TIME AND DEPTH REMIGRATION IN ELLIPTICALLY ANISOTROPIC MEDIA USING IMAGE-WAVE PROPAGATION

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ABSTRACT

The image-wave equations for the problems of depth and time remigration in elliptically anisotropic media are second-order partial differential equations similar to the acoustic wave equation. The propagation variable is the vertical velocity or the medium ellipticity rather than time. In this work, we derive these differential equations from the kinematic properties of anisotropic remigration. The objective is to enable the construction of subsurface images that correspond to different vertical velocity and/or different degrees of medium anisotropy. In this way, "anisotropy panels" can be obtained in a completely analogous way to velocity panels for a migration velocity analysis.

INTRODUCTION

Depending on the velocity model used, seismic migration positions the images of seismic reflectors at different locations in time or depth. To transform these migrated reflector images from one into another in a direct way, i.e., without going back to the original seismic data section, is a seismic imaging task that can be achieved by a process called remigration, also known as residual or cascaded migration (Rothman et al., 1985; Larner and Beasley, 1987) or velocity continuation (Fomel, 1994). In this way, improved seismic reflector images for an improved migration velocity can be obtained by applying a modified migration operator to the already migrated rather then unmigrated section.

Residual migration is based on the fact that a migrated image obtained from migrating a second time (with the migration velocity v_2) a seismic section that has already been migrated (with the migration velocity v_1) is identical to the one that would have been obtained from migrating the original time section once, with the effective migration velocity $v_{eff} = \sqrt{v_1^2 + v_2^2}$ (Rocca and Salvador, 1982). Given the first (wrong) migration velocity v_1 and the desired effective (true) migration velocity v_{eff}, a residual migration is nothing more than a conventional migration with the residual migration velocity $v_2 = \sqrt{v_{eff}^2 - v_1^2}$ (Rothman et al., 1985). Cascaded migration involves an iterative procedure (Larner and Beasley, 1987). By performing n times a migration with a small velocity increment Δv, the desired effective migration velocity $v_{eff} = \sqrt{n\Delta v^2}$ is finally reached.

Is is not difficult to accept that by choosing a large number n of steps and a very small velocity increment Δv, a cascaded migration simulates a quasi-continuous change of the migration velocity. In this situation, the sequence of images of a certain reflector as subsequently migrated with varying migration velocities creates an impression of a propagating wavefront. This "propagating image" was termed an "image wave" by Hubral et al. (1996). The propagation variable, however, is not time as is the case for conventional physical waves, but the migration velocity.

Of course, conventional physical waves and image waves show a different kinematic behaviour. For example, a slanted plane wave in a homogeneous medium preserves its angle to the vertical axis when propagating. On the contrary, the image of a dipping reflector changes its dip angle when migrated with different velocities. For this reason, image-wave propagation cannot be described by a conventional (acoustic or elastic) wave equation. Nontheless, there are partial differential equations that describe the propagation of image waves. In the terminology of Hubral et al. (1996), these are called "image-wave equations".

Inverting the standard ray-theory procedure, such image-wave equations can be derived from the kinematic behaviour of image waves. For homogeneous, isotropic media, image-wave propagation as a function of the (constant) migration velocity has been studied in time (Fomel, 1994; Hubral et al., 1996; Mann, 1998; Fomel, 2003a,b) and in depth (Hubral et al., 1996; Mann, 1998; Schleicher et al., 2004). By treating them in a similar way as conventional acoustic waves, the above authors derived image-wave equations for both, time and depth remigration. Both image wave equations for time and depth remigration are equations similar to the acoustic wave equation (Fomel, 1994; Hubral et al., 1996; Mann, 1998). An independent earlier derivation of the time remigration image-wave equation by Claerbout (1986) has later also been made available to the public.

A first extension of the theory to elliptically anisotropic media was presented by (Aleixo and Schleicher, 2004), who presented the image-wave equation for depth remigration as a function on the medium ellipticity. Here we complement their results with the one for a variation of the vertical (i.e., isotropical background) velocity and the corresponding equations for time remigration. We demonstrate the validity of the theory with a simple numerical example.

DERIVATION OF THE IMAGE-WAVE EQUATIONS

In this section, we describe the variation of the position of a reflector image when the parameters of the elliptically anisotropic medium change. This variation will become the kinematics of the image-wave propagation of the image waves.

Since imaging is a linear operation, we can restrict our study to the behaviour of a single point on the image of a seismic reflector when the medium parameters vary. This situation can be understood in analogy to the propagation of a Huygens wave emanating from a secondary source. The kinematics of the Huygens wave describes the behaviour of a single point on the wave front when time varies. In the same way, the kinematics of the analogous "Huygens image wave" will describe the behaviour of a single point on the reflector image when the velocity model changes.

