2.5D TRUE-AMPLITUDE DIFFRACTION-STACK REDATUMING

M. Pila, J. Schleicher and A. Novais

email: *js@ime.unicamp.br*
keywords: *redatuming, true amplitude, Kirchhoff*

ABSTRACT

The purpose of redatuming is to transform seismic data acquired at a certain measurement surface to simulated data as if acquired at a different measurement surface. Based on the chaining of diffraction-stack migration and isochron-stack demigration, we derive a 2.5D true-amplitude diffraction-stack-type redatuming operator and present its specific form for zero-offset data. The operator consists of performing a single weighted stack along adequately chosen stacking lines. For simple types of media, we derive analytic expressions for the stacking lines and weight functions and demonstrate its functionality with numerical examples.

INTRODUCTION

The purpose of redatuming is to transform seismic data acquired at a certain measurement surface to simulated data as if acquired at a different measurement surface (Wapenaar et al., 1992). Generally, redatuming is used to remove the influence of the surface topography from the data, simulating the acquisition at a planar surface, the so-called *datum* (Berryhill, 1979, 1984). However, in a general treatment of redatuming, the restriction on the output measurement surface to be planar can be dropped.

Many different redatuming techniques have been discussed in the literature. Wiggins (1984) proposed the so-called RKR method, based on the successive aplication of Kirchhoff integration, reciprocal interchange of sources and receivers, and a second Kirchhoff integration. Wapenaar (1993) proposed a recursive one-way Kirchhoff-Helmholtz extrapolation as the basis for true amplitude redatuming in media consisting of homogeneous layers. Tegtmeier et al. (2004) propose a Kirchhoff redatuming that can be applied to sparse data. Other contributions to the theory of wave-equation-based redatuming methods include the works of Yilmaz and Lucas (1986), Bevc (1995), and Schneider et al. (1995). A comprehensive summary about the state of the art in redatuming can be found in Schuster and Zhou (2006).

The correct transformation of the field amplitudes from the original measurement surface to the new datum level is of fundamental importance, if a true-amplitude migration (see, e.g., Schleicher et al., 1993) is to be applied later in the processing sequence. As kinematically discussed in Hubral et al. (1996) and mathematically shown in Tygel et al. (1996), a true-amplitude configuration transform (of which redatuming is a particular case) can be achieved by chaining weighted diffraction-stack migration and isochron-stack demigration. In this work, we derive the true-amplitude redatuming operator resulting from this approach and demonstrate its application in simple situations.

2.5D REDATUMING

In this paper, we derive the redatuming operator for the 2.5D case, i.e., the wave propagates in a 3D medium, which is isotropic and laterally homogeneous in the y direction, but arbitrally inhomogeneous in the (x, z) plane. This situation can be parameterized in a 2D form. Particularly, all reflectors can be parameterized by curves in (x, z). We suppose that all point sources emit identical pulses, and are distributed along the x axis, together with the receivers.

Let the original seismogram section be parameterized by the variable ξ. Thus, the coordinates $S_i = G_i = (\xi, 0, z_i(\xi))$ on the acquisition surface $z = z_i(x)$ and along the seismic line $y = 0$, define the location of the coincident source-receiver pairs. Correspondingly, the output seismogram is parameterized by the variable η, with a acquisition surface described by $z = z_o(x)$ and along the seismic line $y = 0$. Thus, the coordinates $S_o = G_o = (\eta, 0, z_o(\eta))$ describe the source-receivers pairs on the output datum.

We suppose that each real seismic trace in the input section has been transformed into its analytic trace, adding the Hilbert transform of the original trace as the imaginary part. Thus, the output seismogram will be considered analytic too. The analytic traces from input section will be denoted by $U_i(\xi, t)$, where t is the temporal coordinate in this section. Correspondingly, the analytic trace in the output section will be denoted by $U_o(\eta, \tau)$, where τ is the temporal coordinate in the redatumed section.

As kinematically discussed in Hubral et al. (1996) and mathematically proved in Tygel et al. (1996), we know that the migration and demigration operators can be chained to construct operators for a broad range of seismic image transformations. The resulting operator achieves the desired imaging task in a single stacking procedure. In other words, for each point (η, τ) in the redatumed section, we know that $U_o(\eta, \tau)$ can be expressed as a single stacking operator with a weight function acting upon the input data, i.e.,

$$U_o(\eta, \tau) = \frac{1}{\sqrt{2\pi}} \int_A d\xi W_{red}(\xi; \eta, \tau) D_-^{1/2} [U_i(\xi, t)]|_{t=\mathcal{T}_{red}(\xi;\eta,\tau)} \ . \tag{1}$$

The input traces $U_i(\xi, t)$ are weighted by factor $W_{red}(\xi; \eta, \tau)$, and then summed along the stacking curve $t = \mathcal{T}_{red}(\xi; \eta, \tau)$. Both functions depend on the point (η, τ) where the stacking is performed. Moreover, A denotes the aperture of the stack, i.e., the range where the data are available in the input section. At last, the half derivative

$$D_-^{1/2}[f(t)]| = \mathcal{F}^{-1} \left[|\omega|^{1/2} \mathrm{e}^{-i\frac{\pi}{4}\operatorname{sign}(\omega)} \mathcal{F}[f(t)] \right] \ , \tag{2}$$

where $\mathcal{F}$ denotes the Fourier transform, is necessary to correct the pulse shape.

The stacking curve $\mathcal{T}_{red}$ is defined by the kinematics of the problem, while the weight function W_{red} will be determined by the desired amplitude behaviour. So, for a true-amplitute weight function, this is done imposing that, asymptotically, the simulated reflections must have the same geometrical-spreading factor that the reflections would have if they were actually acquired on the new datum. As we will see, the resulting true-amplitude weight function does not depend on any reflector property. Thus, it is possible to evaluate it for any point (η, τ) in the redatumed section using only information about the velocity model.

We are going to construct the stacking curve $\mathcal{T}_{red}$. Given (η, τ) in output section, we want to find the curve in the input section that is kinematically equivalent to (η, τ), i.e., where reflections might be found in the input section that image to (η, τ). For this, we must follow these two steps:

(1) For a given point (η, τ), we determine its isochron $z = \mathcal{Z}_o(x; \eta, \tau)$ in the depth. This isochron is implicity defined by all point $M = (x, \mathcal{Z}_o(x; \eta, \tau))$ in the depth where the sum of the traveltime along two ray segments S_oM and MG_o, connecting M with the source-receiver pair (S_o, G_o), is equal to the given time τ, i.e.,

$$T(S_o, M) + T(M, G_o) = \tau \ . \tag{3}$$

In the same way, we define the isochron $z = \mathcal{Z}_i(x; \xi, t)$ which refers to the input section.

