\ a_2 = 24\eta^2(2\eta^2+5\eta+4)+13\eta+2,\ a_4 = 8\eta(2\eta^2+3\eta+1)(2\eta^2+3\eta+2)+1; \\ b_0 &= 1+8\eta(\eta+1)^2,\ b_2 = 16\eta^2(3\eta^2+7\eta+5)+18\eta+1,\ b_4 = 4\eta(\eta+1)(2\eta+1)^2\,. \end{aligned} \tag{15}$$

Further expressions for the traveltimes can be obtained from approximating the square roots in the above formulas. For small values of $\varepsilon$, we have up to the first order

$$\sqrt{1+\varepsilon} \approx 1+\frac{\varepsilon}{2}\,. \tag{16}$$

If we suppose in equation (10) that $t_h^2(x)$ has a large value compared to $16\eta(1+\eta)x^2/Q$, we can apply the above approximation to find

$$\begin{aligned} t^2(x) &\approx \frac{3+4\eta}{4(1+\eta)}\,t_h^2(x)+\frac{t_h^2(x)}{4(1+\eta)}\left\{1+\frac{16}{2}\frac{\eta(1+\eta)}{Q}\frac{x^2}{t_h^4(x)}\right\} \\ &\approx t_h^2(x)+\frac{2\eta}{Q}\frac{x^2}{t_h^2(x)}. \end{aligned} \tag{17}$$

For small values of $\eta$, we can neglect the term with $\eta^2$ inside the square root of equation (10), thus

$$t^2(x) \approx \frac{3+4\eta}{4(1+\eta)}\,t_h^2(x)+\frac{t_h^2(x)}{4(1+\eta)}\sqrt{1+16\frac{\eta}{Q}\frac{x^2}{t_h^4(x)}} \tag{18}$$

$$\approx t_h^2(x)+\frac{2\eta}{(1+\eta)Q}\frac{x^2}{t_h^2(x)}\,, \tag{19}$$

where the second line is again obtained by approximating the square root according to equation (16).

Other approximations are obtained by replacing $(1+\eta)Q$ in the denominator of equation (19) by either $(1+\eta)^2$ or $Q^2$. This yields

$$t^2(x) = t_h^2(x)+\frac{2\eta}{(1+\eta)^2}\frac{x^2}{t_h^2(x)} \tag{20}$$

and

$$t^2(x) = t_h^2(x)+\frac{2\eta}{Q^2}\frac{x^2}{t_h^2(x)}. \tag{21}$$

Correspondingly, the application of the square root approximation (16) to formula (9) leads to

$$t^2(x) = t_h^2(x)+\frac{A}{Q}\frac{x^2}{t_h^2(x)}. \tag{22}$$

Since this approximation is very similar to equations (17) and (19), it gives us a means of expressing parameter $A$ of Zhang and Uren (2001) for VTI media in terms of $\eta$. The resulting expressions for the approximation (9) are

$$t^2(x) = \frac{1}{2}\left[t_h^2+\sqrt{t_h^4+8\eta\,x^2/Q}\right] \tag{23}$$

and

$$t^2(x) = \frac{1}{2}\left[t_h^2+\sqrt{t_h^4+\frac{8\eta}{1+\eta}\frac{x^2}{Q}}\right]. \tag{24}$$

If we want a approximation for $t(x)$ rather than $t^2(x)$, we use again the square root approximation (16) in equations (17) and (19). This results in equations that exhibit a simple correction term added to the hyperbolic approximation. They read

$$t(x) \approx t_h(x) + \frac{\eta}{Q}\frac{x^2}{t_h^3(x)} \tag{25}$$

and

$$t(x) \approx t_h(x) + \frac{\eta}{(1+\eta)Q}\frac{x^2}{t_h^3(x)}. \tag{26}$$

Actually, other good approximations are obtained by replacing $1+\eta$ in the denominator of the last term in equation (26) by $Q = 1+2\eta$, being

