KGB-PSDM MIGRATION IN CONSTANT GRADIENT VELOCITY MEDIA AND SENSITIVITY ANALYSIS TO VELOCITY ERRORS. A COMPARISON WITH KIRCHHOFF PSDM.

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ABSTRACT

In this work we extend the KGB-PSDM algorithm to the special case of a constant gradient velocity media. Following the same lines as for the homogeneous media, se have teste our operator in some synthetic important geological models and we have observed an increase in the resolution of the seismic images, as well as a great reduction of migration artifacts and noise.

INTRODUCTION

Kirchhoff-type migration has been used as workhorse by the oil industry since the pioneering work of Hagedoorn (1954), whose “maximum convexity surfaces” were later related to the acoustic wave equation and have since then become familiar in the geophysics literature as Kirchhoff migration (Schneider, 1978; Hertweck et al., 2003). However, in the last two decades Kirhhoff migration has evolved from a single imaging operator to an operator that embraces, among others, the structure of an inversion operator. This allowed the development of several others techniques (Tygel et al., 1993; Tygel et al., 1998), such as AVO/AVA operators, important in the characterization and analysis of oil-bearing reservoirs.

The basic algorithm that supports Kirchhoff migration is based on the kinematic and dynamic raytracing (Červený, 2001). However, depending on the velocity model built up through velocity analysis in CMP gathers for the prestack migration, zero order ray theory may not correctly simulate the seismic wavefield in some determined regions of the same velocity field, where raytracing becomes impossible analytically and numerically. In some most common geological models, such as salt dome flanks and thrust faults, shadow zones are formed, which implicates in a lack of ilumination by the simulation of the wavefield, consequently yielding a poor imaging of these parts of the model. In order to solve this dilemma, along the years several alternative methods have been studied, among then the paraxial ray theory (Bortfeld, 1989), where some drawbacks of the ray theory are satisfactorily solved. The paraxial ray theory is essencially a real theory, where wavefront curvatures and and amplitudes are computed by the dynamic raytracing (DRT) procedure (Červený, 2001). Its extension to a complex theory drive us to the Gaussian beam (GB’s) concept, which are paraxial rays with an exponential decay amplitude factor with respect to the central ray. Unlike the central ray itself, which is a mathematical representation of the energy raypath of the wavefield, the beams are physical quantitites that can be measured, and as so, can exist in regions where the rays cannot be traced. And besides that, in some regions where the wavefield is not regular (i.e., amplitudes are infinite), GB’s present an continuation of this wavefield that grants the numerical stability.

To the knowledge of the authors, the first works to use GB’s as imaging operators were due to Raz (1987) and Costa (1989). Later, Hill (1990) developed an GB migration in postack data decomposing the data into plane waves, followed by its downward extrapolation in depth. Although Hill states that its plane wave data uses near surface constant velocities, his methods is able to handle lateral velocity variations.

Beyond discussing and describing quantitatively and qualitatively the abtaintion of the main migration parameters of his imaging, Hill (1990) demonstrates that the variation of the beam half-width does not influence in the final image, considering that the width of this parameter is not larger than the dominant seismic wavelenght.

In Hill (2001), GB migration is extended to 3D prestack data. In this case, by using the Kirchhoff downward extrapolation principle (Schneider, 1978) and the Claerbout (1971) imaging principle, it is shown that in the midpoint coordinates, for each half-offset, the main procedure can be seen as a local slant stack of each element of the wavefield decomposed in beams (Hale, 1992). The final image is obtained by a summation in the half-offset coordinates.

