POSTSTACK TRUE AMPLITUDE WAVE-EQUATION MIGRATION

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keywords: *true amplitude, migration*

ABSTRACT

In homogeneous media, the two-way wave operator can be substituted by the product of two one-way wave operators each of which generates a one-way wave equation. One of these equations has a downgoing wave and the other has an upgoing wave as a solution. Those one-way waves have the same traveltime and amplitudes as the full wave since they satisfy the same eikonal and transport equation. However, in heterogeneous media, the standard one-way waves satisfy only the same eikonal equation as the full wave. Thus, in this case, the amplitudes of the migrated section obtained through a migration method based on the standard wave equations are incorrect. However, the standard one-way waves can be modified in order to produce the true amplitude one-way waves, which not only have the same traveltimes but also the same amplitudes as the full wave. They use these true amplitudes one-way wave equations to preserve the amplitudes in common-shot wave-equation migration. Our goal is to modify phase-shift migration in such a way that it uses the true amplitude one-way wave equations instead of the standard ones, in order to realize a true amplitude wave equation migration for zero-offset data.

INTRODUCTION

Many seismic migration methods, particularly those directly based on the wave equation, take only care of the kinematic aspects of the imaging problem (i.e., the position and structure of the seismic reflectors), while incorrectly treating the dynamics (amplitudes, related to the energy carried by the seismic wavefield). However, as post-migration AVO and AVA studies are becoming more and more important, the correct treatment of migration amplitudes becomes imperative.

In this work, we study wave-equation migration based on one-way wave equations. We are interested in such one-way wave equations that correctly describe not only the traveltime but also the amplitude of the resulting one-way waves. These one-way wave equations are referred to as true-amplitude one-way wave equations.

In homogeneous media, the product of the two differential operators of the two one-way wave equations, which are first-order differential equations, yields the differential operator of the full wave equation. The one-wave wave operators allow to separate the full wavefield into its components traveling in different directions. Generally, the factorization is used to split the wavefield into its up- and downgoing parts. In this form, the one-way wave equations are useful in modeling and, principally, in migration.

In a homogeneous medium, traveltimes and amplitudes of the one-way waves, i.e., the solutions of the so-obtained one-way wave equations, are identical to those of the solution of the full wave equation. However, in inhomogeneous media, the use of the same one-way wave equation leads to different amplitudes than those of the solution of the full wave equation.

Recently, Zhang et al. (2003) showed how to modify the differential operators of the one-way wave equations such that, in zero-order ray approximation, the amplitudes are the same as those governed by the full wave equation. They have shown how to use the modified one-way wave equations in finite-difference true-amplitude common-shot wave-equation migration. Here, we transfer their ideas to poststack (zero-offset) phase-shift migration (Gazdag, 1978) using the true-amplitude one-way wave equations. Since wave-equation migration needs data that are result of a single wave equation, we adopt the strategy of Bleistein et al. (2001) to transform the dynamics of zero-offset data to those of exploding-reflector data.

METHOD

We consider the two-dimensional acoustic wave equation

$$\mathcal{L}u = \nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0 \ , \tag{1}$$

where $u = u(x,z,t)$ is the seismic wave field, and where the propagation velocity c may be constant or depend on one or two spatial coordinates.

Ray equations

Let us start with the simple case of a velocity that depends only on depth, i.e., $c = c(z)$. In this case, the solution of equation (1) can be found using its Fourier transform in time as well as in the horizontal coordinate, viz.

$$\frac{\partial^2 u}{\partial z^2} + \omega^2 p_z^2 u = 0 \ . \tag{2}$$

Here, we have introduced the horizontal and vertical components of the slowness vector,

$$p_x = \frac{k_x}{\omega} \tag{3}$$

and

$$p_z = \frac{k_z}{\omega} = \pm\frac{1}{c(z)}\sqrt{1-(c(z)p_x)^2} \ , \tag{4}$$

where k_x is the wavenumber component relative to coordinate x and

$$k_z = \pm\sqrt{\frac{\omega^2}{c^2} - k_x^2} = \frac{\omega}{c}\sqrt{1-\frac{(ck_x)^2}{\omega^2}} \ . \tag{5}$$

Substitution of the ray ansatz

$$u(x,z,\omega) = A(x,z)\exp\{i\omega\tau(x,z)\} \tag{6}$$

in the Helmholtz equation (2) leads to the eikonal and transport equations

$$\left(\frac{\partial\tau}{\partial z}\right)^2 = p_z^2 \quad : \quad \frac{\partial\tau}{\partial z} = \pm p_z \ , \tag{7}$$

and

$$2\frac{\partial\tau}{\partial z}\frac{\partial A}{\partial z} + \frac{\partial^2\tau}{\partial z^2}A = 0 \ . \tag{8}$$

Taking the derivative of the eikonal equation (7) with respect to z using Snell's law $\frac{\partial p_x}{\partial z} = 0$, we find

$$\frac{\partial^2\tau}{\partial z^2} = \pm\frac{\partial p_z}{\partial z} = \mp\frac{1}{c^3(z)p_z}\frac{\partial c(z)}{\partial z}. \tag{9}$$

Substitution of this result in the transport equation (8) yields

$$\begin{aligned} \pm\left[2p_z\frac{\partial A}{\partial z} - \frac{1}{c^3(z)p_z}\frac{\partial c(z)}{\partial z}A\right] &= 0 \ , \\ \frac{\partial A}{\partial z} - \frac{1}{2c^3(z)p_z^2}\frac{\partial c(z)}{\partial z}A &= 0 \ . \end{aligned} \tag{10}$$

In the above expressions, the upper and lower signs refer to the down- and upgoing waves, respectively. Note that both waves, independently of their predominant propagation direction, must satisfy the same transport equation.

TRUE-AMPLITUDE ONE-WAY WAVE EQUATIONS

The one-way wave equations are obtained by factorization of the above full wave equation (1).

Homogeneous medium

For a constant medium velocity, it is easy to verify that the Helmholtz equation (2) can be factorized as

$$\left[\frac{\partial}{\partial z} \pm ik_z\right]\left[\frac{\partial}{\partial z} \mp ik_z\right]u = \mathcal{L}_0^{\pm}\mathcal{L}_0^{\mp}u = \frac{\partial^2 u}{\partial z^2} + k_z^2 u = 0 \; , \tag{11}$$

where

$$\mathcal{L}_0^{+} = \left[\frac{\partial}{\partial z} + ik_z\right] \; , \quad \mathcal{L}_0^{-} = \left[\frac{\partial}{\partial z} - ik_z\right] \tag{12}$$

are the differential operators of the one-way wave equations, once we fix the sign of k_z according to

$$k_z = \mathrm{sgn}(\omega)\sqrt{\frac{\omega^2}{c^2} - k_x^2} = \frac{\omega}{c}\sqrt{1 - \frac{(ck_x)^2}{\omega^2}} \; . \tag{13}$$

Therefore, any solution of

$$\mathcal{L}_0^{+}u^{+} = 0 \qquad \text{or} \qquad \mathcal{L}_0^{-}u^{-} = 0 \tag{14}$$

is also a solution of the Helmhotz equation (2). This motivates the use of one-wave wave equations in migration, where only downward propagation is required.