(2) Treating the isochron like a reflector, we construct its traveltime curve in the original depth, i.e., we evaluate the reflection time for all source-receiver pairs in ξ. The resulting curve can be written as

$$t = \mathcal{T}_{red}(\xi; \eta, \tau) = T(S_i, M^*) + T(M^*, G_i) \ , \tag{4}$$

where, for each source-receiver pair in ξ, the point M^* represents the position in the isochron $z = \mathcal{Z}_o(x; \eta, \tau)$ where a reflection occurred. The point M^*, supposed to be unique, has the depth coordinates $(x^*, \mathcal{Z}_o(x^*; \eta, \tau))$, where $x^* = x^*(\xi; \eta, \tau)$ is obtained using the stationary condition

$$\frac{\partial}{\partial x} \left[T(S_i, M) + T(M, G_i) \right] |_{x=x^*} = 0 \ . \tag{5}$$

We refrain from detailing the lengthy determination of the true-amplitude weight function $W_{red}(\xi; \eta, \tau)$, because it is completely analogous to the one presented for migration to zero offset (MZO) in Tygel et al.

(1998). The reason is that both MZO and redatuming are part of the general class of configuration transforms. Thus, the arguments apply to both situations in the same way. The final redatuming weight function for an arbitrary medium, configuration and topography reads

$$W_{red}(\xi;\eta,\tau) = \frac{v_{oS}}{v_{iS}}\sqrt{\frac{\sigma_{iS}+\sigma_{iG}}{\sigma_{oS}+\sigma_{oG}}}\frac{\bar{\mathcal{L}}_{iS}\bar{\mathcal{L}}_{iG}}{\bar{\mathcal{L}}_{oS}\bar{\mathcal{L}}_{oG}}\left(\frac{\cos\theta_S}{\bar{\mathcal{L}}_{iS}^2}+\frac{\cos\theta_G}{\bar{\mathcal{L}}_{iG}^2}\right)\frac{1}{\cos\phi}\sqrt{\frac{\cos\theta_{oR}}{v_R^3}} \times\frac{\exp\{i\pi[1-\mathrm{sgn}(K_i-K_o)]/4\}}{\sqrt{2|K_i-K_o|}}\ , \tag{6}$$

where v_{iS} and v_{oS} are the velocities at the sources on the input and output datums, respectively, and v_R is the velocity at M. Also, σ_{iS} and σ_{iG} are the so-called optical lengths of ray segments MS_i and MG_i, respectively, i.e., the integral of squared velocity in traveltime along the ray. Analogously, σ_{oS} and σ_{oG} represent these factors along segments MS_o and MG_o, respectively. These factors represent the out-of-plane geometrical-spreading factors. The in-plane components of the geometrical spreading are given by $\bar{\mathcal{L}}_{iS}$ and $\bar{\mathcal{L}}_{iG}$, along segmentes MS_i and MG_i, and by $\bar{\mathcal{L}}_{oS}$ and $\bar{\mathcal{L}}_{oG}$, along segmentes MS_i and MG_i, respectively. Moreover, symbols θ_{is} and θ_{is} represent the angles that the rays MS_i and MG_i make with the surface normal at S_i and G_i, respectively, and ϕ is the surface dip angle at S_i. Also, θ_{oR} is the reflection angle at M in the output configuration. Finally, K_i and K_o are the curvatures of the input and output isochrons, respectively. For zero-offset, expression (6) simplifies to

$$W_{red}(\xi_R;\eta,\tau) = \frac{v_{oS}}{v_{iS}}\sqrt{2\frac{\sigma_{iS}}{\sigma_{oS}}}\frac{\cos\theta_S}{\cos\phi}\frac{1}{v_R^{3/2}\bar{\mathcal{L}}_{oS}^2}\frac{\exp\{i\pi\kappa/2\}}{\sqrt{|K_i-K_o|}}\ . \tag{7}$$

Below, we construct the stacking curves (4) and weight functions (7) for simple types of velocity models.

Homogeneous medium ($v = v_o$) without topography

We start with redatuming between flat surfaces in a homogeneous medium. For simplicity, we consider $z_i(x) = 0$ and $z_o(x) = z_o$ constant. First we construct the stacking curve (4), and then we specify the weight function (7) for this situation.

A point (η, τ) in the output section determines the isochron $z = \mathcal{Z}_o(x; \eta, \tau)$, which is defined by equation (3). For homogeneous media with ZO configuration, we can write this as

$$T(S_o, M) + T(M, G_o) = 2T(S_o, M) = 2\frac{R_o}{v_o} = \tau\ , \tag{8}$$

where $R_o = \sqrt{(z-z_o)^2 + (x-\eta)^2} = v_o\tau/2$. This equation can be solved in terms of z to yield the semicircle

$$z = \mathcal{Z}_o(x;\eta,\tau) = z_o + \sqrt{R_o^2-(x-\eta)^2}\ . \tag{9}$$

Therefore, the diffraction time $\mathcal{T}_D(x;\xi)$, that is, the sum of the traveltimes along the segments that pass from S_i to an arbitrary point $M = (x, \mathcal{Z}_o(x; \eta, \tau))$ and from this M to G_i, can be written as

$$\mathcal{T}_D(x;\xi) = \frac{2R}{v_o}\ , \tag{10}$$

where

$$R = \sqrt{(x-\xi)^2+z^2} \tag{11}$$

is the distance from S_i and G_i to M, with z given by (9).

However, we are interested only in orthogonal reflections from isochron (9). So, we have to find the stationary point of (10), i.e., the point that satisfies

$$\left.\frac{\partial\mathcal{T}_D}{\partial x}\right|_{x=x^*} = \frac{2}{v_o}\left.\left[\frac{\partial R}{\partial x}+\frac{\partial R}{\partial z}\frac{dz}{dx}\right]\right|_{x=x^*} = 0\ , \tag{12}$$

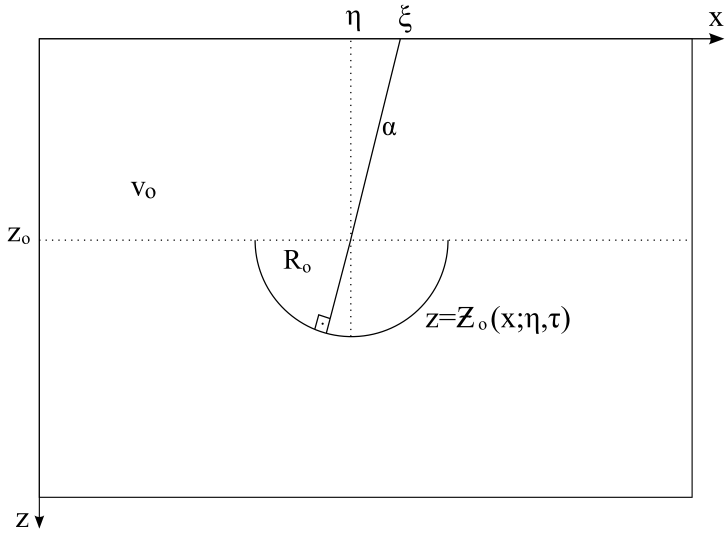

Figure 1: The zero-offset reflection from the source-receiver pair to the isochron $z = \mathcal{Z}_o(x;\eta,\tau)$ crosses the center of semicircle.

with z satisfying the isochron (9). Thus, we obtain the stationary point x^* as

$$x^* = \frac{\gamma R_o}{\alpha} + \eta \; , \tag{13}$$

where $\gamma = \eta - \xi$ and $\alpha = \sqrt{z_o^2 + \gamma^2}$. In other words, γ represents the horizontal displacement between the source-receivers pairs from input and output sections, and α represents the total distance between them (see Figure 1).