In the last WIT meeting, we have presented the theoretical and numerical results of a 3D prestack Kirchhoff-type depth migration operator, where we make use of a weighted superposition integral of GB's as Green's function of the imaging problem. In order to adequately control the parameters that define the half-width of a GB, we use as physical constraint in the determination of the beam parameters the Fresnel volume of the seismic wavefield (Červený and Soares, 1992). Specifically speaking, we use the Fresnel zone of a seismic experiment and its counterpart, projected along the acquisition surface. By using the diffraction stack principle (Schleicher et al., 1993), we calculate the Fresnel volume by DRT, and in doing so, a radius of influence inside the projected Fresnel zone is determined, and the paraxial reflections corresponding to reflector elements in the neighbourhood of a reflection point in depth, over a real reflector, and that influences in the resolution of the final image in depth. At the same time, our process grants (in synthetic data) the true amplitudes of reflection events, correcting the seismic data of geometrical spreading losses. The interaction between the weigh-function of the GB superposition integral and the weigh-function of the true amplitude migration integral yielded in a weigh-function for the superposition integral that is proportional to the value of the Fresnel zone matrix. This has led us to reinterpret in a physical sense the previous, numerical only, weigh-function of Klimĕs (1984), which presented no constraints as for the number of rays to be superposed in a reference geophone. Now, with the idea of the Fresnel zone, it is clear that the number of paraxial rays that influence an observation in a geophone are those that emerges inside the first Fresnel zone of this geophone. These ideas can be seen in Ferreira and Cruz (2004,2005*a*).

In this work we extend the above ideas to a prestack depth migration where velocity varies in a depth-dependent way. The synthetic results demonstrates that in thinking in this way (i.e., using projected Fresnel as delimitators for the paraxial rays), the final images presented a sensible reduction in the number of migration artifacts and noise, besides a good resolution of the final images.

METHODOLOGY

The migration operator to be considered is given by (Ferreira and Cruz, 2005*b*)

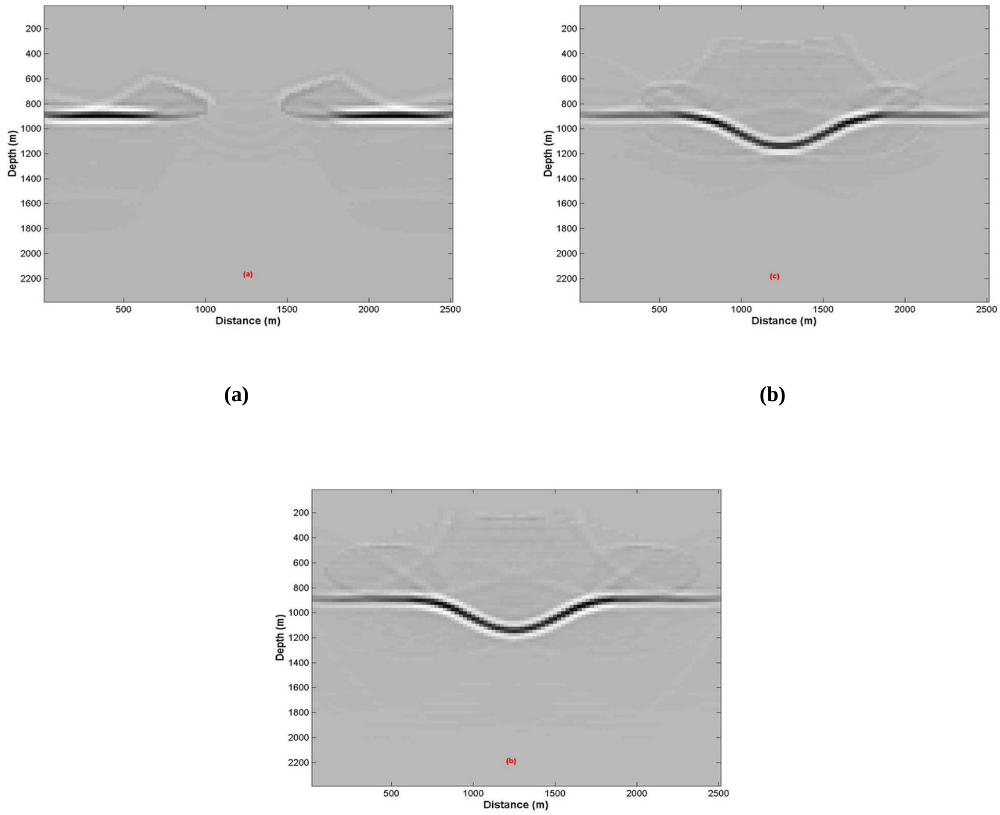
$$I(M,t) = \frac{1}{4\pi^2} \int_A d\xi_1\, d\xi_2\, w(\vec{\xi},M) \int_f df\, \dot{U}[\vec{\xi},f,t+\tau_D(\vec{\xi},M)], \tag{1}$$