Vertically inhomogeneous medium

Let us now look for solutions of the one-way wave equations (14) of the type

$$u^{\pm}(x,z,\omega) = A^{\pm}(x,z)\exp\{i\omega\tau^{\pm}(x,z)\} \; . \tag{15}$$

The resulting eikonal and transport equations read

$$\frac{\partial \tau^{\pm}}{\partial z} = \pm p_z \; , \tag{16}$$

$$\frac{\partial A^{\pm}}{\partial z} = 0 \; . \tag{17}$$

We see that the eikonal equations (7) and (16) are identical, which reflects the well-known fact that even in homogeneous media, the kinematics of the up- and downgoing waves are correctly described by the one-wave wave equations. However, comparing the transport equations (8) and (17), we see that they are identical only in homogeneous media, where $\frac{\partial c}{\partial z} = 0$.

Therefore, for the one-way wave equations to correctly describe the amplitudes of the up- and downgoing waves, at least up to zero-order ray theory, they need to be modified. The simplest way to do so is by adding a new term $\alpha^{\pm}$ to the one-way wave operators $\mathcal{L}_0^{\pm}$. Doing so, we have the modified equations

$$\left[\frac{\partial}{\partial z} \pm ik_z + \alpha^{\pm}\right]u = 0 \; . \tag{18}$$

Searching for solutions of the ray type in equation (15), we find the eikonal and transport equations

$$\frac{\partial \tau^{\pm}}{\partial z} = \pm p_z \; , \tag{19}$$

$$\frac{\partial A^{\pm}}{\partial z} + \alpha^{\pm}A^{\pm} = 0 \; . \tag{20}$$

Comparing these equations with those obtained for the full wave equation [equations (7) and (8)], we recognize that the eikonal equations are still the same. For the transport equations to be identical, both $\alpha^{\pm}$ need to be chosen as

$$\alpha^{\pm} = -\frac{1}{2c^3(z)p_z^2}\frac{\partial c(z)}{\partial z} \ . \tag{21}$$

Thus, the true-amplitude one-way wave equations read

$$\left\{\frac{\partial}{\partial z} \mp i\omega p_z - \frac{1}{2c^3(z)p_z^2}\frac{\partial c(z)}{\partial z}\right\} u = 0 \ , \tag{22}$$

which, by construction, describe up- and downgoing waves that possess, in zero-order ray theory approximation, the same amplitudes and traveltimes as those described by the full wave equation.

GAZDAG MIGRATION

In this section, we adapt the phase-shift migration method of Gazdag (1978, 1980) to the use of the true-amplitude one-way wave equations as reviewed above. Gazdag's method is based on the observation that zero-offset data are kinematically equivalent to those as recorded in a hypothetical experiment with an exploding reflector (Loewenthal et al., 1976). Thus, solving the one-way wave equation for upgoing waves with half the medium velocity and setting in the result the time to zero, the so-called imaging condition, one obtaines a wavefield that is positioned at the reflector.

With the above choice for k_z, the one-way wave equation for homogeneous media reads

$$\frac{\partial U(k_x,z,\omega)}{\partial z} = i\frac{\omega}{c}\sqrt{1-\left(\frac{k_x\, c}{\omega}\right)^2} U(k_x,z,\omega) \ . \tag{23}$$

Its solution, at $t = 0$, can be represented as

$$u(x,z,t=0) = \frac{1}{2\pi}\sum_{k_x}\sum_{\omega} U(k_x,z=0,\omega)\exp\left\{ i\left[k_x x + \frac{\omega}{c}\sqrt{1-\left(\frac{k_x\, c}{\omega}\right)^2} z\right]\right\}\Delta k_x \Delta\omega \ . \tag{24}$$

Stolt (1978) gave a fast way of calculating this expression by a two-dimensional fast Fourier transform, transforming the sum over ω into an inverse discrete Fourier transform over k_z.

Vertically inhomogeneous medium

To apply phase-shift migration in vertically inhomogeneous media, Gazdag (1980) divided the medium into small horizontal layers in such a way that in each layer the velocity can be considered constant. Let us supposed that the z-axis is divided into N_z such layers, the jth layer of which is

$$I_j = \{z | z_j < z < z_{j+1};\ j = 1,2,...,N_z\} \ . \tag{25}$$

Denoting the constant velocity in this layer by c_j, we can express the migration equation as

$$U(k_x,z_{j+1},\omega) = U(k_x,z_j,\omega)\exp\left\{\frac{i\omega}{c_j}\sqrt{1-\left(\frac{k_x c_j}{\omega}\right)^2}(z_{j+1}-z_j)\right\} \ . \tag{26}$$

Since the layers can, in principle, be arbitrarily thin, this procedure can achieve an approximation of any desired precision.

TRUE-AMPLITUDE MIGRATION

We now use the true-amplitude one-way wave equations to introduce amplitude control into Gazdag's phase-shift migration. Thus, we need to solve the true-amplitude one-way wave equation

$$\left\{\frac{\partial}{\partial z} - i\omega p_z - \frac{1}{2c^3(z)p_z^2}\frac{dc(z)}{dz}\right\} u = \left\{\frac{\partial}{\partial z} - i\omega p_z - \alpha\right\} u = 0 \ , \tag{27}$$

instead of equation (23), where p_z is still given by equation (4).

The differential equation (27) can be solved by separation of variables. Integration from initial depth z_0 to final depth z_f yields

$$\int_{z_0}^{z_f} \frac{du}{u} = \int_{z_0}^{z_f} (i\omega p_z + \alpha)\, dz \ : \ \ln\left(\frac{u_f}{u_0}\right) = \int_{z_0}^{z_f} (i\omega p_z) dz + \int_{z_0}^{z_f} \alpha dz \ , \tag{28}$$

from which we find the expression for the wavefield u_f at depth level z_f,

$$\begin{aligned} u_f &= u_0 \exp\left\{\int_{z_0}^{z_f} (i\omega p_z) dz + \int_{z_0}^{z_f} \alpha dz\right\} \\ &= u_0 \exp\left\{\int_{z_0}^{z_f} (i\omega p_z) dz\right\} \exp\left\{\int_{z_0}^{z_f} \alpha dz\right\} \ . \end{aligned} \tag{29}$$

Note that the first exponential term in equation (29) is nothing else but the phase correction term of conventional Gazdag migration. The second exponential term gives rise to the amplitude correction in inhomogeneous media, provenient from the correction term α in the true-amplitude one-way wave equation (27).