Now we can write the redatuming time from isochron (9) in input section. Using the stationary point (13) in equation (10), we find

$$\mathcal{T}_{red}(\xi;\eta,\tau) = \frac{2}{v_o}(R_o + \alpha) = \tau + 2\frac{\alpha}{v_o} \; . \tag{14}$$

This result has a simple geometrical explanation. In Figure 1, we see that for a given ξ, the unique zero-offset ray from the source-receiver pair at ξ to the isochron $z = \mathcal{Z}_o(x;\eta,\tau)$ crosses the center (η, z_o) of the semicircle defined by the isochron $\mathcal{Z}_o$. This must be so because the reflection must be normal to the isochron. The two-way traveltimes of all rays through (η, z_o) define the stacking line in equation (14).

As the next step, we evaluate the weight function (7) of the redatuming operator. First, we will determine the curvatures of the isochrons. We start by determining that of the input isochron. For that purpose, we need the point in depth that has originated the reflection event that arrives at source-receiver pair ξ at time t. In analogy to the previous case, the input isochron is the lower semicircle given by

$$z = \mathcal{Z}_i(x;\xi,t) = \sqrt{(v_o t/2)^2 - (x-\xi)^2} \; . \tag{15}$$

With this, we can evaluate the curvatures of the isochrons (9) and (15) at a point $M = (x,z)$ are

$$K_o = \frac{d^2\mathcal{Z}_o/dx^2}{[1 + (d\mathcal{Z}_o/dx)^2]^{3/2}} = -\frac{1}{R_o} \tag{16}$$

and

$$K_i = \frac{d^2\mathcal{Z}_i/dx^2}{[1 + (d\mathcal{Z}_i/dx)^2]^{3/2}} = -\frac{1}{(v_o t/2)} \; , \tag{17}$$

respectively. In particular, at the stationary point x^*, the traveltime is given by equation (14). Thus,

$$K_i = -\frac{1}{(R_o + \alpha)} \; . \tag{18}$$

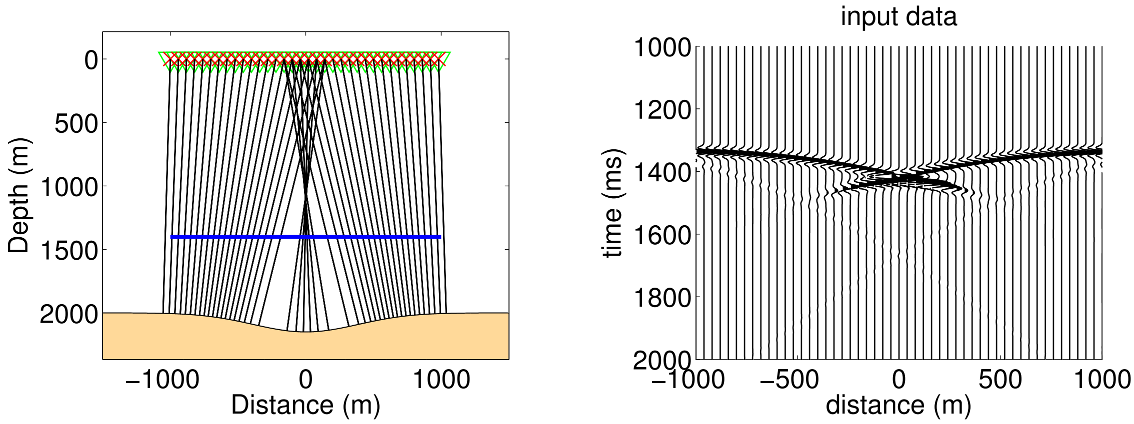

Figure 2: Simulation with the acquisition line at $z_i = 0$ (left) and its respective seismic section (right).

With these two curvatures, we can evaluate the exponential term in the weight function. We have

$$K_i - K_o = \frac{\alpha}{R_o(R_o + \alpha)} \ , \tag{19}$$

and therefore

$$\exp\left\{i\frac{\pi}{2}\kappa\right\} = \exp\left\{i\frac{\pi}{4}[1 - \mathrm{sign}(K_i - K_o)]\right\} = 1 \ . \tag{20}$$

In other words, the weight function for redatuming in a homogeneous medium is always a real quantity. Any phase changes in the events are taken care of by the stacking procedure itself.

To conclude the evaluation of the weight function, we have to determine the geometrical spreading. For homogeneous media, we have $\sigma_{iS} = v_o(R_o + \alpha)$ and $\sigma_{oS} = v_o R_o$. The 2D spreading is the square root of traveltime, i.e., $\bar{\mathcal{L}}_{oS} = \sqrt{R_o/v_o}$.

Using that $\cos\theta_S = z_o/\alpha$, we can thus write the final expression for the weight function of redatuming operator (7) in homogeneous media as

$$W_{red}(\xi; \eta, \tau) = \sqrt{\frac{2}{v_o}} \left(\frac{R_o + \alpha}{R_o}\right) \frac{z_o}{\alpha^{3/2}} \ . \tag{21}$$

Numerical example.— To test the above formulas, we apply them in a simple numerical test. Figure 2 shows the two-layer model used together with the ray family of the zero-offset experiment and the position of the new datum. We model the seismic sections at surface $z_i = 0$ and at new datum z_o, using a software based on the Kirchhoff integral. To the section obtained for $z_i = 0$ (input section), we apply the redatuming and compare this result (output section) with the modelled section obtained at z_o. The source-receiver pairs are equally spaced from -1000 m to 1000 m with displacement 10 m. The new datum (represented by the horizontal line in the left on Figure 2) is $z_o = 1400$ m. The sampling rate was 1 ms. The velocitiy is 3.0 km/s above the reflector and 4.0 km/s below it.

The modelled zero-offset data at $z = z_i = 0$ are depicted on the right side of Figure 2. The left panel of Figure 3 shows the the output section of redatuming applied to the data in Figure 2. For comparison, the right panel of Figure 3 shows the desired output, i.e., the data modelled at the new datum $z = z_o = 1400$ m. Apart from the boundary effects of the stacking procedure, no differences between the two sections are visible. Note that since the caustic occurs above the new datum, redatuming correctly removes the triplication of the reflection event.

To evaluate the quality of the transformation in more detail, we refer to Figure 4. The left side shows the pulse in the traces at $x = 500$ m of the modelled and redatumed sections. The redatumed trace is undistinguishable from the modeled one. As another test, the right side of Figure 4 shows the relative error in amplitude along the event between the redatumed and the modelled sections. Note that outside the boundary region, the amplitude error is about 0.5%.