$$t(x) \approx t_h(x) + \frac{\eta}{Q^2}\frac{x^2}{t_h^3(x)} , \tag{27}$$

or vice versa

$$t(x) \approx t_h(x) + \frac{\eta}{(1+\eta)^2}\frac{x^2}{t_h^3(x)} . \tag{28}$$

Alternative expressions are obtained if we approximate the outer root square in equation (10). This leads to

$$\begin{aligned} t(x) &= t_h(x)\sqrt{1 - \frac{1}{4(1+\eta)} + \frac{1}{4(1+\eta)}\sqrt{1 + 16\frac{\eta(1+\eta)}{Q}\frac{x^2}{t_h^4(x)}}} & (29)\\ &\approx t_h(x)\left(1 - \frac{1}{8(1+\eta)}\right) + \frac{1}{8(1+\eta)}\sqrt{t_h^2(x) + 16\frac{\eta(1+\eta)}{Q}\frac{x^2}{t_h^2(x)}}. & (30)\end{aligned}$$

Note that an approximation of the remaining square root in equation (30) results again in equation (25).

If we put the factor $t_h(x)\sqrt{(3+4\eta)/4(1+\eta)}$ in evidence before approximating the square root, we obtain yet another approximation of equation (10), which reads

$$\begin{aligned} t(x) &= t_h(x)\sqrt{\frac{3+4\eta}{4(1+\eta)}}\sqrt{1 + \frac{1}{3+4\eta}\sqrt{1 + 16\frac{\eta(1+\eta)}{Q}\frac{x^2}{t_h^4(x)}}} & (31)\\ &\approx t_h(x)\sqrt{\frac{3+4\eta}{4(1+\eta)}} + \sqrt{\frac{t_h^2(x)}{16(1+\eta)(3+4\eta)} + \frac{\eta}{(3+4\eta)Q}\frac{x^2}{t_h^2(x)}}. & (32)\end{aligned}$$

The second square root in equation (32) can be further approximated to yield

$$t(x) \approx t_h(x)\left[\sqrt{\frac{3+4\eta}{4(1+\eta)}} + \frac{1}{4\sqrt{(1+\eta)(3+4\eta)}}\right] + \frac{2\eta}{Q}\sqrt{\frac{1+\eta}{3+4\eta}}\frac{x^2}{t_h^3(x)}. \tag{33}$$

The term in brackets can be shown to be very close to one. Thus, we arrive at

$$t(x) \approx t_h(x) + \frac{2\eta}{Q}\sqrt{\frac{1+\eta}{3+4\eta}}\frac{x^2}{t_h^3(x)}. \tag{34}$$

Neglecting the $\eta^2$ term in the inner integral of equation (31) yields

$$t(x) \approx t_h(x)\sqrt{\frac{3+4\eta}{4(1+\eta)}}\sqrt{1 + \frac{1}{3+4\eta}\sqrt{1 + 16\frac{\eta}{Q}\frac{x^2}{t_h^4(x)}}} . \tag{35}$$

Successively approximating the square roots leads to

$$t(x) \approx t_h(x)\sqrt{\frac{3+4\eta}{4(1+\eta)}}\sqrt{1+\frac{1}{3+4\eta}\left(1+8\frac{\eta}{Q}\frac{x^2}{t_h^4(x)}\right)} \tag{36}$$

$$\approx t_h(x)\left[\sqrt{\frac{3+4\eta}{4(1+\eta)}}+\frac{1}{2(3+4\eta)}\right]+\frac{2\eta}{Q\sqrt{(1+\eta)(3+4\eta)}}\frac{x^2}{t_h^3(x)}. \tag{37}$$

Again, the expression in brackets can be shown to be very close to one. Thus, we can write

$$t(x) \approx t_h(x)+\frac{2\eta}{Q\sqrt{(1+\eta)(3+4\eta)}}\frac{x^2}{t_h^3(x)}. \tag{38}$$