Homogeneous medium

Let us now analyse the solution of the true-amplitude one-way wave equation (27) for the case of a constant medium velocity $c = c_0$. In this case, $\alpha = 0$, because $\frac{dc(z)}{dz} = 0$. Thus,

$$\begin{aligned} \int_{z_0}^{z_f} (i\omega p_z) dz &= \int_{z_0}^{z_f} \left(i\frac{\omega}{c_0}\sqrt{1 - \left(\frac{c_0 k_x}{\omega}\right)^2}\right) dz \\ &= \left(i\frac{\omega}{c_0}\sqrt{1 - \left(\frac{c_0 k_x}{\omega}\right)^2}\right) (z_f - z_0) \ . \end{aligned} \tag{30}$$

Assuming that the initial depth is $z_0 = 0$ and the final depth is $z_f = z$, substitution in the solution (29) yields

$$u = u_0 \exp\left\{\left(i\frac{\omega}{c_0}\sqrt{1 - \left(\frac{c_0 k_x}{\omega}\right)^2}\right) z\right\} \ , \tag{31}$$

where $u = u(k_x, z, \omega)$. This is nothing else but the continuous version of Gazdag's migration equation (24) for homogeneous media.

Vertically inhomogeneous media

Let us now suppose that the medium has a vertically varying velocity, i.e., $c = c(z)$. To solve the integral in the first exponential term, we again divied the depth interval $[0, z]$ in N_z subintervals $I_j = \{z | z_j < z < z_{j+1};\ j = 0, 1, 2, ..., N_z - 1\}$. We then apply the solution (29) to each single layer, i.e., $z_0 = z_j$ and $z_f = z_{j+1}$. Denoting the the wavefield at depth z_j as $u_j = u(k_x, z_j, \omega)$, we may thus write

$$\begin{aligned} u_{j+1} &= u_j \exp\left\{\int_{z_j}^{z_{j+1}} (i\omega p_z) dz + \int_{z_j}^{z_{j+1}} \alpha dz\right\} \\ &= u_j \exp\left\{\int_{z_j}^{z_{j+1}} (i\omega p_z) dz\right\} \exp\left\{\int_{z_j}^{z_{j+1}} \alpha dz\right\} \ . \end{aligned} \tag{32}$$

From the Mean Value Theorem for integrals, we can evaluate the integral in the first exponent as

$$\int_{z_j}^{z_{j+1}} (i\omega p_z) dz = i\omega p_z(\xi)(z_{j+1} - z_j) \ , \quad \text{where } \ \xi \in [z_j, z_{j+1}] \ , \tag{33}$$

where $p_z(\xi)$ is the value of the vertical slowness vector component at some point ξ within the interval $[z_j, z_{j+1}]$. If we choose the depth intervals sufficiently small, p_z can be well approximated by its mean value within the depth interval, or by its value as obtained using the mean velocity of the current layer. Denoting the approximate value of the vertical slowness vector component to be used for the phase correction by p_z, equation (32) reads

$$u_{j+1} = u_j \exp\left\{i\omega p_z(z_{j+1} - z_j)\right\} \exp\left\{\int_{z_j}^{z_{j+1}} \alpha dz\right\} . \tag{34}$$

We now need to analyze the integral in the second exponential term of equation (34). For that purpose, we observe that

$$\begin{aligned} \ln(p_z) &= \ln\left(\frac{1}{c(z)}\sqrt{1 - \left(\frac{c(z)k_x}{\omega}\right)^2}\right) : \\ \frac{d\ln(p_z)}{dz} &= -\frac{1}{c^3(z)p_z^2}\frac{dc(z)}{dz}dz = -2\alpha . \end{aligned} \tag{35}$$

Therefore, the integral over α yields

$$\int_{z_j}^{z_{j+1}} \alpha dz = \int_{z_j}^{z_{j+1}} \left(\frac{1}{2c^3(z)p_z^2}\frac{dc(z)}{dz}\right) dz = -\frac{1}{2}\int_{z_j}^{z_{j+1}} \frac{d\ln(p_z)}{dz}dz \tag{36}$$

$$= \frac{1}{2}\int_{z_{j+1}}^{z_j} \frac{d\ln(p_z)}{dz}dz = \frac{1}{2}\ln\left(\frac{p_{z_j}}{p_{z_{j+1}}}\right) , \tag{37}$$

which implies

$$\exp\left\{\int_{z_{j+1}}^{z_j} \alpha dz\right\} = \exp\left\{\frac{1}{2}\ln\left(\frac{p_{z_j}}{p_{z_{j+1}}}\right)\right\} = \sqrt{\frac{p_{z_j}}{p_{z_{j+1}}}} . \tag{38}$$

Thus, the true-amplitude expression for Gazdag's layer-stripping approach to phase-shift migration reads

$$u_{j+1} = u_j \sqrt{\frac{p_{z_j}}{p_{z_{j+1}}}} \exp\left\{i\omega p_z(z_{j+1} - z_j)\right\} , \tag{39}$$

where p_{z_j} and $p_{z_{j+1}}$ denote the vertical slowness vector components at the top and bottom of the current layer, while p_z is some reasonably chosen mean value.

NUMERICAL EXAMPLES

Homogeneous medium

To understand the quality of seismic amplitudes after phase-shift migration, we first look at three models with homogeneous overburden.

Exploding reflector data We began the numerical experiments with synthetic sections that were modeled using an exploding reflector model with unit source strength.

The first model is a simple horizontal reflector at a depth of 1000 m below a constant velocity of 4000 m/s (i.e., 2000 m/s for the exploding reflector modeling). Figure 1 shows the model and the synthetic data. Figure 2 shows the result of phase-shift migration and the peak amplitude along the reflector image. Note that the correct amplitude is recovered by standard phase-shift migration with no amplitude correction, as predicted by the theory.

To demonstrate that in homogeneous media the standard phase-shift migration correctly recovers the seismic amplitudes even for curved reflectors, the second model is a circular reflector with the same homogeneous overburden. Figure 3 shows the model and the synthetic data. Figure 4 shows the result of phase-shift migration and the peak amplitude along the reflector image. Inside the region illuminated by

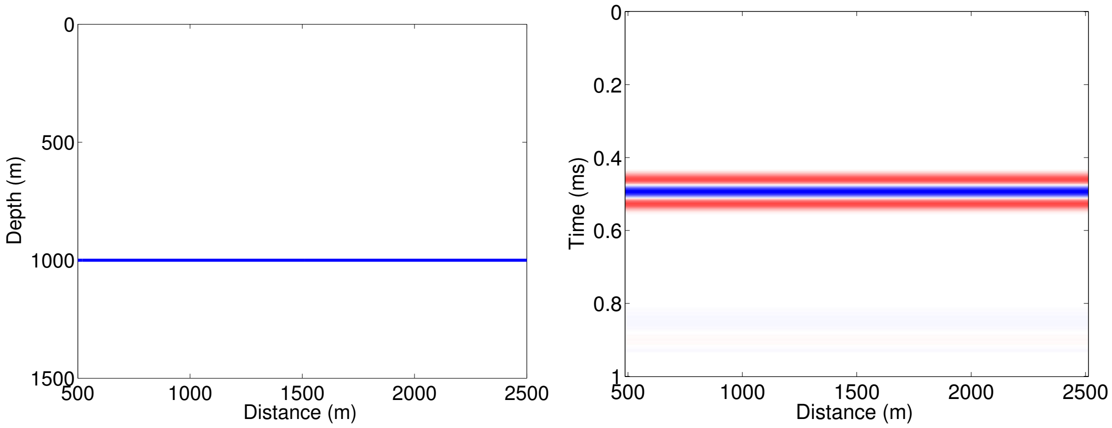

Figure 1: Planar exploding reflector: Model (left) and synthetic data secion (right).