where

$$\dot{U}[\vec{\xi},f,t+\tau_D(\vec{\xi},M)] = \int_{A_P} d\xi_1^P\, d\xi_2^P \sqrt{det\,\mathbf{H}_P(\vec{\xi}^P,f)}\, \ddot{U}[\vec{\xi},f,t+\tau_D(\vec{\xi},\vec{\xi}^P,M)]. \tag{2}$$

In (1) and (2) A is the migration aperture, A_P is the projected Fresnel zone aperture for each seismic trace, $M = M(x,z)$ represents the coordinate of a point in depth, $w(\vec{\xi},M)$ is the migration weigh-function that corrects the seismic amplitudes from geometrical speading losses, $\vec{\xi} = (\xi_1,\xi_2)^T$ is a coordinate vector that parameterizes the positions of sources and geophones along the acquisition surface, $\tau_D(\vec{\xi},\vec{\xi}^P,M)$ is the Huygens surface (Schleicher et al., 1993) relative to the position vector $\vec{\xi}^P$ (points inside the projected Fresnel zone), $\mathbf{H}_P = \mathbf{H}_P(\vec{\xi}^P,f)$ is the projected Fresnel zone matrix and $U(\vec{\xi},t)$ is the seismic data – the dots over it represent time derivatives. In the present case, the dependence of $\mathbf{H}_P$ in frquency f is due to the frequency content of each Fresnel zone that it is summed and serves as input in the Kirchhoff summation; that is why we assign a integration in f in the operator above, but it has nothing to do with Fourier transforms. The imaging condition is given by $t = 0$.

Figure 1 illustrate the fact of imaging a syncline without the frequency summation. In Figure 1*a*, the dominant frequency of the seismic signal is 10 Hz. We can see that only the plane parts of the synclie were



(a) **(b)**

(c)

Figure 1: Frequency content of the data: (a) 10 Hz. (b) 50 Hz. (c) 100 Hz.

actually and correctly imaged; the curved part, correspondng to its trough, was not “seen” by the migration operator. This enphasizes the fact that the Fresnel zones locate don the plane parts of the structure present different values and sizes of Fresnel zones than the values and sizes of the curved parts, inside the trough. It means that these frequencies are higher (i.e., the size of the Fresnel zones in the trough are lower) and the frequency summation was not sufficient to image the referred parts.

In Figures 1*b* and *c*, the frequencies used were 50 Hz and 100 Hz, respectively. In this time, the trough of the syncline was corrected imaged, although the frequency content of 100 Hz did not bring any enhancement to the final image. On the other hand, the planr parts of the model were not well imaged and its resolution were not sufficient to form a image with a higher resolutionin these parts of the structure. The migration artifacts seen in the final images are due to this fact, since when one part of the structure is not imaged due to its frequency content, artifacts imediatelly appear. In the first case (Figure 1*a*), the plane parts of the syncline are correctly imaged, including with no border effects, i.e., no migration smiles. However, artifacts from the trough of the syncline are introduced in the image, since it was not found in the data the informations corresponding to the frequency that is being used in the imaging of that area. In the other examples (Figures 1*b* and *c*), the effects happen again, but this time the artifacts are due to the border, which in turn were not correctly imaged.

SYNTHETIC EXAMPLES

Syncline and anticline models

Now consider the case in which we have a constant gradient velocity medium (Figure 2). In the present case, the velocity is depth dependent only and is given by

$$v_P = v_0 + \beta(z - z_0), \tag{3}$$

where β is the velocity gradient (measured in s^{-1}) and z_0 the value of an initial depth. One common parametrization for the raytracing in this case is to consider the own depth as integrator in ray equations (Portugal, 2002). In this way coordinates x, traveltime τ and the parameter σ, besides the own ray Jacobian (Červený, 2001) are analitically computed.