From the similarity of equations (34) and (38), we conclude that another possible approximation is the intermediate expression

$$t(x) \approx t_h(x)+\frac{2\eta}{Q\sqrt{3+4\eta}}\frac{x^2}{t_h^3(x)}. \tag{39}$$

The first square root in equation (32) can be approximated as

$$\sqrt{\frac{3+4\eta}{4(1+\eta)}} \approx \left(1-\frac{1}{8(1+\eta)}\right). \tag{40}$$

The second square root of equation (32) can again be approximated in two different ways. The first one is obtained by replacing $3+4\eta$ in the denominators of both terms by $4+4\eta$. This results in

$$\sqrt{\frac{t_h^2(x)}{16(1+\eta)(3+4\eta)}+\frac{\eta}{(3+4\eta)Q}\frac{x^2}{t_h^2(x)}} \approx \frac{1}{8(1+\eta)}\sqrt{t_h^2(x)+16\frac{\eta(1+\eta)}{Q}\frac{x^2}{t_h^2(x)}}. \tag{41}$$

The second one is obtained by approximating

$$\sqrt{\frac{4+4\eta}{3+4\eta}} \approx \left(1+\frac{1}{8(1+\eta)}\right). \tag{42}$$

This yields

$$\sqrt{\frac{t_h^2(x)}{16(1+\eta)(3+4\eta)}+\frac{\eta}{(3+4\eta)Q}\frac{x^2}{t_h^2(x)}} \approx \left(1+\frac{1}{8(1+\eta)}\right)\frac{1}{8(1+\eta)}\sqrt{t_h^2(x)+16\frac{\eta(1+\eta)}{Q}\frac{x^2}{t_h^2(x)}}. \tag{43}$$

Using equations (32), (40), (41), and (43) we have two more traveltime approximations, being

$$t(x) \approx t_h(x)\left(1-\frac{1}{8(1+\eta)}\right)+\frac{1}{8(1+\eta)}\sqrt{t_h^2(x)+16\frac{\eta(1+\eta)}{Q}\frac{x^2}{t_h^2(x)}} \tag{44}$$

and

$$t(x) \approx t_h(x)\left(1-\frac{1}{8(1+\eta)}\right)+\left(1+\frac{1}{8(1+\eta)}\right)\frac{1}{8(1+\eta)}\sqrt{t_h^2(x)+16\frac{\eta(1+\eta)}{Q}\frac{x^2}{t_h^2(x)}}. \tag{45}$$

As the numerical experiments will demonstrate, approximation (44) becomes more accurate upon replacing $16\eta(1+\eta)$ by $2\eta(8+7\eta)$, i.e., using the approximation

$$t(x) \approx t_h(x)\left(1-\frac{1}{8(1+\eta)}\right)+\frac{1}{8(1+\eta)}\sqrt{t_h^2(x)+2\frac{\eta(8+7\eta)}{Q}\frac{x^2}{t_h^2(x)}}\ . \tag{46}$$

By substituting $\eta = 0.25$ in the appropriate places in equation (10), we find two more approximations,

$$t(x) \approx \sqrt{\frac{1}{5}\left(4\,t_h^2 + \sqrt{t_h^4 + 20\frac{\eta}{Q}x^2}\right)} \tag{47}$$

and

$$t(x) \approx \sqrt{\frac{1}{5}\left(4\,t_h^2 + \sqrt{t_h^4 + 16\frac{\eta(1+\eta)}{Q}x^2}\right)}. \tag{48}$$

After approximation of the inner square roots, these equations reduce to expression (17) and

$$t(x) \approx \sqrt{t_h^2 + 8\frac{\eta(1+\eta)}{5Q}\,\frac{x^2}{t_h^2(x)}}. \tag{49}$$

An approximation of the remaining square root leads to

$$t(x) \approx t_h(x) + \frac{4}{5}\frac{\eta(1+\eta)}{Q}\,\frac{x^2}{t_h^3(x)}\ . \tag{50}$$

Another observation results from the numerical experiments. As we will see below, the shifted hyperbola, equation (4) can be numerically improved by using different values for parameter $S$ than the ones suggested in the literature. Very good traveltime approximations are obtained for $S = 1 + 3\eta$ and $S = \left(1 - \frac{7}{8}\sqrt{\eta}\right)^{-1}$.