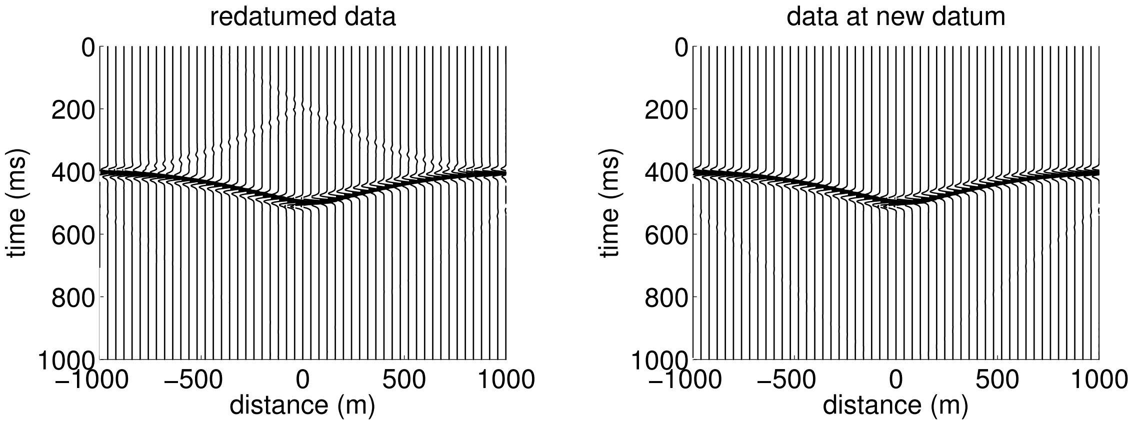

Figure 3: Seismic sections obtained from redatuming (left) and modelling at the datum level $z = z_o$ (right).

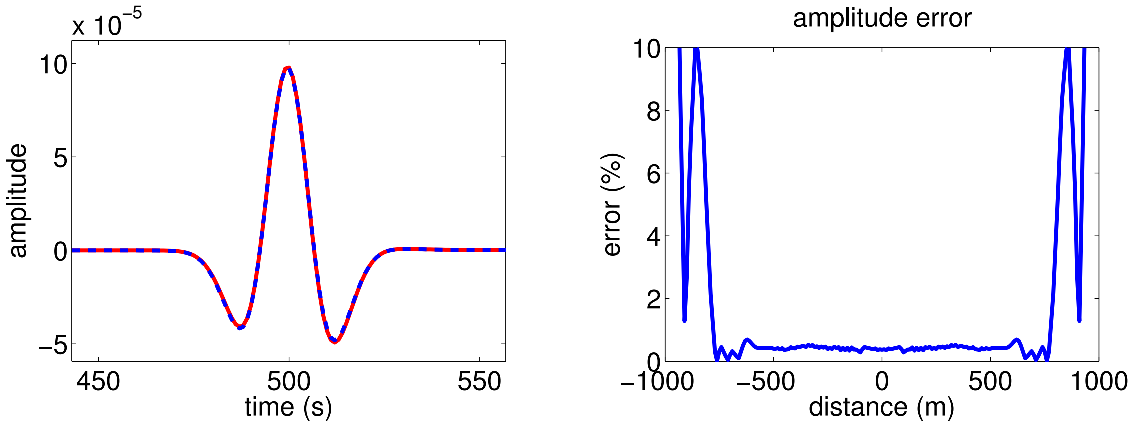

Figure 4: Left: Seismic traces at $x = 500$ m from modelling (continuous line) and redatuming (dashed line). Right: relative amplitude error obtained comparing the peaks of the two seismograms in Figure 3.

Homogeneous layers separated by a flat interface

In this section, we extend the previous results to redatuming to a flat interface separating two homogeneous media with velocities v_1 and v_2. As before, we assume for simplicity that the input data were acquired at $z = z_i = 0$ and that the interface that takes the role of the new datum is at $z = z_o =$ constant (Figure 5). As before, a point (η, τ) in the output section determines the circular output isochron

$$z = \mathcal{Z}_o(x; \eta, \tau) = z_o + \sqrt{R_o^2 + (x - \eta)^2}\ , \tag{22}$$

where now $R_o = v_2\tau/2$.

To find an explicit expression for the reflection traveltime from an input source-receiver pair to the isochron (22), we proceed differently from the homogeneous case. Because of the refacting of the ray at the interface, finding the stationary point x^* of $\mathcal{T}_D(x; \xi)$ involves the solution of a fourth-order equation. Thus, we will localize this point using geometrical arguments. As in the homogeneous case, the unique reflection ray that connects a source-receiver pair to the isochron must cross the center of the semicircle (Figure 5). Therefore, the ray also travels the distance $2(R_o + \alpha)$, however in two different media. Thus we can write the redatuming time as

$$\mathcal{T}_{red}(\xi; \eta, \tau) = \frac{2R_o}{v_2} + \frac{2\alpha}{v_1} = \tau + 2\frac{\alpha}{v_1}\ . \tag{23}$$

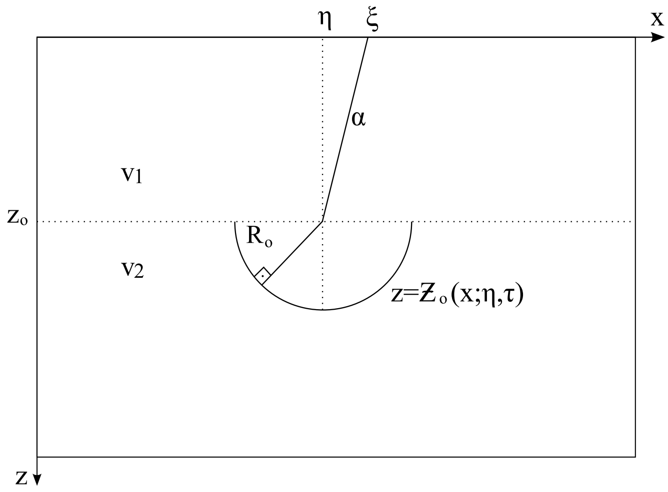

Figure 5: The unique reflection from an input source-receiver pair to the isochron $z = \mathcal{Z}_o(x;\eta,\tau)$ crosses the center of the semicircle.

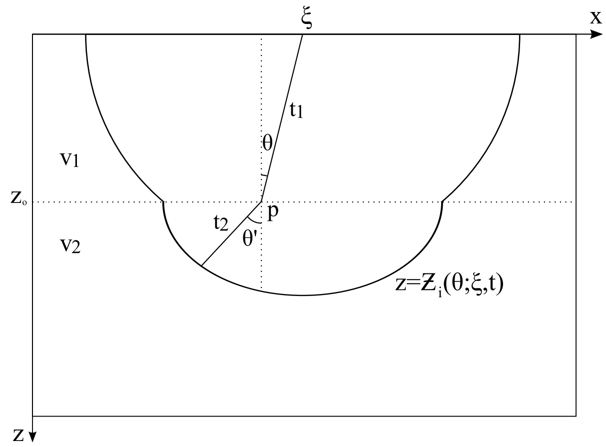

Figure 6: Given (ξ, t) in the input section, we have the isochron $z = \mathcal{Z}_i(\theta;\xi,t)$ in depth.

From the geometry (Figure 5) and Snell's law, we find the respective stationary point where this reflection occurs to be given by

$$x^* = n\frac{\gamma R_o}{\alpha} + \eta \qquad \text{and} \qquad z^* = z_o + \frac{R_o}{\alpha}|\tilde{z}_o| \,, \tag{24}$$

where $n = v_2/v_1$ is the refraction index and $\tilde{z}_o^2 = \alpha^2 - n^2\gamma^2$.