In this particular example, we consider that the P wave velocities in the sediments above the strutures in the form an anticline and in the form of a synclinevaries from $v_0 = 2.0\,Km/s$ to $v_P = 3.5\,Km/s$. Then, the assumed gradient in velocity has the value of $\beta = 0.975\,s^{-1}$. Considering these velocity models as the true ones, we have simulated an common-offset seismic data acquisition, with $2h = 12.5\,m$.

In Figure 3, we see the results of the KGB-PSDM migrations compared to the Kirchhoff PSDM migrations, for the dome model. The KGB-PSDM image is much less noisy than the Kirchhoff PSDM image, but we can also see that the resolution in Kirchhoff is still superior near the dome flanks. In the same figure, we also have the results of comparisons for the case of the syncline model. Again we see that the KGB-PSDM migration is less noisy than Kirchhoff, with some minor differences in some parts of the reflector, such as the borders.

Sensibility with respet to errors in the velocity model

In this section we analyse and compare the results of the migration process for the oparators Kirchhoff and KGB-PSDM, respectively, with respect to errors in the velocity models. We shall use as examples the dome model (anticline) and a trough model in strike (syncline). Their respective velocity models can be seen in Figure 2.

In Figure 4 we see the tests with lower velocity errors (undermigration) in the migration process varying between 5% and 20 %, respectively, assuming a 5% interval. To the left, we have the Kirchhoff migration results; to the right, we have the KGB-PSDM results. Since the velocity model is not correct, the structures were undermigrated to false positions in depth. In the Kirchhoff migration, besides border artifacts and noise, in the top part of the images it can be seen artifacts originated from the plane parts of the model, according to what was explained in Hertweck et al. (2003). With the KGB-PSDM migration these artifacts and noise are no longer seen, but the higher the velocity error the artifacts reappear (cf. below structures in Figures 4*e* e 4*f*). In Figure 5 we test for the overmigration. Again velocities are varying from 5% to 20%, with increments of 5%. Overmigratrion does not collapses diffractions to the apex of their hyperbola (Yilmaz, 1987), such that the final form of the structure does not correspond to its true format in depth. Besides, in the Kirchhoff migration we can still notice the artifacts from the plane parts of the structures and from the borders, which are common in this kind of migration and that are due to the truncation of their diffraction curves on their respective positions. By using the KGB-PSDM migration, all these artifacts disappear, but we can see that the overmigration process is analogous to Kirchhoff, altough less noisy, numerically speaking.

Amplitudes

As for the amplitudes, Figure 6 shows one first comparison of normalized picked amplitudes, obtained by the two migration process, for the dome model (Figure 2*a*), this time the true velocity model. The amplitudes in red represent the amplitudes of interest. In green, we depict the amplitudes obtained by the true amplitude Kirchhoff migration. It must be noticed that the values obtained oscillates along the relfector, mainly around the domic part. With the KGB-PSDM migration, these values does not oscillate like Kirchhoff, but even so they are far unsatisfactory with respect to the true values.

