TRAVELTIME-BASED TRUE-AMPLITUDE MIGRATION IN ANISOTROPIC MEDIA

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ABSTRACT

True-amplitude Kirchhoff migration is a task of high computational effort. A substantial part of this effort is spent on the calculation of Greens functions (GFTs), i.e. traveltime tables and amplitude-preserving weight functions. So far, dynamic ray tracing (DRT) had to be applied to generate the GFTs. However, DRT is an expensive and complicated task if anisotropy has to be considered. Also, storing the GFTs leads to large demands in computer storage in addition to the high requirements in CPU time. In this paper we propose a strategy to compute the weight functions directly from coarsely-gridded traveltimes. Together with a fast and accurate method for the interpolation of the traveltimes onto the required fine migration grid, this leads to savings in storage. Since the traveltimes can be computed with other methods, DRT is no longer required. This traveltime-based approach has already been successfully applied in isotropic media. A synthetic example now demonstrates the extension of the method for anisotropic multi-component data.

INTRODUCTION

True-amplitude prestack depth migration can be implemented as a specific form of Kirchhoff migration. In addition to providing a focused structural image of the subsurface, information on the reflection strength at the discontinuities in the medium is also available from such an image. This information can be used for AVO studies, which play a key role in reservoir characterisation.

True-amplitude migration is carried out in terms of a weighted diffraction stack. For each subsurface point, the seismic traces are stacked along the diffraction time surface for that point. Individual weight functions are applied during the stack to recover the reflection amplitude. The weights depend on dynamic wavefield properties which are usually computed by dynamic ray tracing (DRT) together with the diffraction traveltimes. For a 3D experiment this results in a tremendous amount of auxiliary data which have to be generated and stored, thus making true-amplitude migration a task of high computational costs.

During recent years, multi-component acquisition has become an emerging technology. Since seismic anisotropy has a large impact on the behaviour of shear wave propagation, it must be considered during the processing of such data. For true-amplitude migration of multi-component data in the presence of anisotropy, the weight function must be chosen accordingly (see, e.g., Druzhinin, 2003). In addition to the computational costs of DRT in anisotropic media which are much higher than in the isotropic case, DRT in any type of medium requires continuity of first- and second-order spatial derivatives of the elastic parameters.

We suggest to circumvent the problems associated with DRT by applying the traveltime-based strategy for true-amplitude migration in anisotropic media described in this paper. Here, the requirements in computer storage are significantly reduced, as the only auxiliary quantity required are the diffraction traveltimes, sampled on coarse grids. A fast and accurate interpolation is applied to obtain the stacking surfaces on the fine migration grid. We have originally introduced this strategy for isotropic media (Vanelle et al., 2006). In this paper, we present the extension to anisotropy. As the new anisotropic weight functions can be directly expressed in terms of the interpolation coefficients, this implementation does not require DRT, if other methods are used for the computation of the diffraction traveltime tables. A suitable method is, e.g., kinematic ray tracing which requires only continuity of first-order model derivatives and generates multi-arrival traveltimes also for anisotropic media.

In the next section we will begin with an outline of the method. After a short introduction to true-amplitude migration, we will describe the traveltime-based strategy in detail, including the traveltime interpolation and the determination of the weight functions. We will then illustrate the method with a synthetic multi-component data example, and end the paper with our conclusions.

METHOD

True-amplitude migration in anisotropic media

For simplicity, we will consider a 2.5D symmetry in this paper, where the medium does not vary in the direction perpendicular to the acquisition line. In terms of anisotropy, this also requires that the in-line direction is a symmetry plane for the wave propagation. The method, however, has an equivalent formulation for the 3D case with arbitrary anisotropy, where the restriction to symmetry planes is not necessary.

Following the derivation of Schleicher et al. (1993), Martins et al. (1997) have shown that in 2.5D the true-amplitude migrated output $V(M)$ at the subsurface point M can be obtained from the weighted diffraction stack described by

$$V(M) = \frac{1}{\sqrt{2\pi}} \int_A \mathrm{d}\xi \, W(\xi, M) \, D_t^{-1/2} U(\xi, t) \Big|_{t=\tau_D(\xi,M)} \quad . \tag{1}$$

In Equation (1) the operator $D_t^{-1/2}$ denotes the Hilbert transform of the time half-derivative of the seismic data $U(\xi, t)$, where the data are assumed to consist of analytic traces. The integration is carried out over the aperture A which contains the considered trace positions represented by the so-called configuration parameter ξ. The stacking curve is the diffraction traveltime $t = \tau_D(\xi, M)$ for the depth point M, calculated in a previously determined macro-velocity model.