We observe that redatuming time (23) is the same as obtained in equation (14). This result shows that the kinematics of redatuming are independent of the velocity v_2 below the new datum.

Our next step is to find the weight function (7) for the present situation. First, we need to determine the isochron $\mathcal{Z}_i(x;\xi,t)$ of a point (ξ,t) in the input section. To simplify the considerations, we parameterize the problem using the angle θ between the ray and the surface normal at S_i (see Figure 6) instead of the horizontal coordinate x. The isochron will then be described by $z = \mathcal{Z}_i(\theta;\xi,t)$ $(-\pi/2 < \theta < \pi/2)$.

The given time value t that defines the isochron can be thought of as composed of two contribuitions, i.e., $t = t_1 + t_2$, where t_1 is the two-way time from the source to the interface and where t_2 is the remaining two-way time to the isochron (Figure 6). The geometry implies that

$$\cos\theta = \frac{z_o}{v_1 t_1/2} : t_1 = \frac{2z_o}{v_1\cos\theta} \,. \tag{25}$$

The horizontal coordinate x_p of the refraction point is found correspondingly from a function of θ as

$$\sin\theta = \frac{\xi - x_p}{v_1 t_1/2} : x_p = \xi - z_o \tan\theta \ . \tag{26}$$

The refraction angle θ' is related to θ via Snell's law, i.e.,

$$\frac{\sin\theta}{v_1} = \frac{\sin\theta'}{v_2} \ . \tag{27}$$

Thus,

$$\frac{v_2 \sin\theta}{v_1} = \sin\theta' = \frac{x_p - x}{v_2 t_2/2} \ . \tag{28}$$

Using that $t_2 = t - t_1$ and the result (25), we can write x as

$$x(\theta;\xi,t) = \xi + (n^2-1) z_o \tan\theta - n\frac{v_2 t}{2}\sin\theta \ . \tag{29}$$

Moreover, from Pythagoras' theorem, we have

$$\left(\frac{v_2 t_2}{2}\right)^2 = (z - z_o)^2 + (x_p - x)^2 \ . \tag{30}$$

Combining these two equation together with x_p from equation (26), we obtain the isochron

$$z = \mathcal{Z}_i(\theta;\xi,t) = z_o + \sqrt{1-(n\sin\theta)^2}\left(\frac{v_2 t}{2} - n\frac{z_o}{\cos\theta}\right) \ . \tag{31}$$

Note that for $v_1 = v_2 = v_o$ the isochron, now represented by $x(\theta;\xi,t)$ and $z(\theta;\xi,t)$, reduces to the one for the homogeneous medium, i.e., the semicircle with radius $v_o t/2$.

To evaluate the curvature of this isochron, we need the first and second derivatives of $\mathcal{Z}_i(\theta;\xi,t)$ with respect to x. Because of the chain rule we have

$$\frac{d\mathcal{Z}_i}{dx} = \frac{d\mathcal{Z}_i}{d\theta}\left(\frac{dx}{d\theta}\right)^{-1} \ , \tag{32}$$

where,

$$\frac{d\mathcal{Z}_i}{d\theta} = \frac{n\sin\theta[2z_o(n^2-1) - n v_2 t\cos^3\theta]}{2\cos^2\theta\sqrt{1-(n\sin\theta)^2}} \ , \tag{33}$$

and

$$\frac{dx}{d\theta} = \frac{2z_o(n^2-1) - n v_2 t \cos^3\theta}{2\cos^2\theta} \ . \tag{34}$$

Thus, we obtain

$$\frac{d\mathcal{Z}_i}{dx} = n\frac{\sin\theta}{\sqrt{1-(n\sin\theta)^2}} = \tan\theta' \ . \tag{35}$$

Correspondingly, the second-order derivative with respect x is

$$\frac{d^2\mathcal{Z}_i}{dx^2} = \frac{d}{dx}\frac{d\mathcal{Z}_i}{dx} = \frac{d}{d\theta}\left(\frac{d\mathcal{Z}_i}{dx}\right)\left(\frac{dx}{d\theta}\right)^{-1} \ , \tag{36}$$

where

$$\frac{d}{d\theta}\left(\frac{d\mathcal{Z}_i}{dx}\right) = \frac{n\cos\theta}{[1-(n\sin\theta)^2]^{3/2}} \ . \tag{37}$$

We find

$$\frac{d^2\mathcal{Z}_i}{dx^2} = \frac{n\cos^3\theta}{[z_o(n^2-1) - n(v_2 t/2)\cos^3\theta][1-(n\sin\theta)^2]^{3/2}} \ . \tag{38}$$

With these expressions, we obtain the curvature as

$$K_i = \frac{n \cos^3 \theta}{z_o(n^2 - 1) - n(v_2 t/2) \cos^3 \theta} \; . \tag{39}$$

We are interested in the curvature at the stationary point. According to Figure 5, θ^* must satisfy $\cos\theta^* = z_o/\alpha$ and $\sin\theta^* = -\gamma/\alpha$, where α and γ continue as defined in the beginning of the section. Using this fact and that t is given by (23), we can write

$$K_i = -\frac{n z_o^2}{\alpha \tilde{z}_o^2 + n R_o z_o^2} \; . \tag{40}$$

Again, we observe that for $v_1 = v_2 = v_o$ this curvature reduces to the previous result (18) for a homogeneous medium.

With this curvature and the curvature of the isochron (22), we can now calculate the exponential term in the weight function. First, we observe that

$$K_i - K_o = \frac{\alpha \tilde{z}_o^2}{R_o[\alpha \tilde{z}_o^2 + n R_o z_o^2]} \; . \tag{41}$$

Thus, the value of $\mathrm{sign}(K_i - K_o)$ depends on the sign of $\tilde{z}_o^2 = \alpha^2 - n^2\gamma^2 = \alpha^2(1 - n^2 \sin^2\theta) = \alpha^2 \cos^2\theta'$, which is positive for all nonevanescent transmissions. Thus, as for the homogeneous case,

$$\kappa = \frac{1}{2}[1 - \mathrm{sign}(K_i - K_o)] = 0 \; , \tag{42}$$

which means that the weight function for redatuming in this situation is still a real quantity and any phase changes in the events are taken care of by the stacking procedure itself.

Next, we will investigate the factors that constitute the geometrical spreading. As the medium is homogeneous in each layer, we have $\sigma_{iS} = v_1\alpha + v_2 R_o$ and $\sigma_{oS} = v_2 R_o$. The 2D spreading in the second layer is again $\bar{\mathcal{L}}_{oS} = \sqrt{R_o/v_2}$.