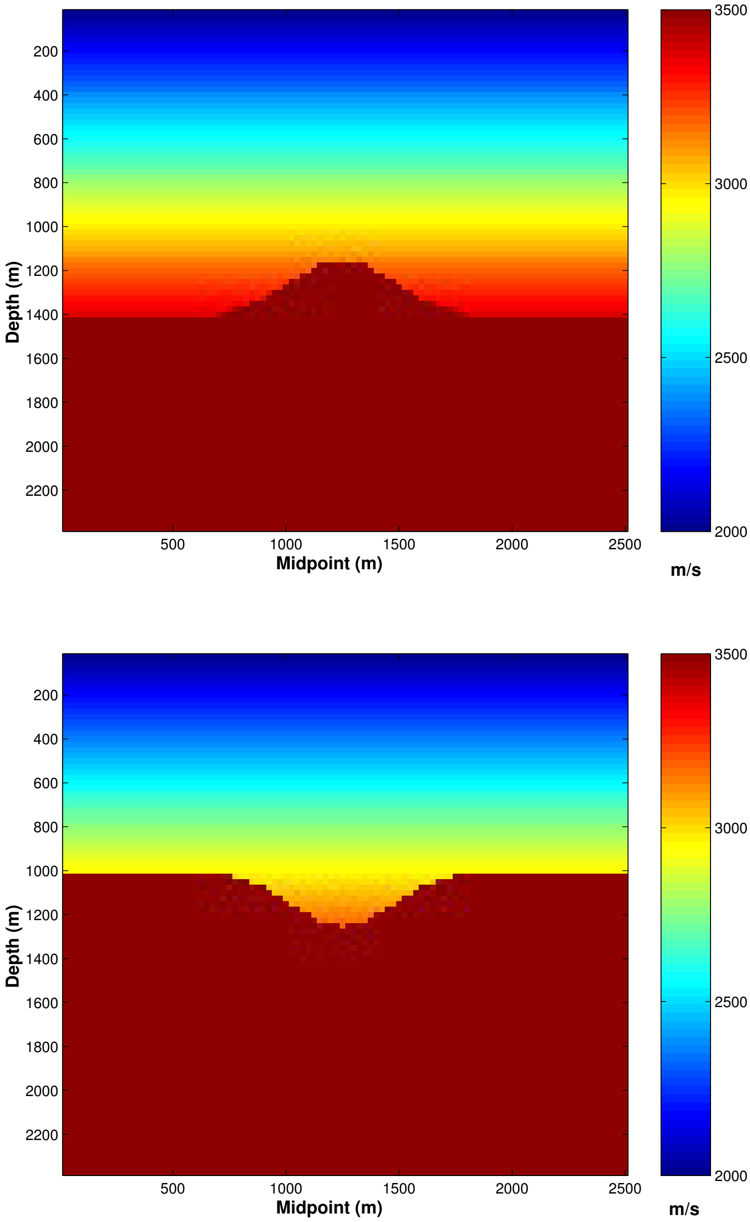


Figure 2: Velocity models with constant vertical gradient. In these models we consider $\frac{dv}{dz} = 0.975\, s^{-1}$. (a) Anticline model (dome). (b) Syncline model (trough or channel).