The procedure follows the lines applied by Hubral et al. (1996) to derive the time and depth image-wave equations in isotropic media. It starts by the construction of the Huygens image wave, that is, the set of points that describes the possible location of the original point on the reflector after a variation of the propagation variable. In a second step, the coordinates of the original image point are replace by derivatives, in this way constructing an image eikonal equation the solution of which is the Huygens image wave. In a last step, the most simple of all second-order partial differential equations that generate this image eikonal equation is identified as the searched-for image-wave equation.

Elliptically anisotropic medium

An elliptically anisotropic medium is characterized by possessing a vertical symmetry. Its density-normalized elastic tensor, i.e., $A_{ik} = C_{ik}/\rho$, with C_{ik} being the elements of the elastic tensor organized in matrix form, can be written as a 6×6-matrix of the form (Vanelle, 2002)

$$\boldsymbol{A} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & 0 & 0 & 0 \\ A_{12} & A_{11} & A_{13} & 0 & 0 & 0 \\ A_{13} & A_{13} & A_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{66} \end{pmatrix}, \tag{1}$$

with the additional restrictions that

$$\begin{aligned} A_{12} &= A_{11} - 2A_{66}, \\ (A_{13} + A_{44})^2 &= (A_{11} - A_{44})(A_{33} - A_{44}). \end{aligned} \tag{2}$$

In this way, an elliptically anisotropic medium is described by four independent elastic parameters.

Propagation velocity. For seismic imaging purposes, the most important medium parameter is the velocity of seismic wave propagation. Here, we need expressions for this parameter in elliptically anisotropic media. For more information on elliptically anisotropic media, the reader is referred to Helbig (1983) or Vanelle (2002).

In a homogeneous elliptically anisotropic medium, the propagation of a quasi-P wave takes place in a plane (Helbig, 1983). For simplicity, we assume this plane to be the (x, z)-plane. Therefore, we can treat the problem as a two-dimensional one. All formulas below can readily be extended to 3D by adding corresponding y components. The group velocity vector of the quasi-P wave, $\vec{v}$, depends only on two of the components of the elasticity tensor. Within the (x, z)-plane, it can be written as

$$\vec{v} = \left(\frac{A_{11}}{V} \sin\phi, 0, \frac{A_{33}}{V} \cos\phi \right), \tag{3}$$

where A_{11} and A_{33} are components of the elastic tensor and ϕ is the angle between the normal to the wavefront and the vertical z-axis. Moreover, quantity

$$V = \sqrt{A_{11} \sin^2\phi + A_{33} \cos^2\phi} \tag{4}$$

denotes the phase velocity of the quasi-P wave.

From equations (3) and (4), we conclude that the modulus of the group velocity can be represented as

$$|\vec{v}| = v(\phi) = \frac{\sqrt{A_{11}^2 \sin^2\phi + A_{33}^2 \cos^2\phi}}{V} \tag{5}$$

However, in anisotropic media, the wavefront normal does not generally point into the propagation direction of the wave. For our purposes, we need the propagation velocity as a function of the propagation direction. Therefore, we need to introduce the propagation angle θ, i.e., the angle between the group velocity vector $\vec{v}$ (which points into the propagation direction) and the vertical z-axis. The relationship between ϕ and θ is given by (Vanelle, 2002)

$$\tan\theta = \frac{A_{11}}{A_{33}} \tan\phi. \tag{6}$$

Introducing this relationship in equation (5), we find that the modulus of the group velocity depends on the propagation direction according to

$$v(\theta) = \left[\frac{\sin^2\theta}{A_{11}} + \frac{\cos^2\theta}{A_{33}} \right]^{-1/2}. \tag{7}$$

As a consequence of the medium anisotropy, the propagation velocities of the quasi-P wave depend on the propagation direction. In particular, there are different wave velocities in the vertical and horizontal directions. From equation (7), we recognize that the vertical ($\theta = 0$) and horizontal ($\theta = \pi/2$) velocities are given by

$$v = \sqrt{A_{33}} \qquad \text{and} \qquad u = \sqrt{A_{11}}, \tag{8}$$

respectively.

At this point, it turns out to be convenient to introduce a new parameter called "medium ellipticity", defined as

$$\varphi = \frac{A_{33}}{A_{11}} = \frac{v^2}{u^2}. \tag{9}$$

Upon the introduction of the medium ellipticity φ as well as the vertical velocity v in equation (7), the angle-dependent wave velocity in an elliptically anisotropic media can be recast into the form

$$v(\theta) = v \left[\varphi \sin^2\theta + \cos^2\theta \right]^{-1/2}. \tag{10}$$

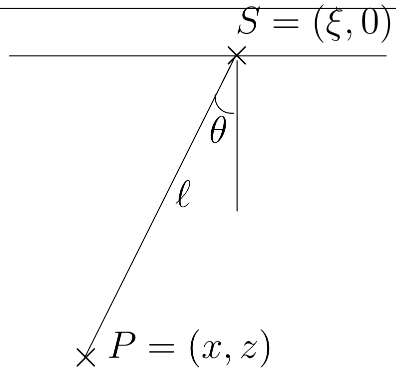

Figure 1: Geometry of the zero-offset ray connecting a source at $S = (\xi, 0)$ to a point $P = (x, z)$ on the seismic reflector image.