With all its individual contributions calculated, we can express the weight function of the stacking operator (1) for redatuming to a planar interface as

$$W_{red}(\xi;\eta,\tau) = \sqrt{\frac{2}{v_2}} \frac{n z_o}{\alpha^{3/2} R_o} \sqrt{\frac{(\alpha + nR_o)(\alpha \tilde{z}_o^2 + n R_o z_o^2)}{n \tilde{z}_o^2}} \; . \tag{43}$$

Unlike in homogeneous media, the aperture A in stack (1) is now restricted if $v_2 > v_1$. The reason is that supercritical rays don't penetrate the interface. Thus, the corresponding source positions do not contribute to the stack. The critical angle θ_c satisfies

$$\frac{\sin\theta_c}{v_1} = \frac{\sin 90^o}{v_2} = \frac{1}{v_2} \; . \tag{44}$$

Thus, the aperture condition can be formulated as

$$|\sin\theta| \leq |\sin\theta_c| : \frac{|\xi - \eta|}{\alpha} \leq \frac{v_1}{v_2} \; , \tag{45}$$

or, in other words, for a given η, only those sources will contribute to the stack whose coordinates ξ satisfy

$$|\xi - \eta| \leq \frac{z_o}{\sqrt{n^2 - 1}} \; . \tag{46}$$

Note that the maximum aperture tend to infinity when n tends to one, which is the case of the homogeneous medium discussed above.

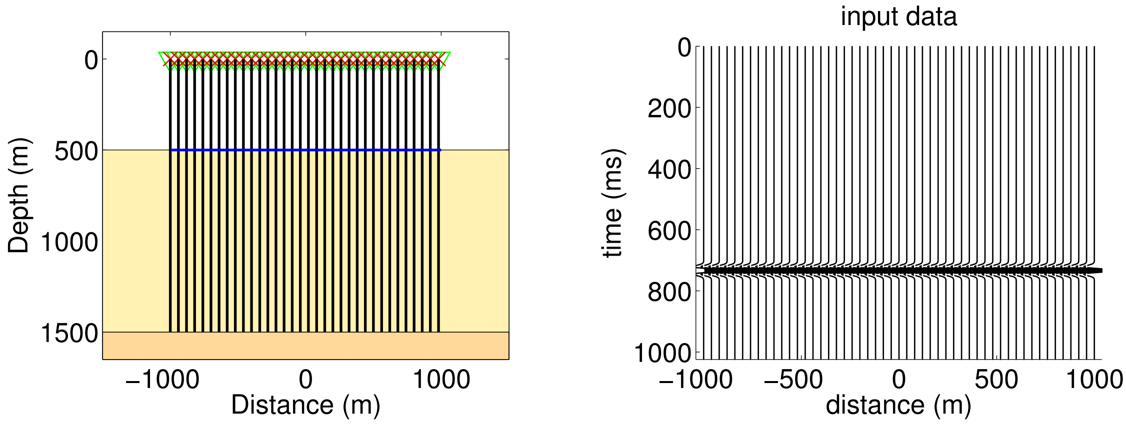

Figure 7: Simulation with the acquisition line at $z_i = 0$ (left) and its respective seismic section (right).

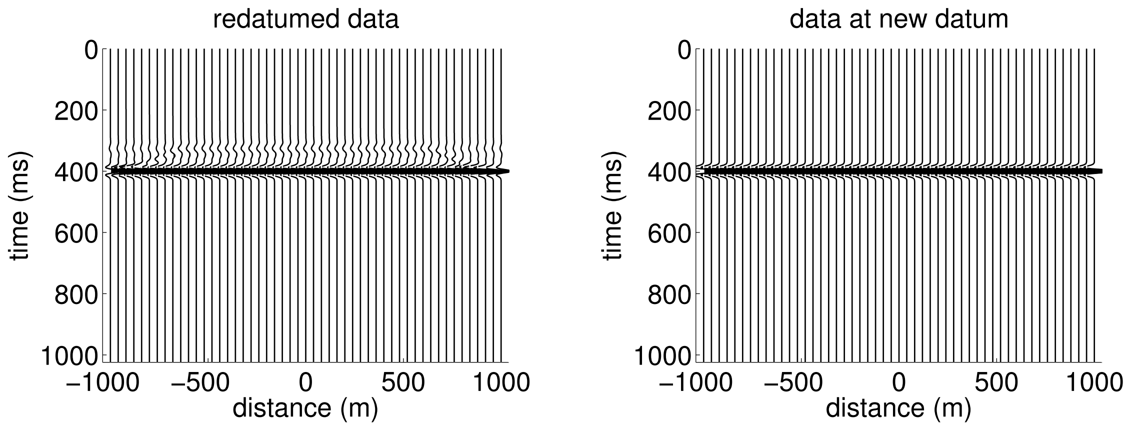

Figure 8: Seismic sections obtained from redatuming (left) and from modelling considering the acquisition line at $z = z_o$ (right).

Numerical example.— In the following numerical test, we considered a planar reflector without topographies in input and output datum (see Figure 7). The acquisistion parameters are the same as used in the previous example. The input data are modeled at $z = z_i = 0$ and the output datum is at $z = z_o = 500$ m depth. The medium velocities are 3000 m/s above the datum and 5000 m/s below it. As before, the seismic section on the right side in Figure 7 is the modelled input data. We use it to apply the redatuming operator. The resulting seismogram is depicted in the left panel of Figure 8 and compared to the synthetic data modelled at the datum level (right panel). Except for a weak precursor to the reflection event, the modelled and redatumed sections are identical. The detailed quality analysis was done in the same way as before. The results are depicted in the Figure 9. The left panel compares the pulses of the trace at $x = 500$ m in the modelled (continuous) and redatumed (dashed) sections. The actual reflection event is practically identical. As already observed in Figure 8, the redatumed trace has a small precursor which is due to imperfect tapering of the stacking operator. The right panel of Figure 9 shows the amplitude error along the redatumed reflection event. Except for the boundary region, the error is under 2%.

Homogeneous medium ($v = v_o$) with topography

In this section, we calculate the redatuming operator for the ZO configuration in a homogeneous medium ($v = v_o$), now allowing for topography both at the acquisition surface $z_i = z_i(x)$ and at new datum $z_o = z_o(x)$.

The derivation is very similar to the one of the first section. Therefore, we will restrict the discussion

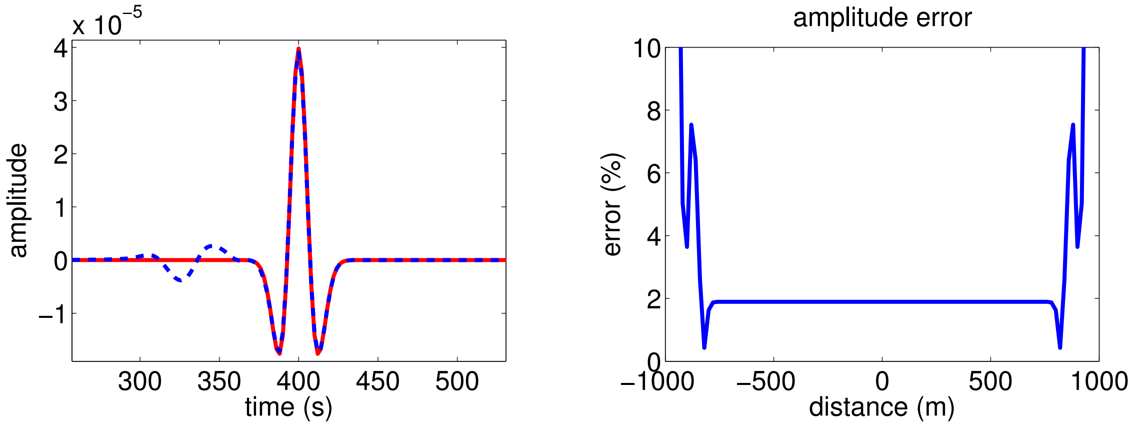

Figure 9: Left: Seismic traces at $x = 500$ m from modelling (continuous line) and redatuming (dashed line). Right: relative amplitude error obtained comparing the peaks of the two seismograms in Figure 8.