After a transformation to the frequency domain, the integral (1) can be solved in the high frequency limit by applying the method of stationary phase (Bleistein, 1984). This leads to the weight function

$$W_{2.5\mathrm{D}}(\xi, M) = \sqrt{\frac{\rho(G)}{\rho(S)}} \, \frac{\sqrt{v_z(G)\; v_z(S)}}{V(S)} \, \frac{1}{\mathcal{G}(S,\gamma)} \times$$
$$\frac{|N_{SM}^{(r)}\Sigma + N_{GM}^{(r)}\Gamma|}{\sqrt{|N_{SM}^{(r)}\, N_{GM}^{(r)}|}} \, \sqrt{\sigma_{SM} + \sigma_{GM}} \; \mathrm{e}^{-i\frac{\pi}{2}(\kappa_{SM}+\kappa_{GM})} \quad , \tag{2}$$

where $\rho(S)$ and $\rho(G)$ are the densities at the source and the receiver, respectively. The phase velocity at the source is denoted by $V(S)$, and $v_z(S)$ and $v_z(G)$ are the vertical components of the group velocities at source and receiver. The measurement configuration (e.g., common shot) is described by the scalars Σ and Γ, the radiation function of the source with the take-off angle γ is given by $\mathcal{G}(S, \gamma)$, and the κ_I are the KMAH-indices of the two ray branches, from the source to the image point, and from the receiver to the image point. Finally, the σ_I describe the out-of-plane geometrical spreading (see, e.g., Ettrich et al., 2002, for anisotropic media), and the $N_I^{(r)}$ are second-order derivatives of the traveltime with respect to the source (index $_{SM}$) or receiver (index $_{GM}$) and the tangent plane of the reflector (indicated by the superscript (r)).

Note that the $N_I^{(r)}$ are closely related to the DRT quantities Q describing the in-plane geometrical spreading. It is also possible to express the weight function (2) in terms of DRT quantities, however, the above formulation is better-suited for the traveltime-based approach suggested in this paper.

The traveltime-based approach

In this section, we will explain how the weight function (2) can be computed from traveltimes only. We use the following expression for the traveltime from a source S at the position $\mathbf{s}$ to a subsurface point M at the position $\mathbf{m}$ (Vanelle and Gajewski, 2002):

$$\begin{aligned} T^2(\mathbf{s},\mathbf{m}) &= (T_{SM} - \mathbf{p}_{SM}\Delta\mathbf{s} + \mathbf{q}_{SM}\Delta\mathbf{m})^2 \\ &\quad -2\,T_{SM}\,\Delta\mathbf{s}\cdot\underline{\mathbf{N}}_{SM}\Delta\mathbf{m} \\ &\quad -T_{SM}\,(\Delta\mathbf{s}\cdot\underline{\mathbf{S}}_{SM}\Delta\mathbf{s} - \Delta\mathbf{m}\cdot\underline{\mathbf{G}}_{SM}\Delta\mathbf{m}) \quad . \end{aligned} \tag{3}$$

Equation (3) is a hyperbolic expansion of the traveltime in source and subsurface point coordinates. The traveltime T_{SM} is that from the source at $\mathbf{s}_0$ to the subsurface point at $\mathbf{m}_0$. We will refer to the combinations of $(\mathbf{s}_0, \mathbf{m}_0)$ as the expansion points that are represented by the coarse grid. The slowness vectors $\mathbf{p}_{SM}$ and $\mathbf{q}_{SM}$ are the first-order traveltime derivatives at the source and subsurface point, respectively. The three matrices

$$\begin{aligned} S_{SM_{ij}} &= -\frac{\partial^2 T}{\partial s_i\,\partial s_j} \quad , \qquad G_{SM_{ij}} = \frac{\partial^2 T}{\partial m_i\,\partial m_j} \quad , \\ N_{SM_{ij}} &= -\frac{\partial^2 T}{\partial s_i\,\partial m_j} \quad , \end{aligned} \tag{4}$$

are the second-order derivatives of the traveltime. Since the traveltimes are in any event required for the stacking surface, we assume that these are available and sampled on coarse grids. As described in Vanelle and Gajewski (2002), the coefficients in Equation (3) can be determined from the traveltime tables, with the exception of the vertical slowness at the source. This coefficient can be directly obtained following Červený and Pšenčík (1972) for the 2.5D case, or Vanelle and Gajewski (2004) for the 3D situation with arbitrary anisotropy. Once the slowness vector is known, we can compute the group velocity and its vertical component (Červený and Pšenčík, 1972). We apply a corresponding expression for the traveltime from a receiver G at the position $\mathbf{g}$ to a subsurface point M.