Zero-offset configuration

We assume that the migrated section to be remigrated was obtained from zero-offset (or stacked) data under application of a zero-offset migration. The coincident source-receiver pairs where localized at a planar horizontal surface ($z = 0$) at points $S = (\xi, 0)$ (Figure 1).

We denote by x and z the coordinates of a certain point P within the medium under consideration. Moreover, we denote by ℓ its distance from a source S, such that $\ell^2 = (x-\xi)^2 + z^2$. The propagation angle of a wave that propagates from $S = (\xi, 0)$ to $P = (x, z)$ thus satisfies

$$\cos\theta = \frac{z}{\ell} \qquad \text{and} \qquad \sin\theta = \frac{x-\xi}{\ell}. \tag{11}$$

For the geometry, see again Figure 1.

Using equations (8) and (11) in expression (10), we find the following alternative representation for the modulus of the group velocity vector in explicit dependence of the coordinates of point P rather than the propagation angle θ,

$$v(x, z) = \ell v \left[\varphi(x-\xi)^2 + z^2\right]^{-1/2}. \tag{12}$$

Traveltime. With these results on the propagation velocity, we are now ready to describe the traveltime T of a wave that was emitted and registered at S and reflected at P. From formula (12) for the propagation velocity as a function of the coordinates of P, we obtain for the desired traveltime

$$T(\xi; x, z) = \frac{2\ell}{v(x, z)} = \frac{2}{v}\left[\varphi(x-\xi)^2 + z^2\right]^{1/2}. \tag{13}$$

The factor 2 is due to the fact that in equation (10), we have $v(\theta) = v(\theta + \pi)$. Therefore, the traveltime for the wave to arrive at the depth point P is the same as the time it takes from there back to the receiver at S.

Remigration

Seismic remigration tries to establish a relationship between to media of wave propagation in such a way that identical seismic surveys on their respective surfaces would yield the same seismic data. One of these media is the wrong velocity model used for the original migration. The other medium represents the updated model within which a new image of the subsurface needs to be constructed.

Variation of vertical velocity

Let us suppose that the original migration has been realized with a model M_0 with same ellipticity φ as used in the updated model M, but a different vertical velocity v_0. In this old model, the same diffraction traveltime T of equation (13) is consumed by a different wave, reflected at a different point $P_0 = (x_0, z_0)$. It is therefore given by the modified equation

$$T(\xi; x, z) = \frac{2\ell_0}{v(x_0, z_0)} = \frac{2}{v_0}\left[\varphi(x-\xi)^2 + z^2\right]^{1/2}. \tag{14}$$

Huygens image wave. To derive the desired image-wave equation, we follow the lines of Hubral et al. (1996). Firstly, we need to find the set of all points $P = (x, z)$ in medium M for which the diffraction traveltime of equation (13) is equal to the corresponding diffraction traveltime of point $P_0 = (x_0, z_0)$ in medium M_0. In other words, we are interested in localizing the so-called Huygens wave for this kind of image-wave propagation. This Huygens image wave then describes the position $z(x)$ of the image at the "instant" v that "originated" at the "instant" v_0 at point P_0. For this purpose, we equal the times T of P and P_0, resulting in

$$F(x, z, \xi, v) = \frac{\varphi}{v^2}(x-\xi)^2 + z^2 - \frac{\varphi}{v_0^2}(x_0-\xi)^2 - z_0^2 = 0. \tag{15}$$

This equation represents a family of curves $z(x; \xi)$ that, for a fixed ξ, connect all points P in model M that possess the same diffraction traveltime $T(\xi; x, z)$ as P_0 in model M_0 for the same ξ.

The set of points P such that $T(\xi; x, z)$ is equal to $T(\xi; x_0, z_0)$ for *all* values of x and z is given by the envelope of this family of curves described by $F(x, z, \xi, v)$. This envelope is the mentioned Huygens image wave that represents the image in model M of point P_0 in model M_0. Application of the envelope condition

$$\frac{\partial F}{\partial \xi} = 0, \tag{16}$$

to equation (15) yields the condition for the stationary value

$$2\frac{\varphi}{v^2}(\xi - x) + 2\frac{\varphi}{v_0^2}(\xi - x_0) = 0, \tag{17}$$

which can be solved to yield

$$\xi = \frac{v^2 x_0 - v_0^2 x}{v^2 - v_0^2}. \tag{18}$$

It is interesting to observe that the stationary point ξ does not depend on the medium ellipticity φ. Equation (18), when substituted back in equation (15), leads to

$$z = \frac{v}{v_0}\sqrt{z_0^2 - \varphi v_0^2 \frac{(x-x_0)^2}{v^2 - v_0^2}}, \tag{19}$$

where the negative square root has been discarded as unphysical.