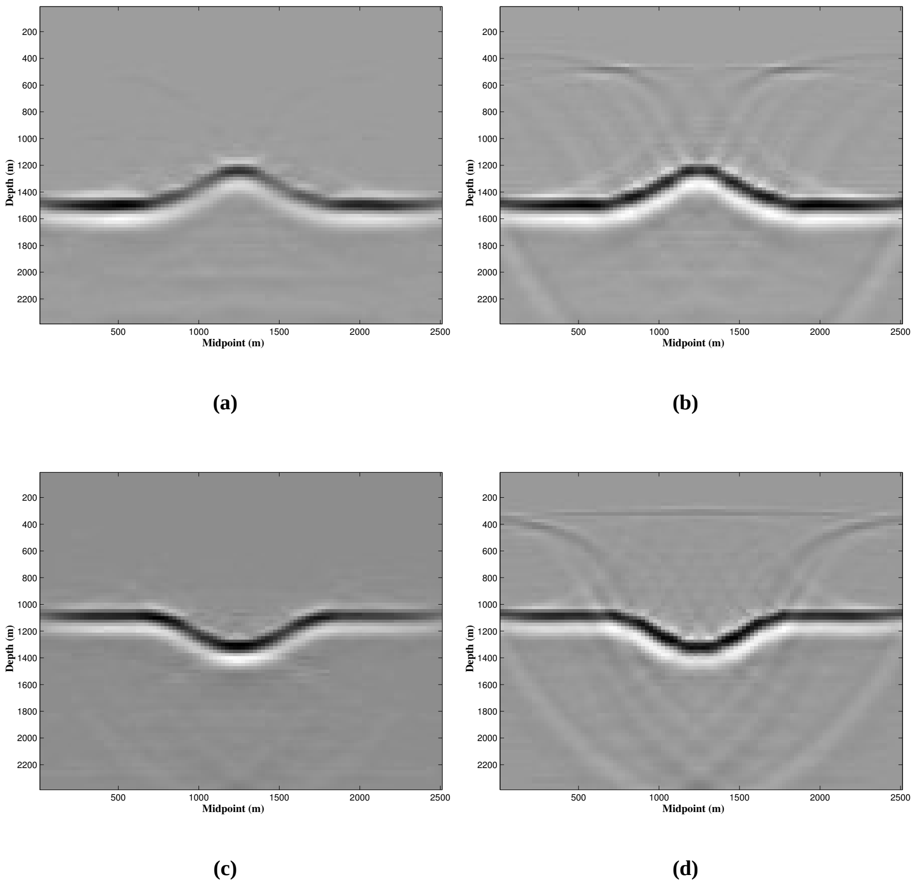
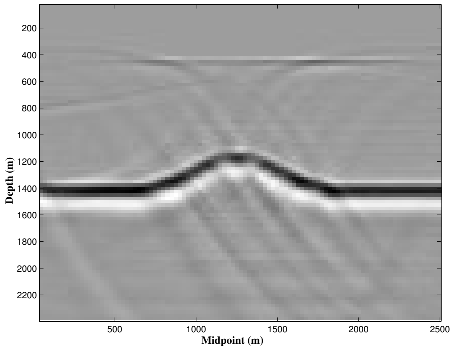
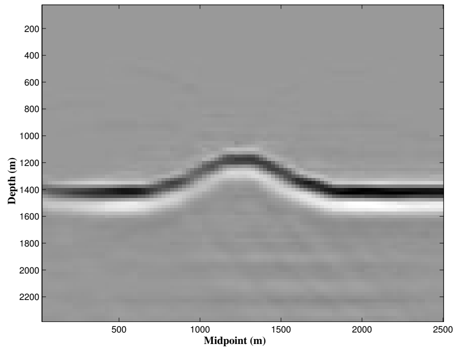


Figure 3: Constant gradient migration for the velocity models of Figure 2. Left: KGB-PSDM migration. Right: Kirchhoff PSDM migration.



(a)



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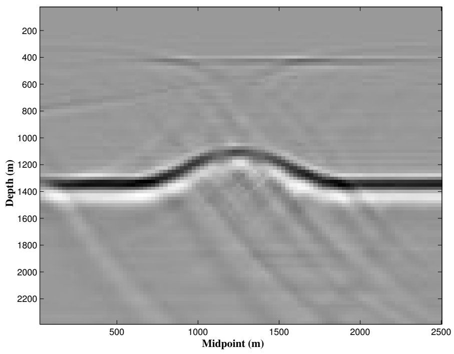

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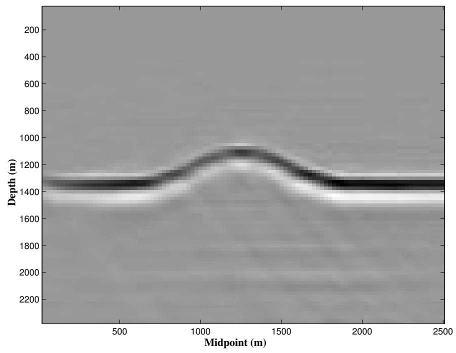

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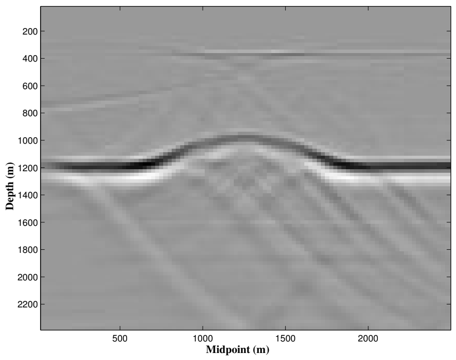
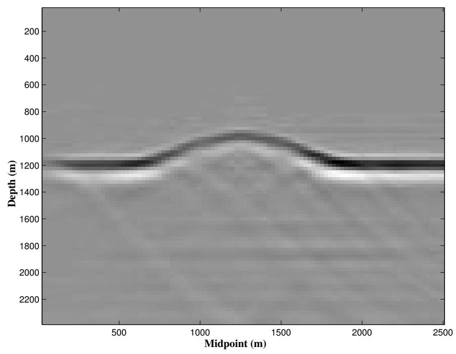