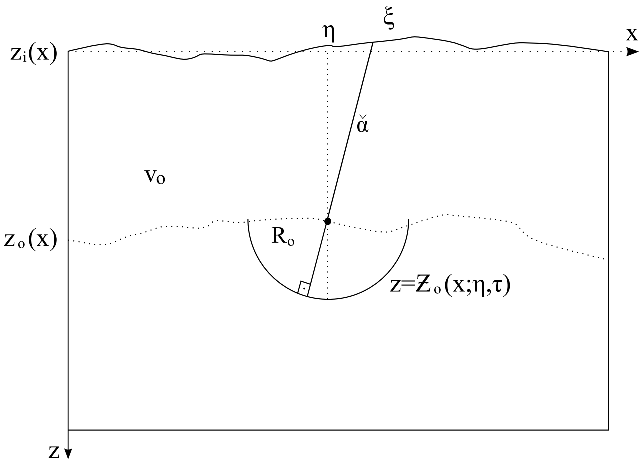

Figure 10: The unique point that go up, arrive at isochron $z = \mathcal{Z}_o(x; \eta, \tau)$ and go back to the same place, is the ray that cross the center of the semicircle.

to the geometrical arguments. Given a point (η, τ) in the output section, the isochron (9) must now be modified to

$$z = \mathcal{Z}_o(x; \eta, \tau) = z_o(\eta) + \sqrt{R_o^2 - (x - \eta)^2} \ , \tag{47}$$

where still $R_o = v_o\tau/2$.

We are only interested in the orthogonal reflections to isochron (47). As before, (see Figure 10) the ray path covers a distance of $2(R_o + \alpha)$ on its way from the source back to the receiver. Thus, the redatuming time of isochron (47) in the input section is

$$\mathcal{T}_{red}(\xi; \eta, \tau) = \frac{2}{v_o}(R_o + \alpha) = \tau + 2\frac{\alpha}{v_o} \ , \tag{48}$$

where now $\alpha = \sqrt{\gamma^2 + [z_o(\eta) - z_i(\xi)]^2}$. Moreover, the respective stationary point where this reflection occurs is

$$x^* = \frac{\gamma R_o}{\alpha} + \eta \ . \tag{49}$$

Now, we calculate the curvature of the isochrons. To take the topography into account, the input isochron of equation (15) must be modified to

$$z = \mathcal{Z}_i(x; \xi, t) = z_i(\xi) + \sqrt{(v_o t/2)^2 - (x - \xi)^2} \ . \tag{50}$$

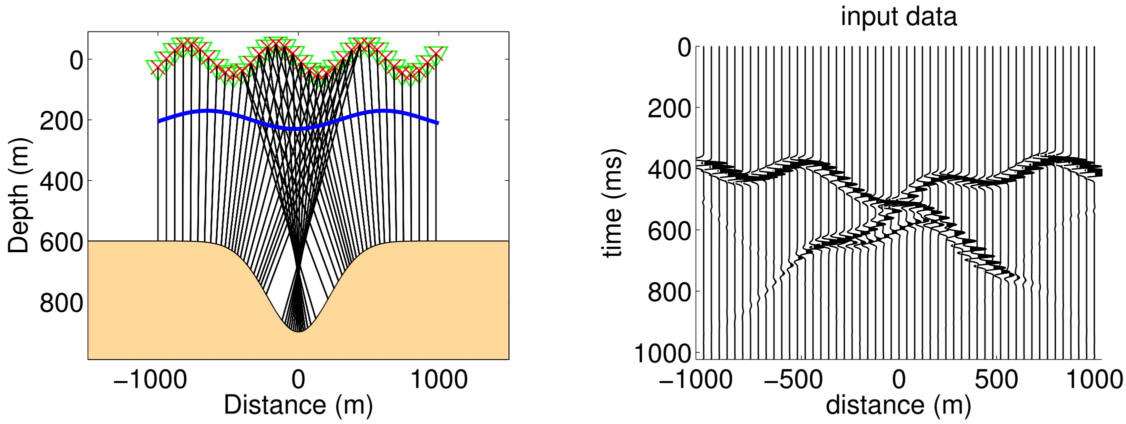

Figure 11: Simulation with the acquisition line at $z_i(x)$ (left) and its respective seismic section (right).

This isochron is different of (15), just on the fact that now the topography influences in the depth of the isochron. So, the Since the modifications of the isochrons (47) and (50) only affect their vertical position, their curvatures are still given by equations (16) and (18), respectively. Thus, the exponential factor in the weight function continues to be given by equation (20).

We still need expressions for the geometrical-spreading factors. Analogously to the previous results, $\sigma_{iS} = v_o(R_o + \alpha)$ and $\sigma_{oS} = v_o R_o$, and the 2D spreading is the square root of the traveltime, i.e., $\bar{\mathcal{L}}_{oS} = \sqrt{R_o/v_o}$.

The last quantity to be specified in terms of model quantities is the emergence angle θ_S^* of the stationary ray at S_i. Because of the surface topography, this angle no longer equals the ray propagation angle θ. From geometrical considerations, it is not difficult to see that

$$\cos\theta_S^* = \cos\phi(z_o(\eta) - z_i(\xi) - \gamma z_i'(\xi))/\alpha \ , \tag{51}$$

where $z_i' = dz_i/dx$ denotes the derivative of the function z_i that describes the surface topography.

Combining these results, we have the weight function given by

$$W_{red}(\xi;\eta,\tau) = \sqrt{\frac{2}{v_o}}\left(\frac{R_o+\alpha}{R_o}\right)\frac{[z_o(\eta) - z_i(\xi) - \gamma z_i'(\xi)]}{\alpha^{3/2}} \ . \tag{52}$$

Numerical example.— As before, we have tested the expressions for the redatuming stacking operator with a simply synthetic data set. We considered a synclinal reflector below a homogeneous overburden. The acquisition surface and datum have different topographies (see Figure 11). The acquisition parameters are the same as before. Again, the synthetic seismic section at right in the Figure 11 is the input to the redatuming operator. Figure 12 compares the resulting redatumed section (left) with the synthetic data as modelled at the datum level (right). As before, the sections look very similar except for a few boundary effects. The quality analysis in Figure 13 compares two traces at $x = 250$ m from these sections (left) and depicts the relative amplitude error along the first arrival (right). Again, the traces are virtually identical. The error is below 1% except in the boundary region and in the center of the caustic, where the events intersect.

CONCLUSIONS

In this work, we have developed a true-amplitude theory for a diffraction-stack type redatuming operation. Such an operation can be conceived of as being composed of a true-amplitude diffraction-stack migration and a true-amplitude isochron-stack demigration, as described in the unified approach to seismic reflection imaging (Hubral et al., 1996; Tygel et al., 1996).