## NUMERICAL COMPARISONS

In this section, we compare the above traveltime approximations for a homogeneous VTI medium above a horizontal reflector with the exact traveltime. Since we are comparing normalized traveltimes, no values for reflector depth and NMO velocity need actually to be specified. The elastic parameters of the VTI medium are that of the Greenhorn shale (Jones and Wang, 1981), i.e., $c_{11} = 14.47$ km$^2$/s$^2$, $c_{33} = 9.57$ km$^2$/s$^2$, $c_{13} = 4.51$ km$^2$/s$^2$, and $c_{55} = 2.28$ km$^2$/s$^2$, which were also used by Fomel (2004). In this medium, we have $\eta = 0.34068$.

To make sure that the approximations still hold for other amounts of anisotropy, we also run corresponding tests with values of $\eta$ between 0.1 and 0.5. All deviations from the true traveltime discussed below behave very similar for all tested values of $\eta$.

The first comparisons involve the traveltime approximations from the literature. In Figure 1, we compare the conventional hyperbolic approximation with NMO velocity (1), approximation (6) of Tsvankin and Thomsen (1994), the hyperbolic approximation with horizontal velocity (11), and Bolshix' Taylor-series approximations of 4th and 6th orders for normalized offsets up to 3 to the exact traveltime. We see that most of these approximations are rather poor approximations for large offsets. The exceptions are approximation (6) of Tsvankin and Thomsen (1994) and the hyperbolic approximation (11) using the horizontal velocity. However, the latter show already a deviation at smaller offsets.

In Figure 2 we present the most recent and more accurate traveltime approximations in the literature. These are good approximations up to much larger offsets. Since approximation (6) of Tsvankin and Thomsen (1994) also has a reasonable quality, it is repeated in Figure 2.

To better appreciate the quality of these approximations, Figure 3 shows the relative error between the approximations of Figure 2 and the exact traveltime. We see that approximation (10) is the best of these, with its error never exceeding 4% in the depicted offset range between 0 and 3. The second-best is approximation (6) with a relative error below 6%. The errors of the other approximation exceeds 6% for rather small offsets.

The next figures show the relative errors for our collection of new traveltime approximations. In Figure 4 we present the Padé approximations of equation (10) of Fomel (2004). Observe that the best approximation is achieved with the Padé [4,2] approximation, which already has rather complicated expressions for the coefficients [see equations (14)].

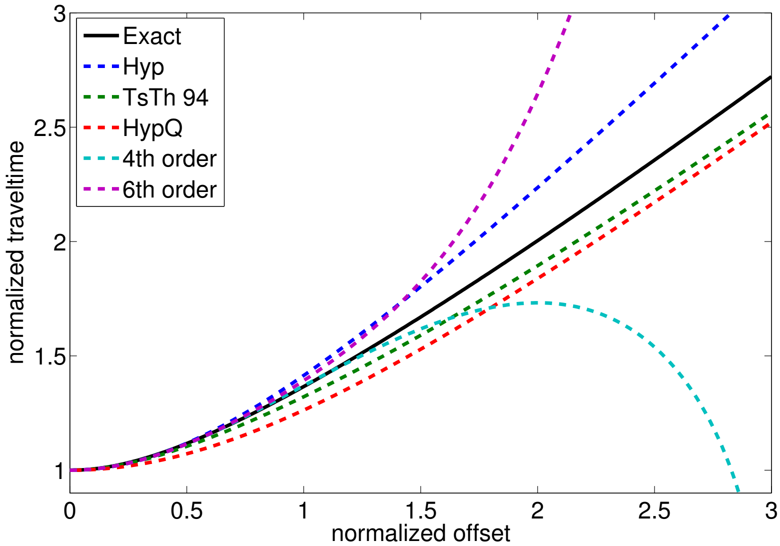


**Figure 1:** Comparison of traveltime approximations (1) [Hyp], (6) [TsTh 94], (11) [HypQ], as well as the 4th order and 6th order approximations of Bolshix (1956) with the exact VTI traveltime.