(e)



(f)

Figure 4: Sensibility comparison with respect to errors in the velocity model for the Kirchhoff (left) and KGB-PSDM (right) migrations, respectively. Errors in velocity percentage: (a) and (b) 5% lower, (c) and (d) 10% lower, (e) and (f) 20 % lower.

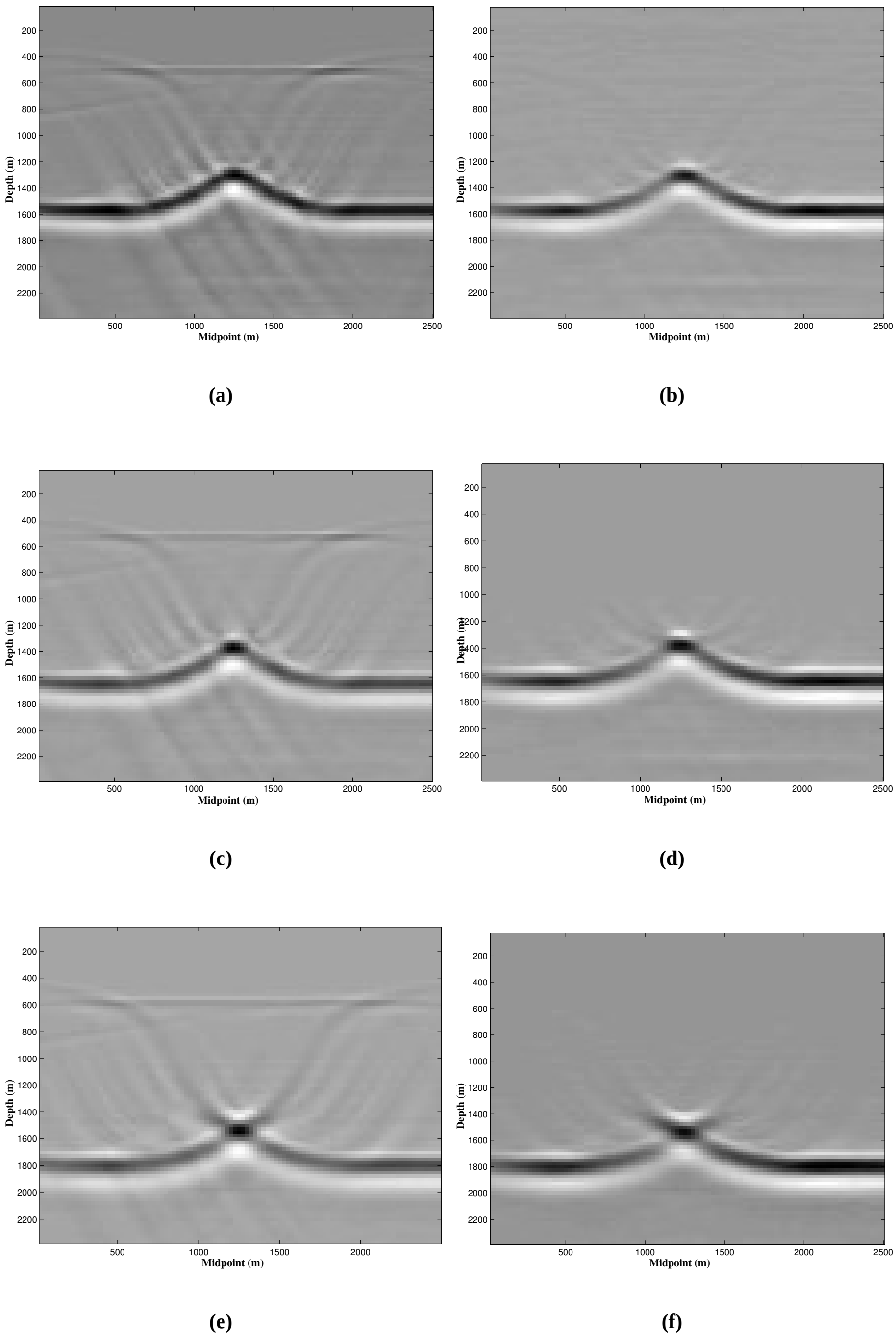


Figure 5: Sensibility comparison with respect to errors in the velocity model for the Kirchhoff (left) and KGB-PSDM (right) migrations, respectively. Errors in velocity percentage: (a) and (b) 5% higher, (c) and (d) 10% higher, (e) and (f) 20 % higher.

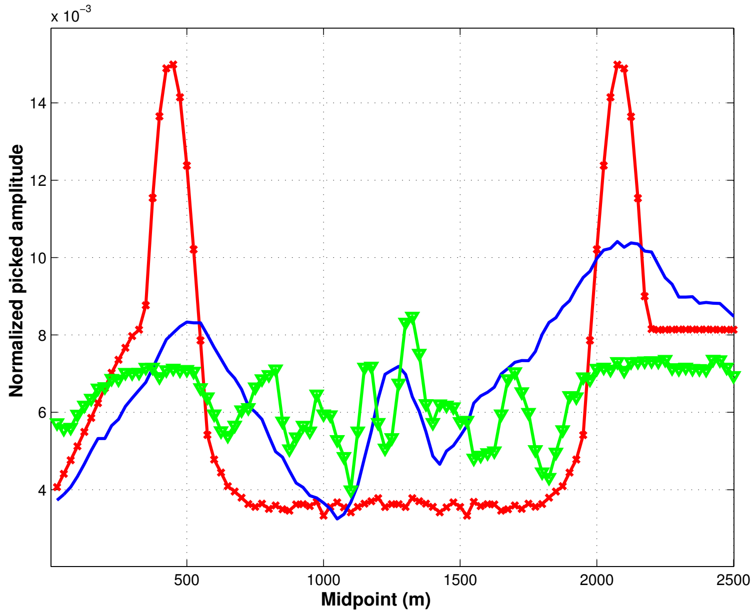


Figure 6: Normalized picked amplitude comparison by the two types of migration (green-Kirchhoff, blue-KGB-PSDM) with respect to true amplitudes (red curve).

CONCLUSIONS

We have developed a Kirchhoff-type prestack depth migration in which we considered as Green function for the imaging problem an superposition integral of Gaussian beams. In the present work, this approach was extended to the special case of a constant gradient velocity media in depth.

We have tested the KGB-PSDM algorithm in some curved geologic models, such as salt domes (anticline) and a channel (syncline). The velocity gradient with depth considered was $0.975\,s^{-1}$. In both examples, the KGB-PSDM algorithm considerably eliminated the presence of migration artifacts, as well as migration noise most common in the tradicional Kirchhoff migration. In the case of the salt dome, we have observed some lack of resolution in the parts representing the flanks of the structure, where we firmly believe that this a direct consequence of thetrunction of its impulse response. But, above all, both images using the KGB-PSDM algorithm showed themselves less aliased when compared to the Kirchhoff process.

One test regarding the sensibility of our algorithm to possible errors in the velocity models showed that the KGB-PSDM operator is equally sensible to errors in the velocity model, considering the cases os undermigration or overmigration. However, in terms of artifacts and noise commonly present in the seismic data, the KGB-PSDM migration showed less aliased a less noisy results.

A preliminary comparison of normalized picked amplitudes of both methods, considering the true migration velocity, showed that the amplitudes recovered by the KGB-PSDM process does not oscillate as much as the ones recovered by Kirchhoff. These amplitude were abtained along the domic part of the structure. Altohugh these are good results, they are still unsatisfactorally good for the present case, when compared to their true values to be recovered.

PUBLICATIONS

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