The coefficients can then be applied for the interpolation of the traveltimes onto the fine migration grid. If only first-arrival traveltimes are given, the KMAH indices are zero. If later arrivals are considered, it is convenient to generate the tables for the individual arrivals with an algorithm that outputs them sorted by KMAH, e.g., with the wavefront-oriented ray tracing technique by Coman and Gajewski (2005), and its extension to anisotropic media (Kaschwich and Gajewski, 2003).

Note that the second-order derivative matrices in Eq. (4) are taken in Cartesian coordinates (x, z), whereas the $N_I^{(r)}$ are taken in the reflector tangent plane. The $N_I^{(r)}$ are obtained from the $N_{I_{ij}}$ by

$$N_I^{(r)} = N_{I_{xx}} \cos\beta - N_{I_{xz}} \sin\beta \quad , \tag{5}$$

where

$$\beta = \frac{q_{SM_x} + q_{GM_x}}{q_{SM_z} + q_{GM_z}} \quad . \tag{6}$$

Finally, with

$$\sigma_I = \frac{1}{N_{I_{yy}}} \tag{7}$$

the weight (2) can be computed from the coefficients of Equation (3).

In the following section, we will apply the traveltime interpolation (3) and the weight functions (2) to a synthetic data set.

EXAMPLE

We have applied the method to the simple anisotropic velocity model displayed in Figure 1. In both layers we chose $\epsilon = \delta = 0.1$ because this enabled us to compute the analytical reflectivity for comparison with the migration results. Ray synthetic seismograms were generated for an explosion source. The required traveltime tables for the qP- and SV-waves were computed analytically. These were the only input data needed for the computation of the true-amplitude weight functions (2).

The depth-migrated PP section is shown in Figure 2a; the PS section in Figure 2b. Since we have used the correct elastic parameters for the generation of the traveltimes, the migration result shows the reflector imaged in the correct location.

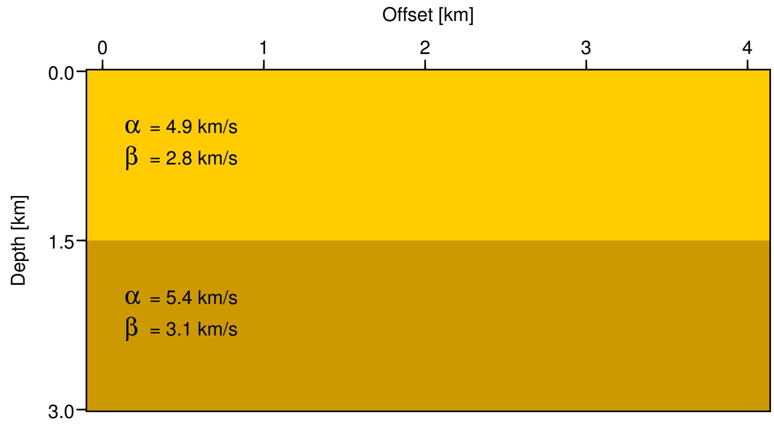

Figure 1: The anisotropic velocity model. Thomsen's parameters were chosen as $\epsilon = \delta = 0.1$ and $\gamma = 0.1$.

Since another aim of the true-amplitude migration is to recover the reflectivity, we have picked the amplitudes from the image gathers. The results are shown together with the analytic solutions in Figure 3a for the PP result, and in Figure 3b for the PS result. As we can see, both reconstructed AVO curves coincide with the analytic values. The deviations at higher offsets are caused by the limited extent of the receiver line. Due to the asymmetry of the ray paths, the offset range is different for the PP and the PS case.

CONCLUSIONS

By applying the traveltime-based strategy for true-amplitude migration in anisotropic media we can obtain a dynamically correct depth migrated image without the need for dynamic ray tracing. Although the example shown was simple, we conclude that the method is equally suited for complex situations. This conclusion is supported by the application of our method to complex isotropic examples (Vanelle et al., 2006), as well as the accuracy of the determination of geometrical spreading from traveltimes for 3D anisotropic models (Vanelle and Gajewski, 2003), using the same coefficients as the weight function introduced in this paper. The extension to three dimensions is straightforward.