We have derived the general expressions for the stacking line and weight function for such a redatuming operator. The quantities involved can be calculated by ray tracing in a given macro-velocity model. For

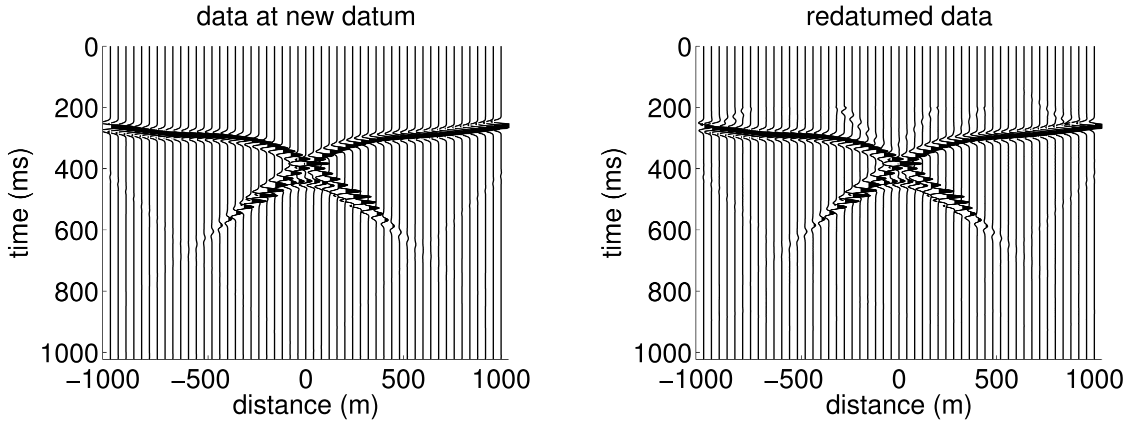

Figure 12: Seismic sections obtained from redatuming to $z = z_o(x)$ (left) and from modelling considering the acquisition line at $z = z_o(x)$ (right).

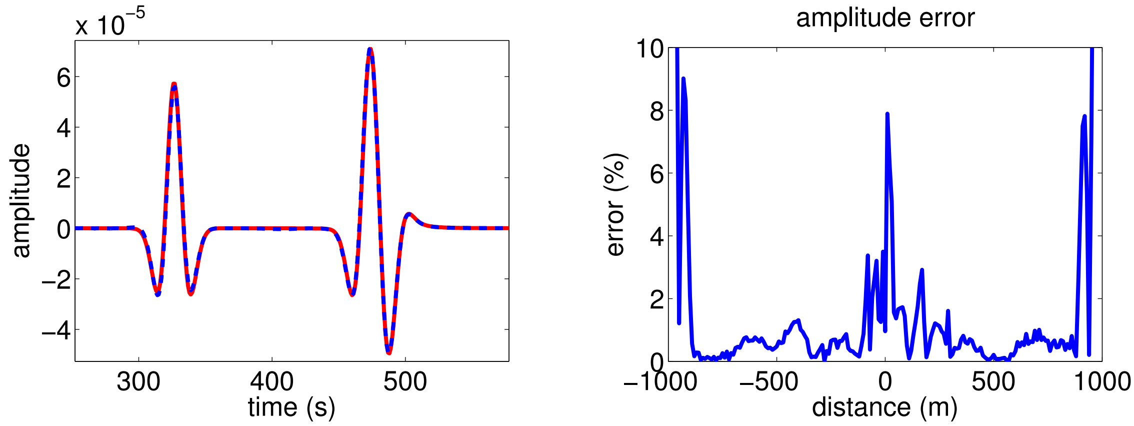

Figure 13: Left: Seismic traces at $x = 250$ m from modelling (continuous line) and redatuming (dashed line). Right: relative amplitude error obtained comparing the peaks of the two seismograms in Figure 12.

zero-offset data in two simple situations, being a homogeneous overburden and a horizontal interface at the datum level, we have explicitly expressed the stacking line and weight function. We have seen that in both situations, no contributions from the redatuming process to the number of caustics in the data need to be taken into account. In other words, the weight function is a real quantity. All phase shifts in the wavelet due to the new acquisition level are automatically taken into account by the stacking procedure.

In simple numerical examples, we have demonstrated the quality of the redatuming operation by comparing redatumed data to the ideal output obtained from modelling. We have seen that the proposed diffraction-stack type redatuming indeed correctly transforms traveltimes, amplitudes and wavelet shapes of the simulated data.

Ongoing research will extend the applicability of the method to more general situations. Of particular interest is the situation where the new datum is a reflector with topography. Moreover, we intend to address the question about the degree of medium inhomogeneity that is acceptable for constant-velocity redatuming to work with a tolerable error.

ACKNOWLEDGMENTS

This research has been supported in part by the São Paulo State Research Foundation (FAPESP, proc. 05/00305-0), the Brazilian National Research Council (CNPq), and the sponsors of the Wave Inversion Technology (WIT) Consortium.

REFERENCES

Berryhill, J. R. (1979). Wave-equation datuming. *Geophysics*, 44:1329–1344.

Berryhill, J. R. (1984). Wave equation datuming before stack. *Geophysics*, 49(11):2064–2067.

Bevc, D. (1995). Imaging under rugged topography and complex velocity struture. *Ph.D. dissertation, Stanford University*.

Hubral, P., Schleicher, J., and Tygel, M. (1996). A unified approach to 3-D seismic reflection imaging – Part I: Basic concepts. *Geophysics*, 61(3):742–758.

Schleicher, J., Tygel, M., and Hubral, P. (1993). 3-D true-amplitude finite-offset migration. *Geophysics*, 58(8):1112–1126.

Schneider, W. A., Phillip, L. D., and Paal, E. F. (1995). Wave-equation velocity replacement of the low-velocity layer for overthrust-belt data. *Geophysics*, 60:573–580.

Schuster, G. T. and Zhou, M. (2006). A theoretical overview of model-based and correlation-based redatuming methods. *Geophysics*, 71(4):SI103–SI110.

Tegtmeier, S., Gisolf, A., and Verschuur, E. (2004). 3D sparse data Kirchhoff redatuming. *Geophysical Prospecting*, 52(6):509–521.

Tygel, M., Schleicher, J., and Hubral, P. (1996). A unified approach to 3-D seismic reflection imaging – Part II: Theory. *Geophysics*, 61(3):759–775.

Tygel, M., Schleicher, J., Hubral, P., and Santos, L. (1998). 2.5D true-amplitude Kirchhoff migration to zero offset in laterally inhomogeneous media. *Geophysics*, 63(2):557–573.

Wapenaar, C. P. A. (1993). Kirchhoff-helmholtz downward extrapolation in a layered medium with curved interfaces. *Geophys J. Int.*, 115:445–455.

Wapenaar, C. P. A., Cox, H. L. H., and Berkhout, A. J. (1992). Elastic redatuming of multicomponent seismic data. *Geophysical Prospecting*, 40(04):465–482.

Wiggins, J. W. (1984). Kirchhoff integral extrapolation and migration of nonplanar data. *Geophysics*, 49:1239–1248.

Yilmaz, O. and Lucas, D. (1986). Prestack layer replacement. *Geophysics*, 51:1355–1369.