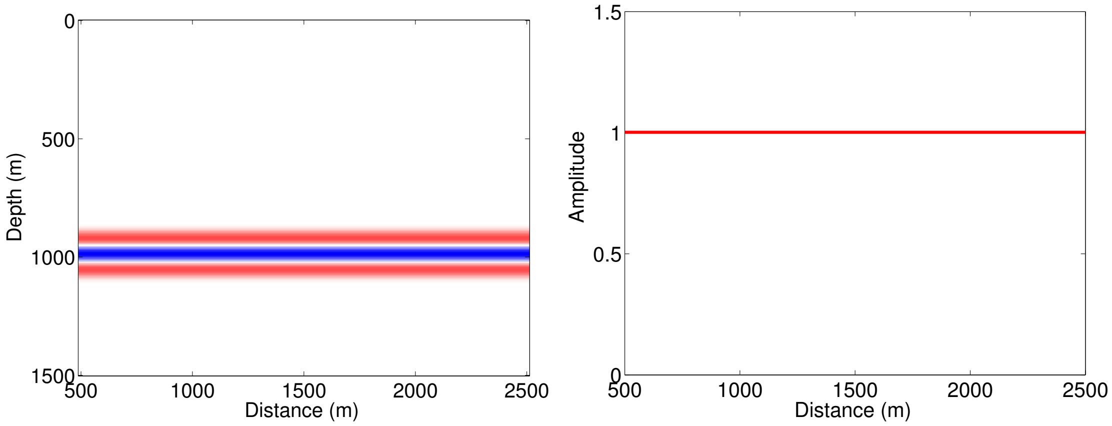

Figure 2: Planar exploding reflector: Migrated section (left) and amplitdes along the reflector image (right).

the chosen experiment, the correct amplitude is recovered by standard phase-shift migration with no amplitude correction.

Our last homogeneous example is a dome-like reflector with varying curvature. Figure 5 shows the model and the synthetic data. Figure 6 shows the result of phase-shift migration and the peak amplitude along the reflector image. While the correct amplitude is recovered reasonably well by standard phase-shift migration, the presence of an incomplete bow-tie structure due to a caustic close to the aquisition surface seems to slightly perturb the amplitude on the flanks of the dome. Moreover, the effect of a crossing boundary event is clearly visible.

Zero-offset data Zero-offset data are kinematically equivalent to exploding-reflector data (Loewenthal et al., 1976). Although their dynamics are different, there exists a transformation to correct for this (Bleistein et al., 2001). For constant velocity, this transformation amounts to a multiplication with $4\pi vt$. Again, we demonstrate the correctness of this amplitude transformation for a planar and a circular reflector.

Figure 7 shows the model and the synthetic data for the planar reflector. Figure 8 shows the result of phase-shift migration and the peak amplitude along the reflector image. The desired amplitude is perfectly recovered by standard phase-shift migration after amplitude scaling for zero-offset data. For comparison, Figure 9 shows the corresponding results using true-amplitude Kirchhoff migration (Hubral et al., 1991), applied to the unscaled data. Also Kirchhoff migration correctly recovers the unit amplitude. For this simple model, phase-shift migration does not suffer from boundary effects because it takes advantage of the periodicity of the discrete Fourier transform.

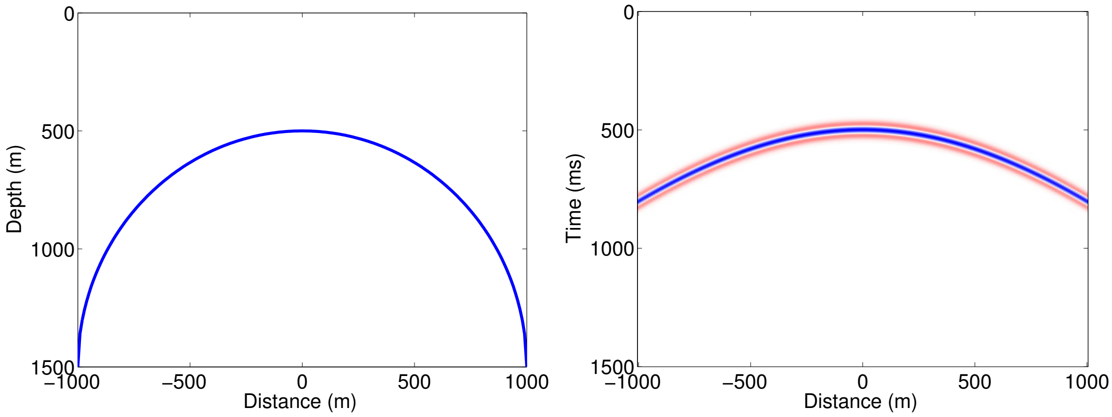

Figure 3: Circular exploding reflector: Model (left) and synthetic data (right).

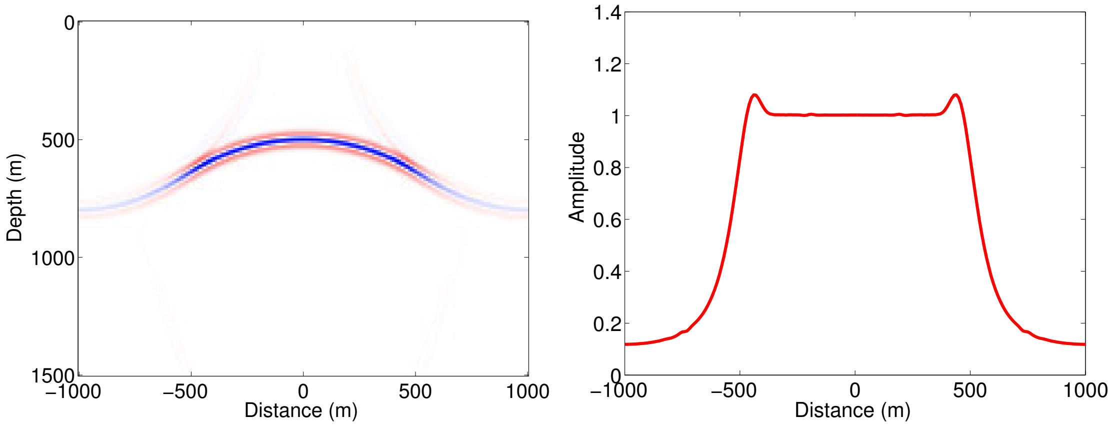

Figure 4: Circular exploding reflector: Migrated section (left) and amplitudes along the reflector image (right).

Figure 10 shows the model and the synthetic data for the circular reflector. Figure 11 shows the result of phase-shift migration and the peak amplitude along the reflector image. Inside the illuminated part of the reflector, the desired amplitude is perfectly recovered by standard phase-shift migration after amplitude scaling for zero-offset data. For comparison, Figure 12 shows the corresponding results using true-amplitude Kirchhoff migration (Hubral et al., 1991), applied to the unscaled data. For this model, the boundary effects are almost identical.