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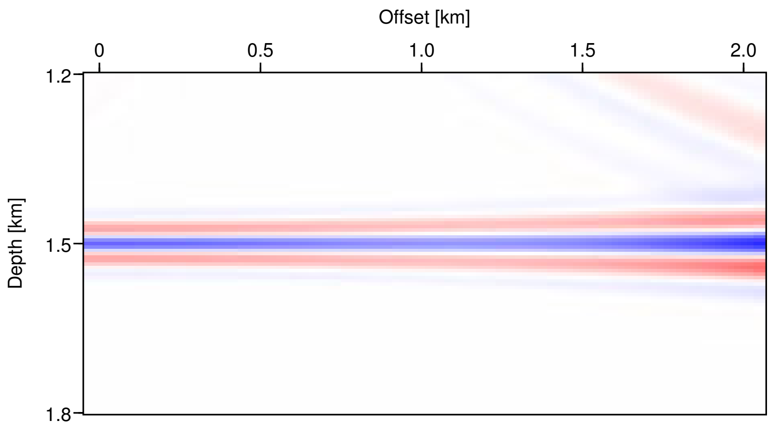
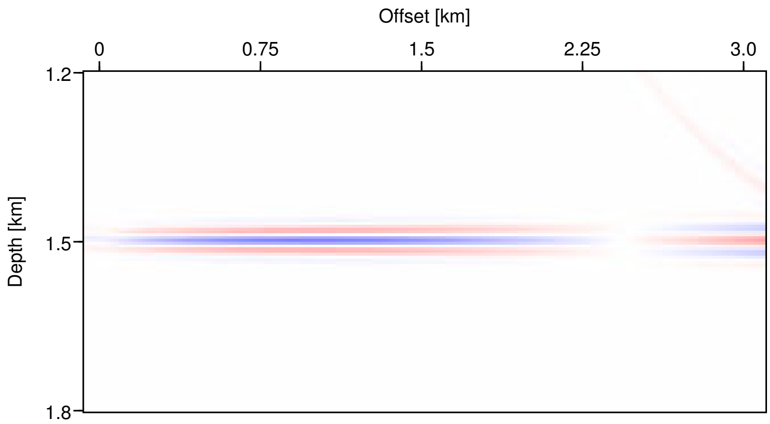
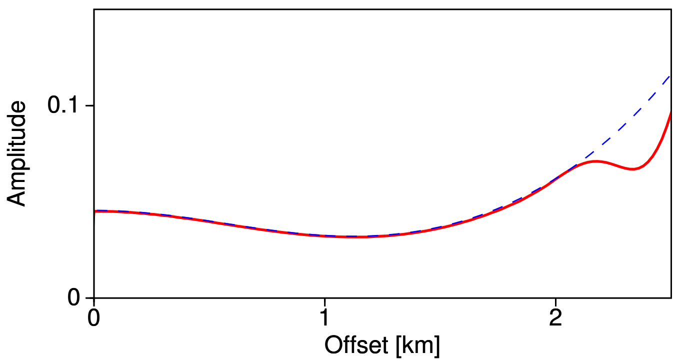


(a)

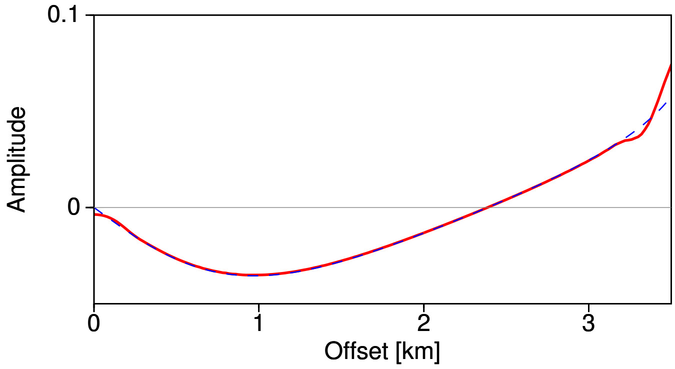


(b)

Figure 2: (a) The depth-migrated PP section, and (b) the depth-migrated PS section



(a)



(b)

Figure 3: AVO for (a) PP reflections and (b) PS reflections: the reconstructed reflectivity (solid red line) coincides with the analytic solution (blue dashed line).

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