Equation (19) describes the position of the Huygens image wave for depth remigration that was excited with the initial conditions $(x_0, z_0; v_0)$. For an isotropic medium, where $\varphi = 1$, the above expression reduces to the one derived by Fomel (1994) or Hubral et al. (1996).

The corresponding position of the Huygens image wave for time remigration can be obtained from equation (19) by converting depth to vertical time according to $z = v\tau/2$ and $z_0 = v_0\tau_0/2$. The time domain version of equation (19) reads then

$$\tau = \sqrt{\tau_0^2 - 4\varphi \frac{(x-x_0)^2}{v^2 - v_0^2}}. \tag{20}$$

Again, for $\varphi = 1$, the above expression reduces to the known one for an isotropic medium.

Eikonal equation. The Huygens image wave of equation (19) describes the variation of a single point P_0 on a reflector image under variation of the vertical velocity v, starting at an initial velocity v_0. To transform this expression into one that describes the variation of any arbitrarily shaped reflector image for arbitrary velocity variations, we need to eliminate these initial conditions from equation (19). In other words, we need to replace the constants x_0, z_0, and v_0 in equation (19) by derivatives, so as to describe image-wave propagation for any set of initial conditions.

For this purpose, we introduce the image-wave eikonal $v = \mathcal{V}(x, z)$. An explicit expression for $\mathcal{V}(x, z)$ can be found by solving equation (19) for v. The image-wave eikonal equation is then found by replacing v by $\mathcal{V}(x, z)$ in equation (19), taking the derivatives with respect to x and z of the resulting expression, and using them to eliminate the constants x_0, z_0, and v_0 from equation (19). Since this procedure has beed detailed in Hubral et al. (1996) and Aleixo and Schleicher (2004), here we just state the result for the present case. The searched-for differential equation for $\mathcal{V}$ is

$$\mathcal{V}_x^2 + \varphi \mathcal{V}_z^2 - \frac{\varphi \mathcal{V}}{z} \mathcal{V}_z = 0, \tag{21}$$

where $\mathcal{V}_x$ and $\mathcal{V}_z$ stand for the partial derivatives of $\mathcal{V}$ with respect to x and z, respectively. Its solution for initial conditions $(x_0, z_0; v_0)$ is equation (19) solved for v. This differential equation (21) is the image-wave eikonal equation for depth remigration in elliptically anisotropic media under variation of the vertical velocity. In other words, it describes the kinematics of image-wave propagation for any arbitrary set of initial conditions as a function of the vertical velocity.

The corresponding procedure applied to equation (20) yields the image-wave eikonal equation for time remigration,

$$\mathcal{V}_x^2 - \frac{4\varphi}{\tau \mathcal{V}} \mathcal{V}_\tau = 0, \tag{22}$$

where now $\mathcal{V} = \mathcal{V}(x, \tau)$. Both of the above equations (21) and (22) reduce to their isotropic counterparts when substituting $\varphi = 1$.

Image-wave equation. Now we want to find a partial differential equation such that equation (21) is its associated eikonal equation. In other words, upon substitution of the ray-theory ansatz $p(x, z, v) = p_0(x, z) f[v - \mathcal{V}(x, z)]$ into our desired differential equation, the leading-order terms need to provide equation (21). From the leading-order terms of the second derivatives of this expression, we recognize that the second-order partial differential equation

$$p_{xx} + \varphi p_{zz} + \frac{\varphi v}{z} p_{vz} = 0 \tag{23}$$

is the simplest one to fulfill this condition. Any additional terms involving arbitrary combinations of p and its first derivatives with respect to x, z, or v, do not alter the associated eikonal equation. Therefore, we refer to equation (23) as the image-wave equation for depth remigration in elliptically anisotropic media under variation of the vertical velocity.

Correspondingly, equation (22) leads to an image-wave equation for time remigration,

$$p_{xx} + \frac{4\varphi}{v\tau} p_{v\tau} = 0. \tag{24}$$

Again, both of the above equations (23) and (24) reduce to their isotropic counterparts when substituting $\varphi = 1$.

It is to be observed that a change of variables $\omega = v/\sqrt{\varphi}$ transforms equation (24) into

$$p_{xx} + \frac{4}{\omega\tau} p_{\omega\tau} = 0, \tag{25}$$

which is the corresponding equation in isotropic media (Fomel, 1994; Hubral et al., 1996). Thus, time remigration under variation of the vertical velocity in elliptically anisotropic media can be realized by the same computational program as in isotropic media upon reinterpretation of the velocity variable.

Variation of medium ellipticity

In elliptically anisotropic media, a remigration can be realized upon the variation of a second parameter, the medium ellipticity. The corresponding image-wave equation for depth remigration has been derived in Aleixo and Schleicher (2004). We include its derivation here for completeness and add the one for time remigration.