Vertically inhomogeneous medium

Exploding reflector data Now we are ready to test the amplitude behaviour of phase-shift migration in inhomogeneous media. We applied phase-shift migration with and without amplitude correction according to the theory of true-amplitude one-way wave equations. The test models were again a horizontal, a circular, and a dome-like reflector, buried in a background medium with the velocity varying with a constant vertical gradient of 1/s from 2000 m/s at the acquisition surface. Again, the data were generated using an exploding reflector using a unit source strength.

Figure 13 shows the synthetic data and the migrated section from true-amplitude phase-shift migration for the planar reflector. Figure 14 shows the resulting peak amplitudes along the reflector image as obtained from phase-shift migration without and with amplitude correction. While standard phase-shift migration produces an amplitude bias, true-amplitude phase-shift migration correclty recovers the unit amplitude.

The second example uses a circular reflector. Figure 15 shows the synthetic data and the migrated sec-

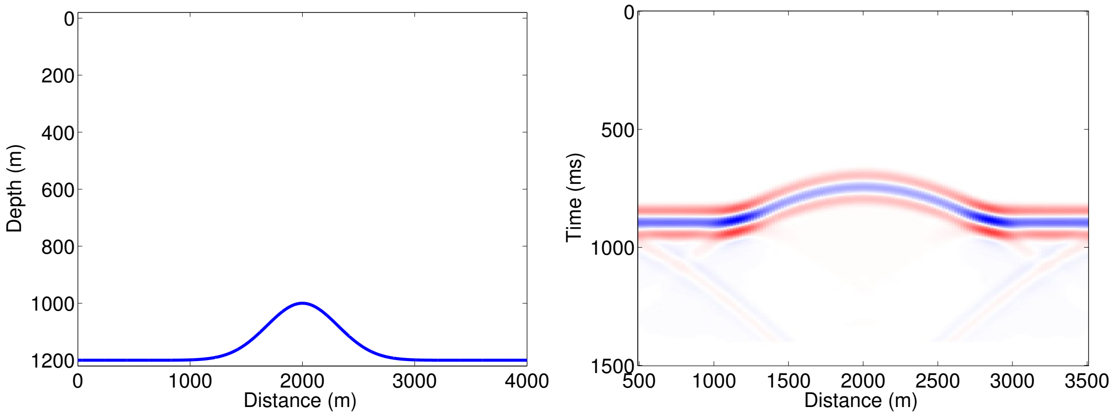

Figure 5: Dome-like exploding reflector: Model (left) and synthetic data (right).

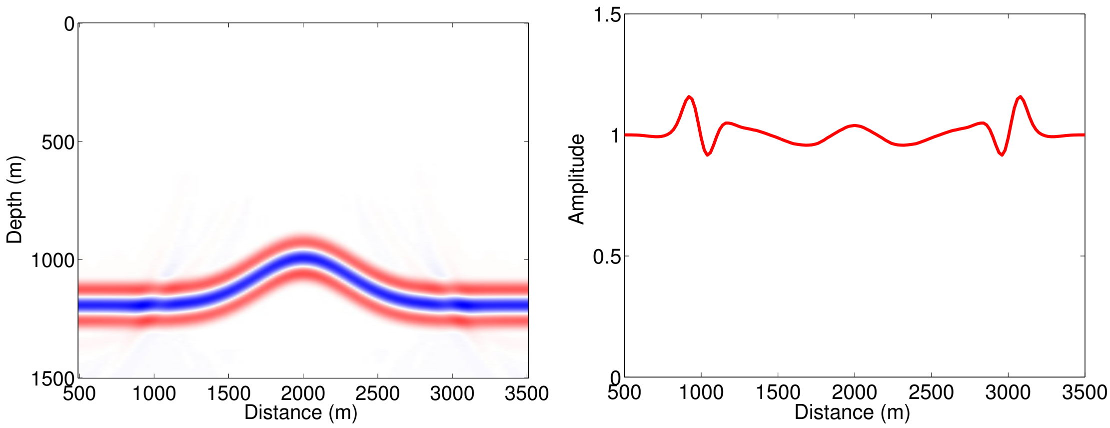

Figure 6: Dome-like exploding reflector: Migrated section (left) and amplitudes aong the reflector image (right).

tion from true-amplitude phase-shift migration for the planar reflector. Figure 16 shows the resulting peak amplitudes along the reflector image as obtained from phase-shift migration without and with amplitude correction. Except for the boundary region, we observe the same amplitude recovery due to the amplitude correction as obtained for the planar reflector.

Our last example makes use of the dome-like reflector. Figure 17 shows the synthetic data and the migrated section from true-amplitude phase-shift migration for the planar reflector. Figure 18 shows the resulting peak amplitudes along the reflector image as obtained from phase-shift migration without and with amplitude correction. Again, the amplitude recovery due to the amplitude correction corresponds to those obtained for the planar and circular reflectors. The effects of the caustic and the crossing boundary event are the same as in the homogeneous case.

CONCLUSIONS

Separation of the full wave equation into one-way wave equations that describe only up- or downgoing waves is a standard tool for seismic migration. As shown by Zhang et al. (2003, 2005), the solutions of the standard one-way wave equations do not have the same amplitude as those of the full wave equations. By adding a correction term to the standard one-way wave equations, the cited papers derive modified true-amplitude one-way wave equations that correctly treat the amplitudes in a zero-order ray-theory sense. Moreover, the authors demonstrate how these one-way wave equations can be used for true-amplitude wave-equation migration using finite differences.

In this paper, we have transferred the concepts of Zhang et al. (2003) to phase-shift migration of Gazdag

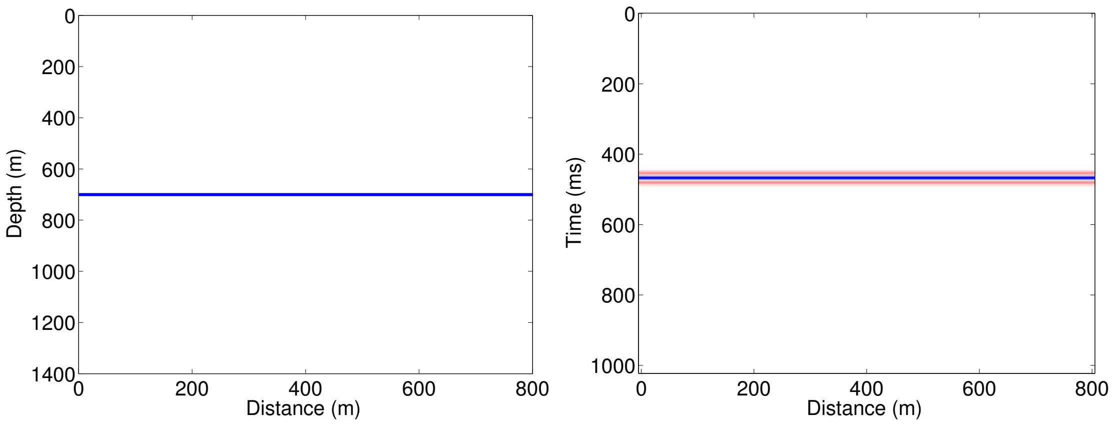

Figure 7: Planar reflector: Model (left) and synthetic zero-offset data (right).