Figure 5 depicts the approximations of the type

$$t^2(x) \approx t_h^2(x) + B(\eta)\,\frac{x^2}{t_h^2(x)}. \tag{51}$$

As we can see, these are rather accurate approximations. None of these approximations exceeds a relative error of 5%, the best one being equation (49), the error of which remains below 3%.

The approximations for $t^2(x)$ that are not of the type of equation (51) are more disperse (see Figure 6). Equations (47) and (48) are rather good approximations with relative errors below 4%. The error of equation (18) remains below 5%. On the other hand, the error of equations (23), (24), and (36) increase rapidly, exceeding the 6% threshold already for rather small offsets.

In Figure 7 we present the approximations of the type

$$t(x) \approx t_h(x) + B(\eta)\,\frac{x^2}{t_h^3(x)}. \tag{52}$$

Again, all of these approximations are rather accurate. None of them exceeds a relative error of 5% in the chosen range of offsets. Moreover, these approximations possess quite simple expressions that may be advantageous for theoretical considerations. The best of these approximations with a maximum error of about 2.5% is the one given in equation (34).

Figure 8 shows the remaining traveltime approximations for $t(x)$. All of them are rather accurate with relative errors below 4%. Equations (32), (33), (37), and (45), do not start with a zero error at zero offset, but still have a good overall error behaviour.

Finally, in Figure 9 we compare the shifted hyperbola approximations from the literature with the ones obtained with our suggested choices for $S$. Note that the choices $S = 1 + 3\eta$ and $S = (1 - \frac{7}{8}\sqrt{\eta})^{-1}$ yield highly accurate approximations with maximum errors below 2%.

## CONCLUSION

Accurate traveltime approximations for large offsets are very important for many tasks of seismic processing. The conventional hyperbolic approximation, which is still used by many processing algorithms for

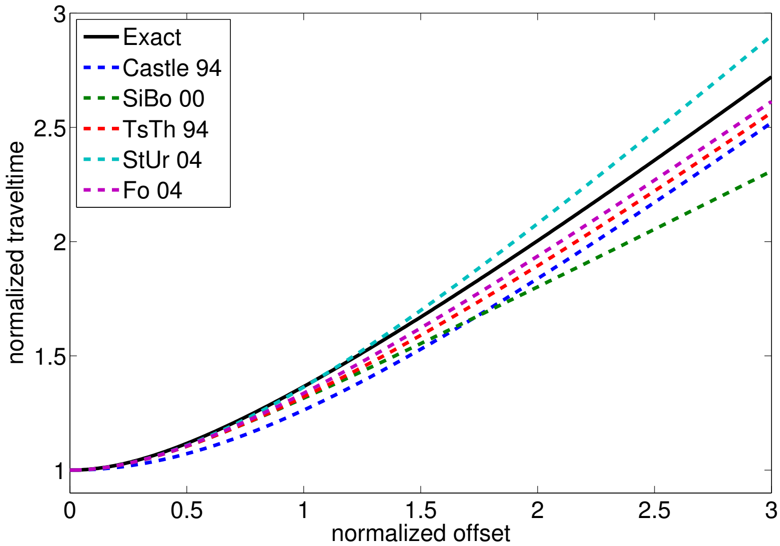


**Figure 2:** Comparison of traveltime approximations (4) (shifted hyperbola) with $S = S(x)$ [Castle 94] and $S = 1 + 8\eta$ [SiBo 00], (6) [TsTh 94], (7) [StUr 04], and (10) [Fo 04] with the exact VTI traveltime.