We now suppose that the original migration has been realized with a model M_0 with same vertical velocity v as used in the updated model M, but a different ellipticity φ_0. As before, the same diffraction traveltime T of equation (13) corresponds to a $P_0 = (x_0, z_0)$ in the old model and a set of points $P = (x, z)$ in the new model. Therefore, the modified traveltime reads

$$T(\xi; x_0, z_0) = \frac{2\ell_0}{v(x_0, z_0)} = \frac{2}{v} \left[\varphi_0 (x_0 - \xi)^2 + z_0^2\right]^{1/2}. \tag{26}$$

Huygens image wave. Again, to derive the desired image-wave equations, we need to find the Huygens image wave for this problem, i.e., the set of all points $P = (x, z)$ in medium M for which the diffraction traveltime of equation (13) is equal to the diffraction traveltime (26) of point $P_0 = (x_0, z_0)$ in medium M_0. This Huygens image wave then describes the position $z(x)$ of the image at the "instant" φ that "originated" at the "instant" φ_0 at point P_0. Equal the times T of equations (13) and (26), we find

$$F(x, z, \xi, \varphi) = \varphi(x - \xi)^2 + z^2 - \varphi_0(x_0 - \xi)^2 - z_0^2 = 0. \tag{27}$$

This equation represents a family of curves $z(x; \xi)$ that, for a fixed ξ, connect all points P in model M that possess the same diffraction traveltime $T(\xi; x, z)$ as P_0 in model M_0 for the same ξ.

The set of points P such that $T(\xi; x, z)$ is equal to $T(\xi; x_0, z_0)$ for *all* values of x and z is given by the envelope of this family of curves described by $F(x, z, \xi, \varphi)$. This envelope is the mentioned Huygens image wave that represents the image in model M of point P_0 in model M_0. Application of the envelope condition (16) to equation (27) yields the stationary point

$$\xi = \frac{\varphi x - \varphi_0 x_0}{(\varphi - \varphi_0)}, \tag{28}$$

which, upon substitution in equation (27), leads to

$$z = \sqrt{z_0^2 + \varphi\varphi_0 \frac{(x - x_0)^2}{\varphi - \varphi_0}}. \tag{29}$$

Equation (29) describes the position of the Huygens image wave that was excited with the initial conditions $(x_0, z_0; \varphi_0)$.

As for the velocity variation, the substitution $z = v\tau/2$ and $z_0 = v\tau_0/2$ transfers the Huygens image wave to the time-migrated domain, resulting in

$$\tau = \sqrt{\tau_0^2 + \frac{4\varphi\varphi_0}{v^2} \frac{(x - x_0)^2}{\varphi - \varphi_0}}. \tag{30}$$

Eikonal equation. The Huygens image wave of equation (29) describes the variation of a single point P_0 on a reflector image under variation of the medium ellipticity φ, starting at an initial ellipticity φ_0. To transform this expression into one that describes the variation of any arbitrarily shaped reflector image for arbitrary ellipticity variations, we need to eliminate these initial conditions from equation (29). In other words, need to replace the constants x_0, z_0, and φ_0 in equation (29) by derivatives, so as to describe image-wave propagation for any set of initial conditions.

For this purpose, we introduce the image-wave eikonal $\varphi = \Phi(x, z)$. An explicit expression for $\Phi(x, z)$ can be found by solving equation (29) for φ. By replacing φ by $\Phi(x, z)$ in equation (29) and taking the derivatives with respect to x and z of the resulting expression, we find a differential equation for Φ, the solution of which for initial conditions $(x_0, z_0; \varphi_0)$ is equation (29) solved for φ. This differential equation is the image-wave eikonal equation,

$$\Phi_x^2 - \frac{2\Phi^2}{z}\Phi_z = 0, \tag{31}$$

which describes the kinematics of the propagation of a reflector image as a function of the medium ellipticity for any arbitrary set of initial conditions, not only of that of a single initial point (x_0, z_0).

The same procedure applied to equation (30) yields the corresponding image-wave eikonal equation for time remigration,

$$\Phi_x^2 + \frac{8\Phi^2}{\tau v^2}\Phi_\tau = 0, \tag{32}$$

where now $\Phi = \Phi(x, \tau)$.

Image-wave equation. Again, the last step is to find a partial differential equation such that equation (31) is its associated eikonal equation. In other words, upon substitution of the ray-theory ansatz

$p(x,z,\varphi) = p_0(x,z) f[\varphi - \Phi(x,z)]$ into our desired differential equation, the leading-order terms need to provide equation (31). From the leading-order terms of the second derivatives of this expression, we recognize that the second-order partial differential equation

$$p_{xx} + \frac{2\varphi^2}{z} p_{z\varphi} = 0 \tag{33}$$

is the simplest one to fulfill this condition. Therefore, we refer to equation (33) as the image-wave equation for depth remigration in elliptically anisotropic media under variation of the ellipticity.