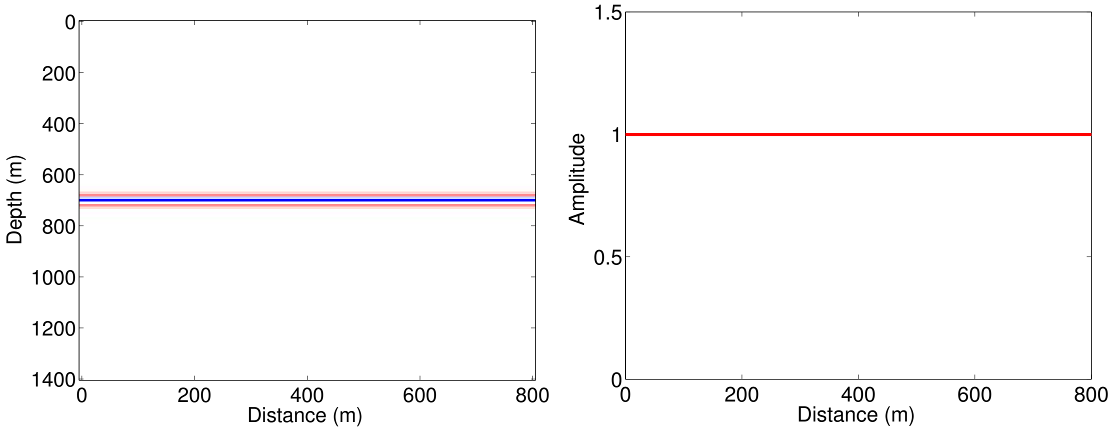

Figure 8: Planar reflector: Phase-shift migrated zero-offset data (left) and amplitudes along the reflector image (right).

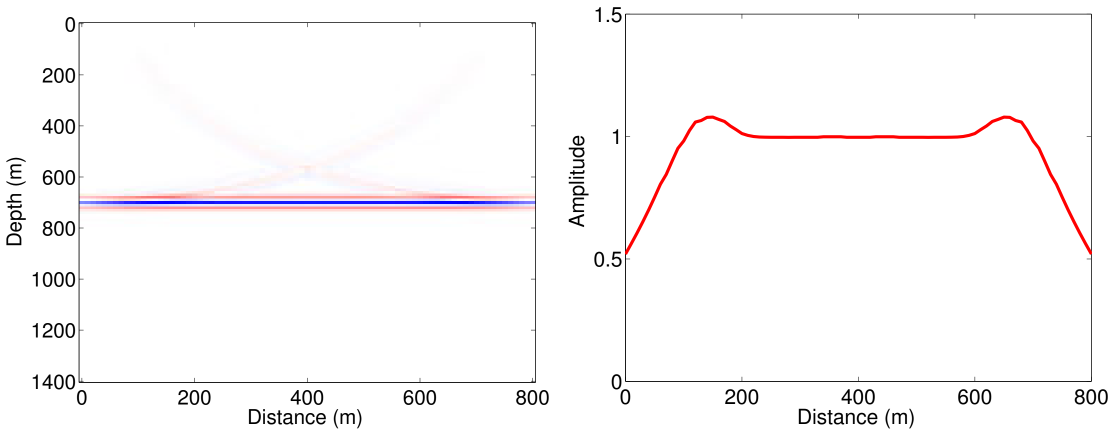

Figure 9: Planar reflector: Kirchhoff migrated zero-offset data (left) and amplitudes along the reflector image (right).

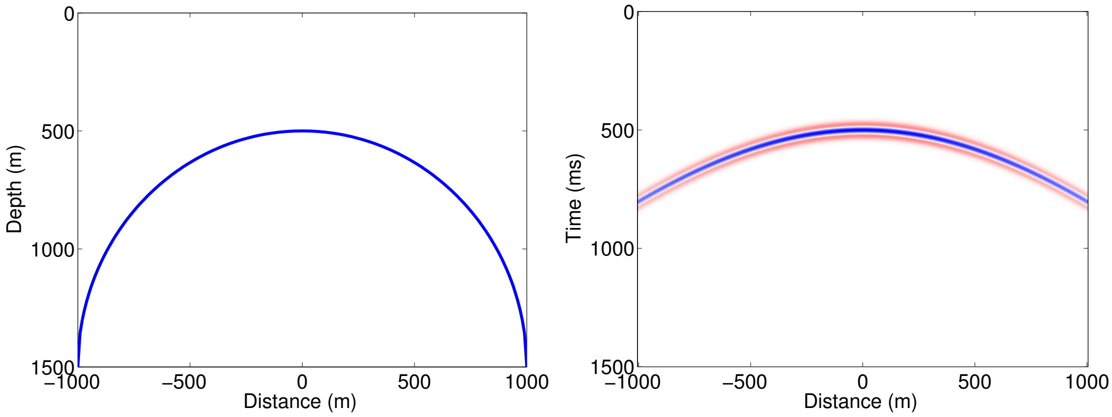

Figure 10: Circular reflector: Model (left) and synthetic zero-offset data (right).

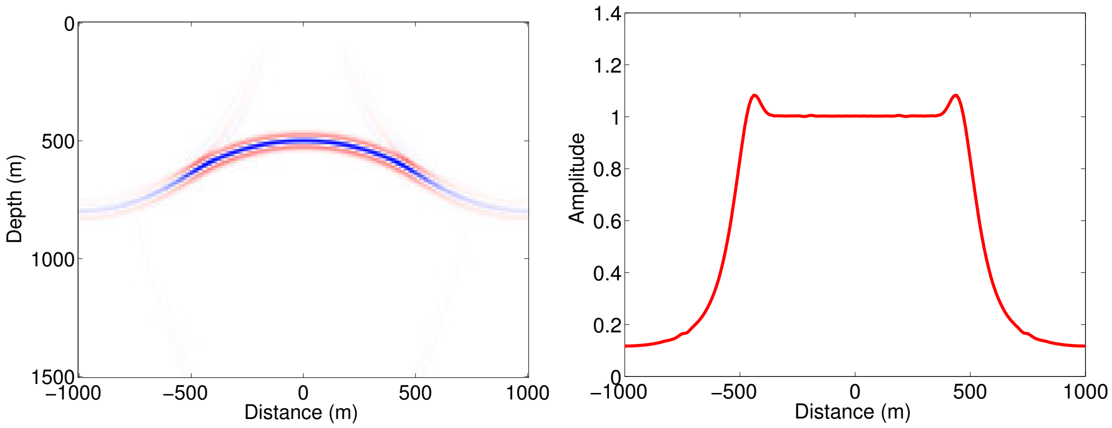

Figure 11: Circular reflector: Phase-shift migrated zero-offset data (left) and amplitudes along the reflector image (right).