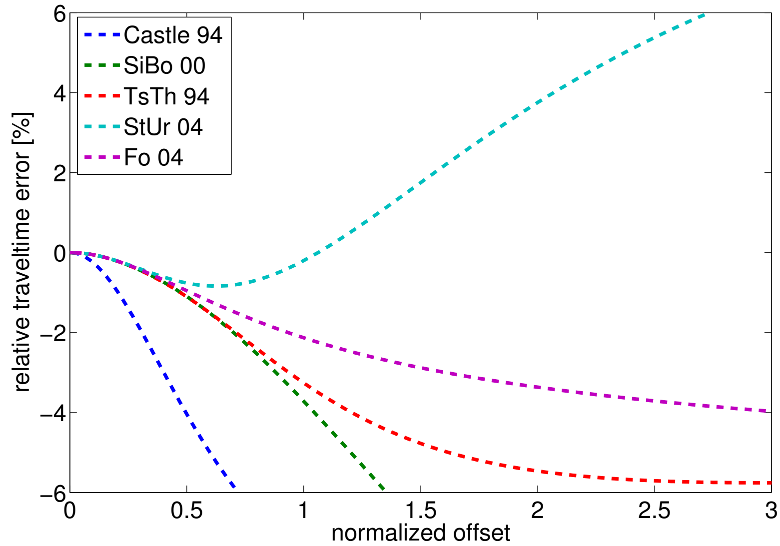


**Figure 3:** Relative error of traveltime approximations (4) (shifted hyperbola) with $S = S(x)$ [Castle 94] and $S = 1 + 8\eta$ [SiBo 00], (6) [TsTh 94], (7) [StUr 04], and (10) [Fo 04].

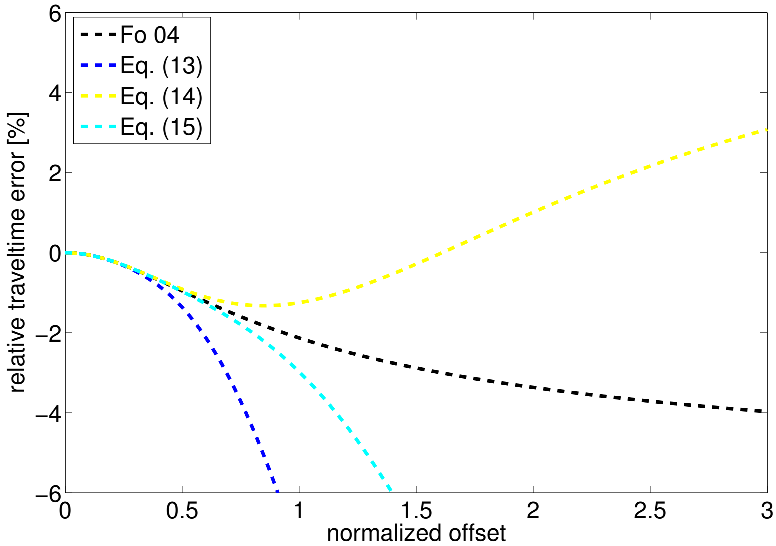


**Figure 4:** Relative error of traveltime approximation (10) [Fo 04] and its Padé approximations [2,2], [4,2] and [4,4].