It is important to observe that the image-wave equation (33) can be transformed into a partial differential equation with constant coefficients. Upon the introduction of the new variables $\gamma = 1/\varphi$, and $\zeta = z^2/4$, the mixed derivative becomes

$$p_{z\varphi} = p_{\zeta\gamma}\, \zeta_z\, \gamma_\varphi = p_{\zeta\gamma}\, \frac{z}{2}\, \frac{-1}{\varphi^2}. = -\frac{z}{2\varphi^2}\, p_{\zeta\gamma}. \tag{34}$$

Under this variable transformation, the image-wave equation (33) thus takes the form

$$p_{xx} - p_{\gamma\zeta} = 0\ . \tag{35}$$

The transformation into equation (35) is meaningful from an implementational point of view, since for differential equations with constant coefficients, it is generally much easier to find stable FD implementations.

As a final word on the image-wave equation (33) or its constant-coefficient version (35), let us mention that both equations do not depend on the vertical velocity v but only on the medium ellipticity φ. Thus, it can be expected that depth image-wave remigration in elliptically anisotropic media should be relatively insensitive to the actual value of the vertical velocity. This, in turn, points towards a potentially broad applicability of the image-wave concept for elliptically anisotropic remigration even in inhomogeneous media.

Correspondingly, equation (32) leads to an image-wave equation for time remigration,

$$p_{xx} - \frac{8\varphi^2}{\tau v^2} p_{\varphi\tau} = 0. \tag{36}$$

It is to be observed that the same change of variables as before, $\omega = v/\sqrt{\varphi}$, now with varying φ, also transforms this equation into the corresponding equation (25) for isotropic media. In other words, also time remigration under variation of the medium ellipticity can be realized by the same computational program as in isotropic media.

In fact, a careful analysis of time remigration under a simultaneous variation of both, vertical velocity v and medium ellipticity φ shows that even in this situation, the final image-wave equation can be transformed into equation (25) that depends on the above combined parameter ω only. By substitution of the definition of the medium ellipticity φ into the above expression for the transformed variable ω, we observe that

$$\omega = \frac{v}{\sqrt{\varphi}} = \frac{v}{\sqrt{v^2/u^2}} = u \tag{37}$$

is nothing else than the horizontal velocity. In other words, time remigration in elliptically anisotropic media is independent of the vertical velocity and depends only on the variation of the horizontal velocity.

NUMERICAL EXAMPLE

To validate the above theoretical results, we present a numerical example for time remigration in elliptically anisotropic media using the image-wave equation (25). The model, depicted in Figure 2, is a simple synclinal structure with a homogeneous, elliptically anisotropic overburden. Synthetic zero-offset data where modeled using an elliptically anisotropic finite-differences code and an exploding-reflector model. 767 source-receiver pairs were positioned at every 12 m between $x = 0$ km and $x = 9.192$ km, at a depth of 250 m. The synthetic data are depicted in Figure 3. The reflector shadow is due to incomplete damping of the surface reflection by the absorbing boundary conditions.

These data are the input for the implementation of Novais et al. (2005) of an implicit finite-difference scheme for image-wave remigration. A few snapshots of the resulting image-wave propagation for different

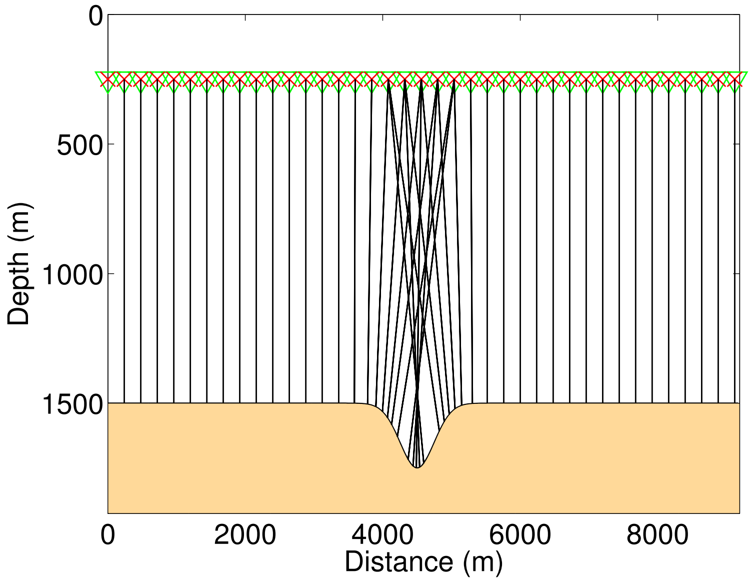

Figure 2: Model for the numerical example. The overburden of the synclinal reflector is homogeneous, elliptically anisotropic with a horizontal velocity of 4.5 km/s and vertical velocity of 3.0 km/s. Also shown is the ray field for the used configuration.