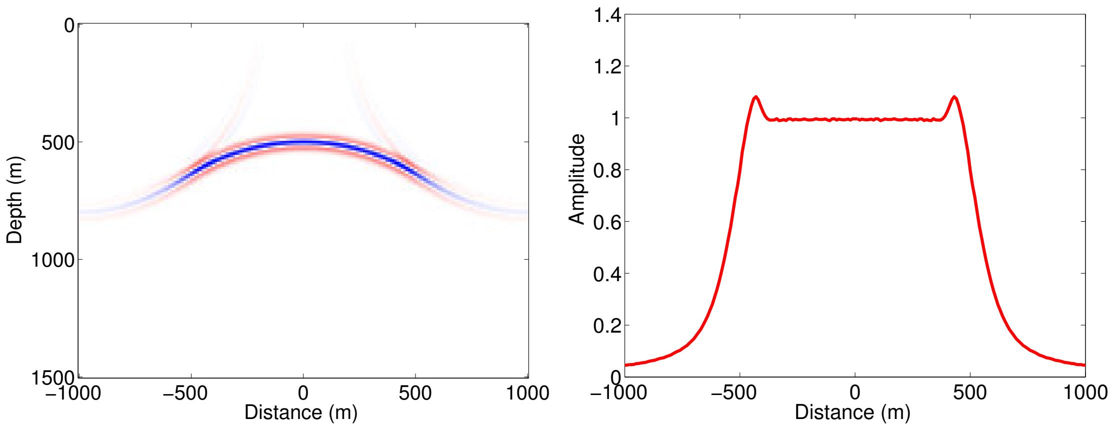

Figure 12: Circular reflector: Kirchhoff migrated zero-offset data (left) and amplitudes along the reflector image (right).

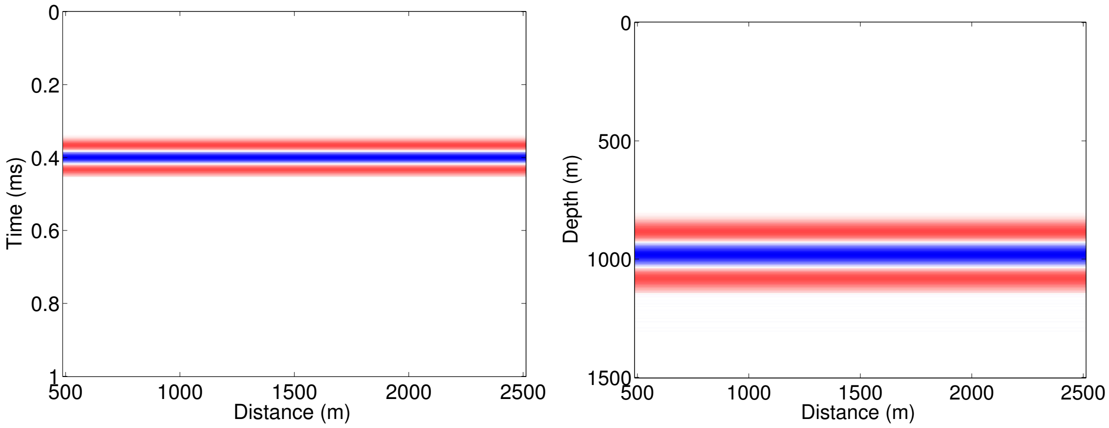

Figure 13: Planar exploding reflector in vertical-gradient model: synthetic data (left) and migrated section (right).

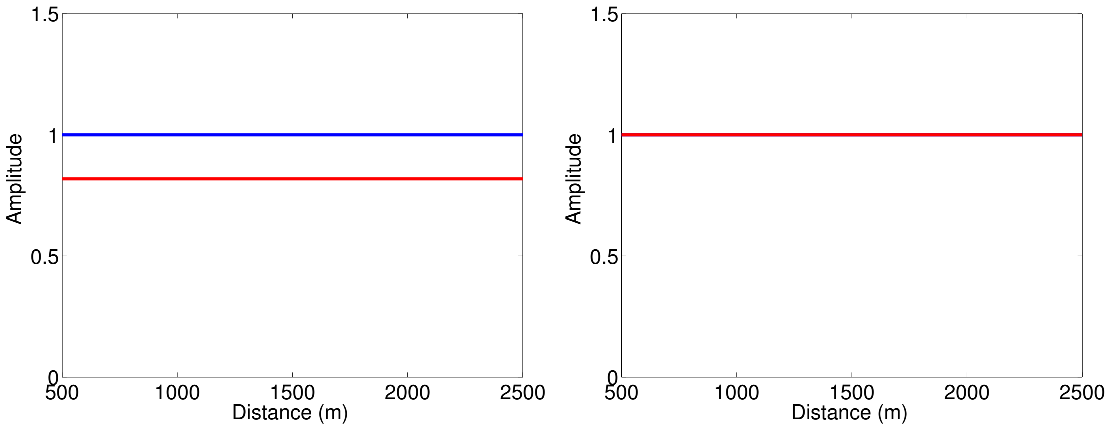

Figure 14: Planar exploding reflector in vertical-gradient model: peak amplitude along the reflector image (red line) after phase-shift migration without (left) and with (right) amplitude correction. Also shown is the correct unit amplitude (blue line).

(1978, 1980). By analytically solving the true-amplitude one-way wave equations in vertically inhomogeneous media, we have shown that a true-amplitude phase-shift migration consists of the same phase correction as in standard phase-shift migration, plus an amplitude correction that can be applied at each depth level. Simple numerical examples demostrate the improvement of the amplitudes in vertically inhomogeneous media.

To apply the amplitude correction in poststack migration, we propose to apply the dynamic transform to exploding-reflector data as introduced by Bleistein et al. (2001).

Since the solution of the true-amplitude one-way wave equations provides an amplitude-correction factor for each depth level, an extension of the present true-amplitude wave equation migration to laterally varying media using split-step and/or phase-shift plus interpolation (PSPI) migrations is straigtforward.

ACKNOWLEDGMENTS

This work is supported by FAPESP and CNPq (Brazil) as well as by the sponsors of the Wave Inversion Technology (WIT) Consortium.

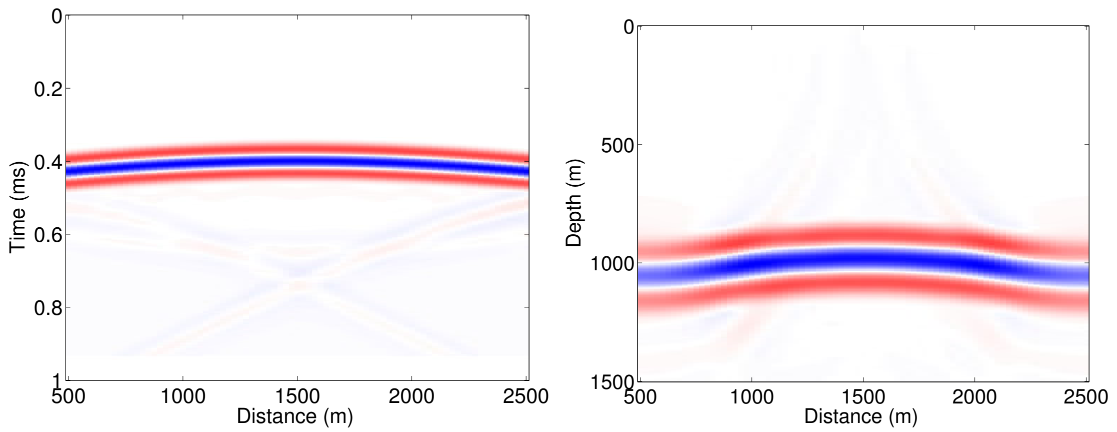

Figure 15: Circular exploding reflector in vertical-gradient model: synthetic data (left) and migrated section (right).