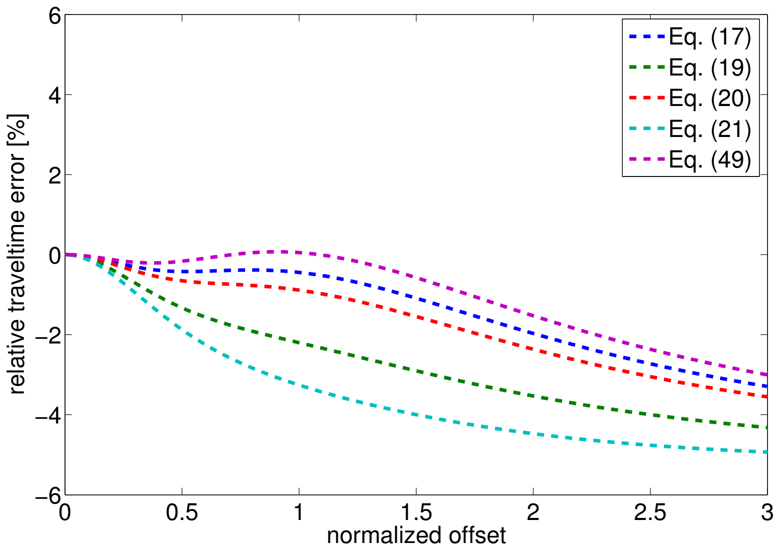


**Figure 5:** Relative error of new traveltime approximations for $t^2(x)$ of the type $t^2(x) = t_h^2(x) + B(\eta)\dfrac{x^2}{t_h^2(x)}$.

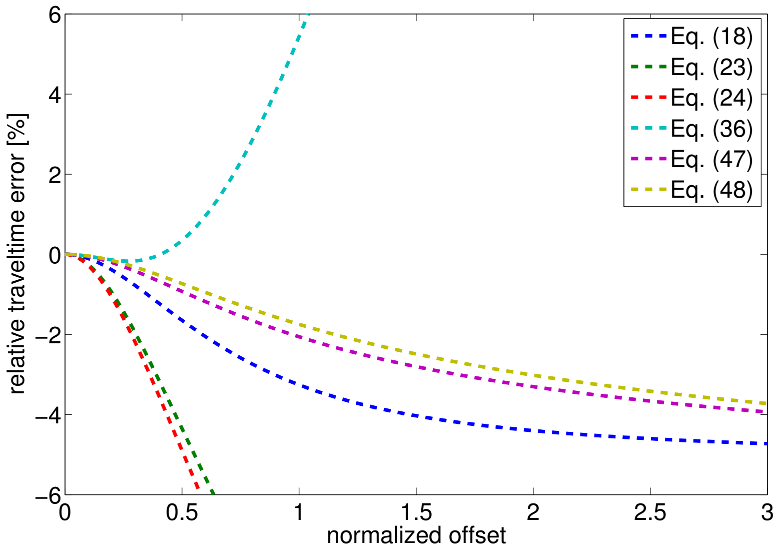


**Figure 6:** Relative error of other traveltime approximations for $t^2(x)$.

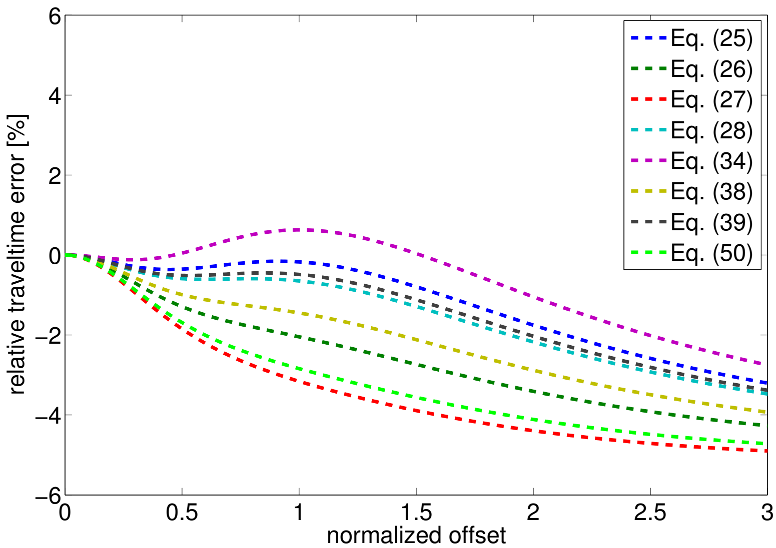


**Figure 7:** Relative error of new traveltime approximations for $t(x)$ of the type $t(x) \approx t_h(x) + B(\eta)\,\dfrac{x^2}{t_h^3(x)}$.