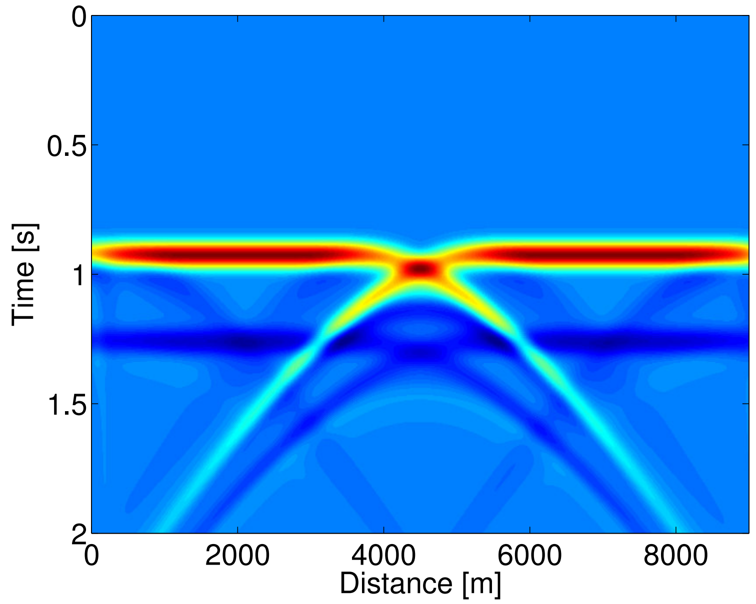

Figure 3: Synthetic zero-offset for the model of Figure 2.

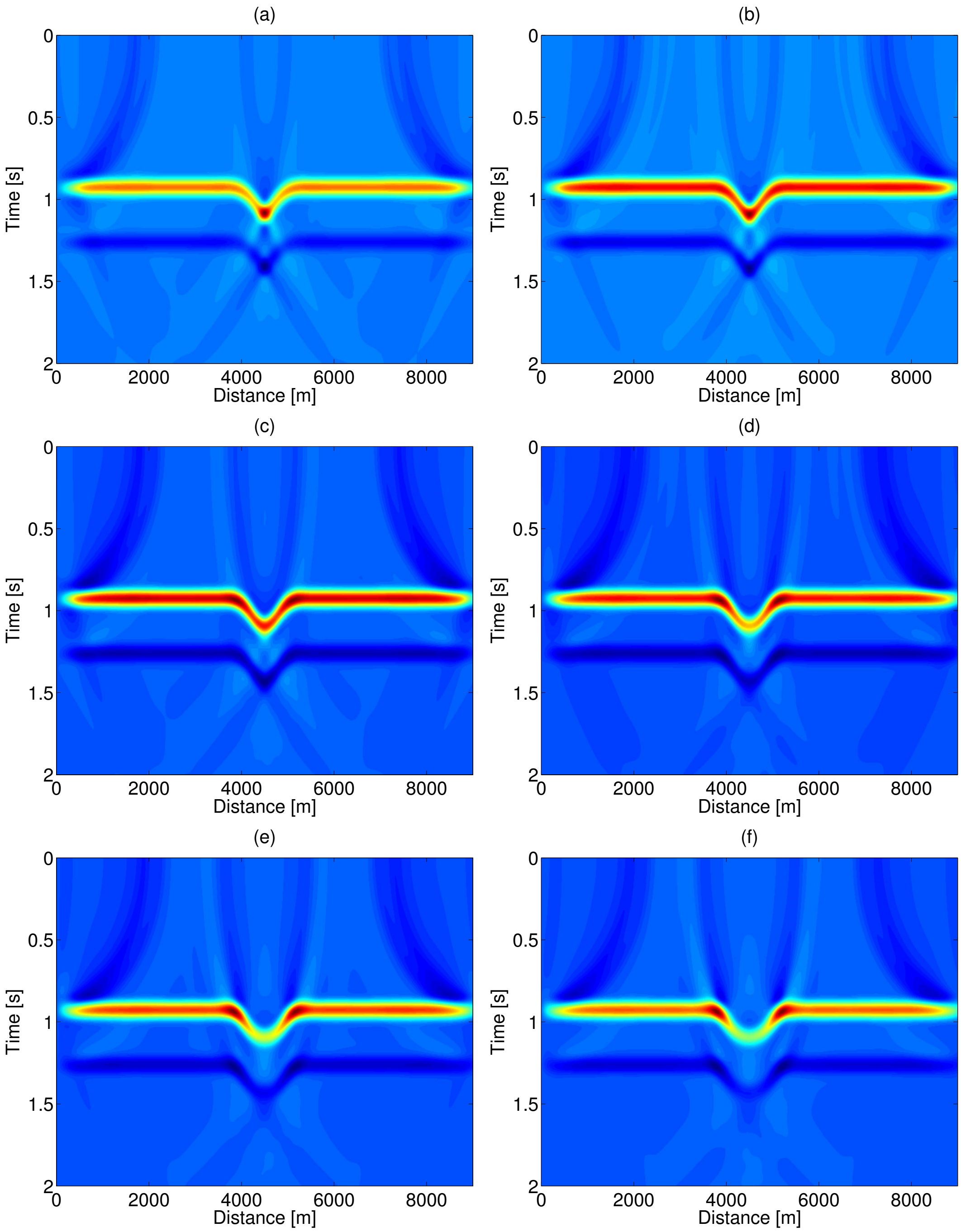

Figure 4: Snapshots of image-wave propagation at (a) $\omega = 4.2$ km/s, (b) $\omega = 4.3$ km/s, (c) $\omega = 4.4$ km/s, (d) $\omega = 4.5$ km/s, (e) $\omega = 4.6$ km/s, (f) $\omega = 4.7$ km/s.

values of the propagation variable ω are shown in Figure 4. As predicted by equation (37), the synclinal structure assumes its correct shape at where the value of ω is equal to the horizontal velocity of the model.