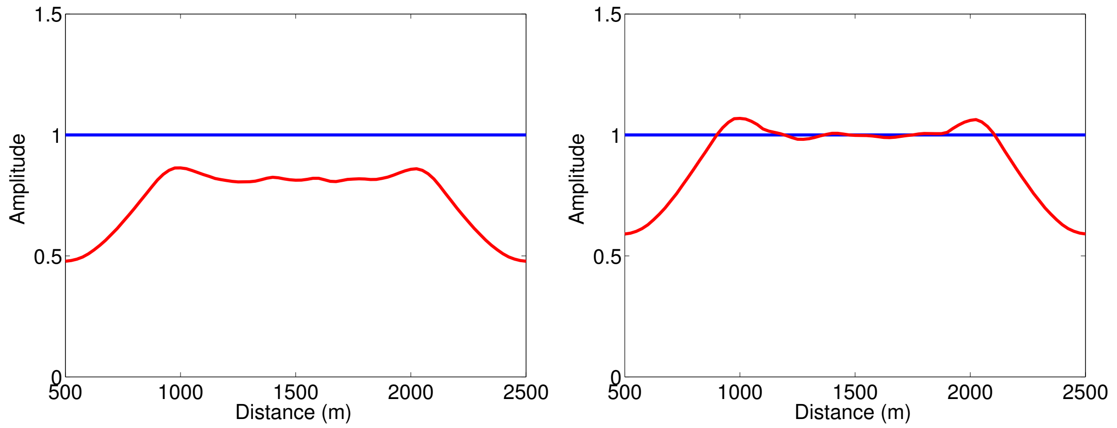

Figure 16: Circular exploding reflector in vertical-gradient model: peak amplitude along the reflector image (red line) after phase-shift migration without (left) and with (right) amplitude correction. Also shown is the correct unit amplitude (blue line).

REFERENCES

Bleistein, N., Cohen, J. K., and Stockwell Jr., J. W. (2001). *Mathematics of Multidimensional Seismic Migration, Imaging, and Inversion*. Springer, New York.

Gazdag, J. (1978). Wave equation migration with the phase-shift method. *Geophysics*, 43(07):1342–1351.

Gazdag, J. (1980). Wave equation migration with the accurate space derivative method. *Geophys. Prosp.*, 28(01):60–70.

Hubral, P., Tygel, M., and Zien, H. (1991). Three-dimensional true-amplitude zero-offset migration. *Geophysics*, 56(1):18–26.

Loewenthal, D., Lu, L., Roberson, R., and Sherwood, J. W. C. (1976). The wave equation applied to migration. *Geophysical Prospecting*, 24:380–399.

Stolt, R. H. (1978). Migration by Fourier transform. *Geophysics*, 43(1):23–48.

Zhang, Y., Zhang, G., and Bleistein, N. (2003). True amplitude wave equation migration arising from true amplitude one-way wave equations. *Inverse Problems*, 19:1113–1138.

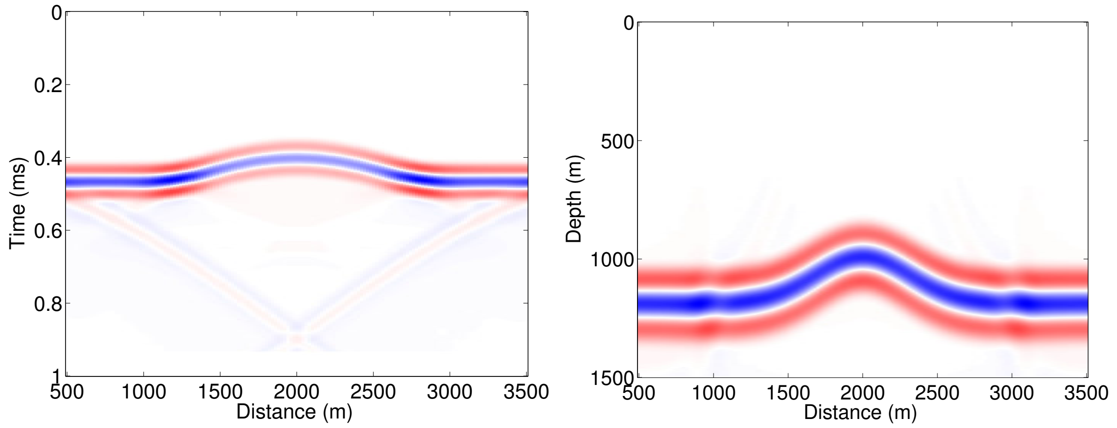

Figure 17: Dome-like exploding reflector in vertical-gradient model: synthetic data (left) and migrated section (right).

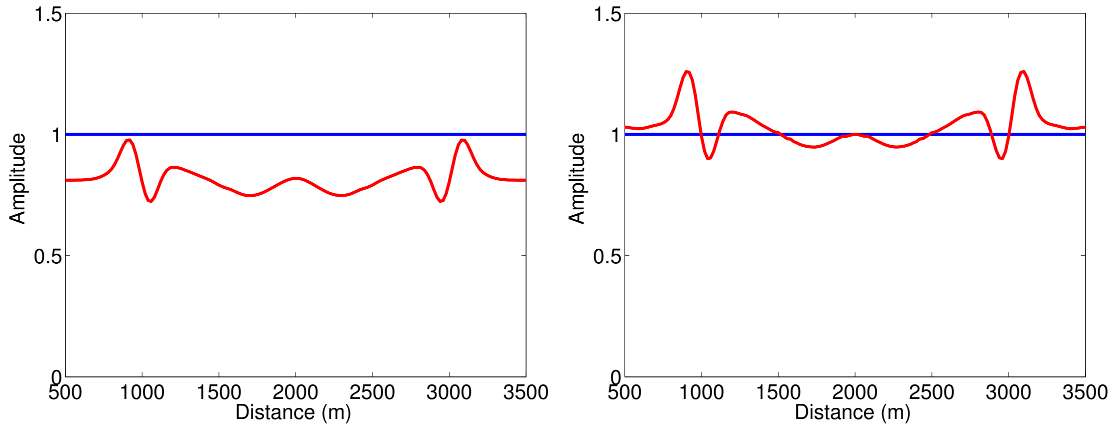

Figure 18: Dome-like exploding reflector in vertical-gradient model: peak amplitude along the reflector image (red line) after phase-shift migration without (left) and with (right) amplitude correction. Also shown is the correct unit amplitude (blue line).

Zhang, Y., Zhang, G., and Bleistein, N. (2005). Theory of true-amplitude one-way wave equations and true-amplitude common-shot migration. *Geophysics*, 70(4):E1–E10.