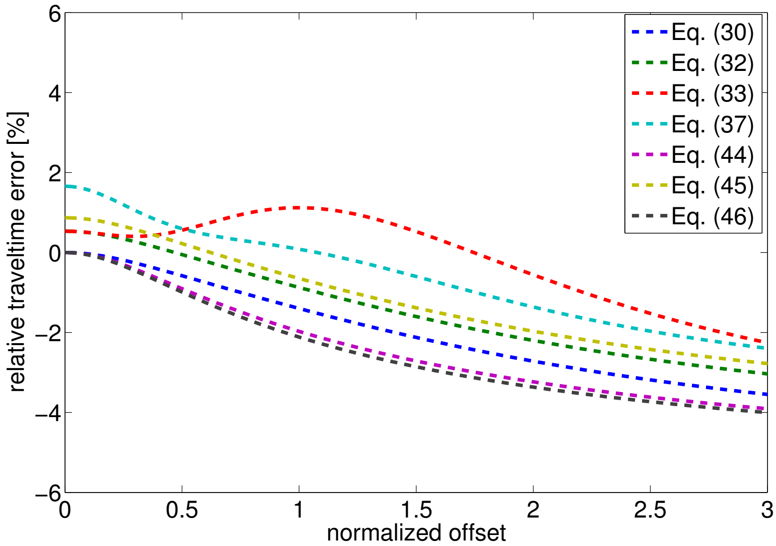


**Figure 8:** Relative error of other traveltime approximations for $t(x)$.

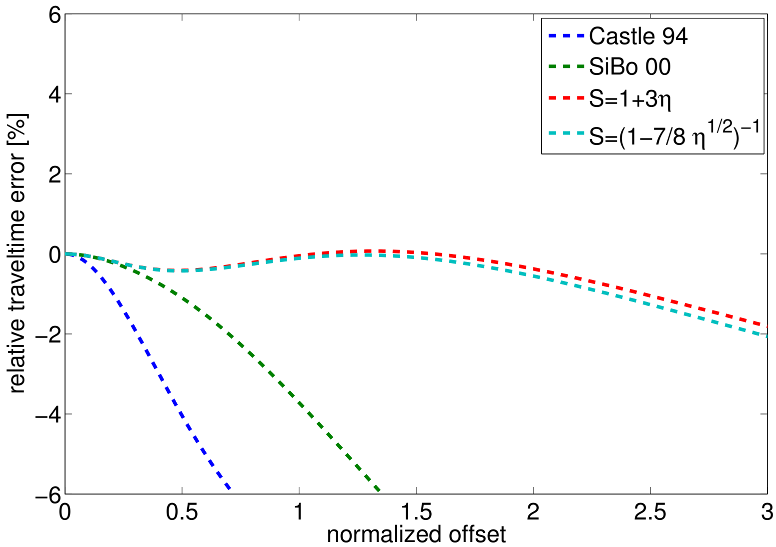


**Figure 9:** Relative error of shifted hyperbola with $S = S(x)$ [Castle 94], $S = 1 + 8\eta$ [SiBo 00], and new shifted hyperbola approximations with $S = 1 + 3\eta$ and $S = \left(1 - \frac{7}{8}\sqrt{\eta}\right)^{-1}$.

moveout correction, time migration, multiple attenuation and velocity analysis, is inaccurate as soon as anisotropy, wave-mode conversions or significant medium heterogeneity are involved.

Many different formulas to approximate far-offset traveltimes have been proposed in the literature (see, e.g., Tsvankin and Thomsen, 1994; Fomel, 2004, and the references therein). Most of these are rather complicated algebraic expressions that are hard to use.

In this paper, we have studied the quality of many of these approximations for a homogeneous VTI medium above a horizontal reflector. Moreover, by further approximation of the formulas from the literature, as well as by combining some of their properties, we have presented a host of new traveltime approximations. Our numerical comparisons show that it is possible to find traveltime formulas of a much simpler type that provide equal or even better approximations to the true traveltime than those proposed in the literature. The formulas that provided the best approximations to the true traveltime are the shifted hyperbola with a different choice for the free parameter and the hyperbolic traveltime with a rather simple correction term.

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