Moreover, it is interesting to observe that from the unfolding of the bow-tie structure, together with the distribution of the amplitude along the reflector image and the focussing of energy at the flanks of the trough, it is even possible to establish bounds for an estimate of the horizontal velocity. Clearly, in Figure 4a, the bowtie structure of Figure 3 is not fully resolved, while in Figure 4f, the trough is already overmigrated. So, even if the velocity were unknown, it would be possible to determine that it must be between 4.2 km/s and 4.7 km/s.

CONCLUSIONS

As discussed by Fomel (1994) and Hubral et al. (1996), the changing position of a seismic reflector image under variation of the migration velocity model can be understood in an analogous way to the propagation of a physical wave.

In this work, we have derived a set of second-order partial differential equations that work as image-wave equations for remigration in elliptically anisotropic media. They describe the propagation of a reflector image in time and depth remigration as a function of the vertical velocity and the medium ellipticity. To this end, we have studied the kinematics of the image wave in such media to derive the corresponding eikonal equations. From an inverted ray procedure, we have then inferred the desired image-wave equations, the solutions of which exhibit this correct kinematic behaviour.

The description of the position of the reflector image as a function of the medium ellipticity can be very useful for the detection of this parameter. A set of migrated images for different medium ellipticities can be obtained from a single migrated image without the need for multiple anisotropic migrations. From additional information on the correct reflector position, focusing analysis, or the like, the best fitting value of the medium ellipticity can then be determined.

The probably most interesting application of this procedure would start with an initial condition of an isotropic medium, described by unit ellipticity, i.e., $\varphi_0 = 1$. Since isotropic migration is a very well understood field, the image-wave equation could then be used to transform an isotropically migrated image, which can be obtained with one of the highly sophisticated migration methods that are nowadays available, into an image that corresponds to an elliptically anisotropic medium.

In the case of time remigration, the image-wave equation shows that the position of the reflector image in elliptically anisotropic media depends on the horizontal velocity only. This theoretical prediction, which is in agreement with the findings of Alkhalifah and Tsvankin (1995), was confirmed with a simple numerical example. This implies that a migration velocity analysis based on time migration can only detect this parameter. In particular, this means that there is no way to distinguish an elliptically anisotropic medium from an isotropic one on the basis of time migration only.

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REFERENCES

Aleixo, R. and Schleicher, J. (2004). Image-wave remigration in elliptically anisotropic media. *Annual WIT Report*, 8:131–139.

Alkhalifah, T. and Tsvankin, I. (1995). Velocity analysis for transversely isotropic media. *Geophysics*, 60(05):1550–1566.

Claerbout, J. (1986). Velocity extrapolation by cascaded 15 degree migration. *Stanford Exploration Project*, SEP-48:79–84.

Fomel, S. (1994). Method of velocity continuation in the problem of seismic time migration. *Russian Geology and Geophysics*, 35(5):100–111.

Fomel, S. (2003a). Time migration velocity analysis by velocity continuation. *Geophysics*, 68(5):1662–1672.

Fomel, S. (2003b). Velocity continuation and the anatomy of residual prestack time migration. *Geophysics*, 68(5):1650–1661.

Helbig, K. (1983). Elliptical anisoptropy—its significance and meaning. *Geophysics*, 48:825–832.

Hubral, P., Tygel, M., and Schleicher, J. (1996). Seismic image waves. *Geophysical Journal International*, 125(2):431–442.

Larner, K. and Beasley, C. (1987). Cascaded migration: Improving the accuracy of finite-difference migration. *Geophysics*, 52(5):618–643.

Mann, J. (1998). Derivation and implementation of the seismic image wave theory and its application to seismic reflection data. Master's thesis, Universität Karlsruhe (TH).

Novais, A., Costa, J., and Schleicher, J. (2005). Velocity determination by image-wave remigration. *Annual WIT Report*, 9:66–75. This issue.

Rocca, F. and Salvador, L. (1982). Residual migration. In *52nd Ann. Internat. Mtg., SEG*, Dallas. SEG.

Rothman, D., Levin, S., and Rocca, F. (1985). Residual migration: Applications and limitations. *Geophysics*, 50(1):110–126.

Schleicher, J., Novais, A., and Munerato, F. P. (2004). Migration velocity analysis by depth image-wave remigration: First results. *Geophysical Prospecting*, 52:559–573.

Vanelle, C. (2002). A tutorial on elliptical anisotropy. *Annual WIT Report*, 6:267–275.