Filling Gaps in Ray Traveltime Maps

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ABSTRACT

Seismic traveltime maps computed with ray methods can exhibit gaps at isolated grid points, e.g. in caustic regions. For some applications, like migration, these gaps can lead to problems. In this paper we suggest a method to fill the gaps. It is based on a hyperbolic traveltime expansion and leads to a simple formula for the estimated traveltime at the gap. We illustrate the accuracy of the technique with two examples.

INTRODUCTION

The calculation of seismic traveltimes is fundamental for many applications within subsurface imaging, such as tomography, or Kirchhoff migration. Due to their flexibility and accuracy, wavefront construction (WFC) methods, a class of ray-based traveltime algorithms, are often applied to generate gridded traveltime maps. However, in certain situations blank spaces occur in the traveltime maps, meaning that for isolated grid points a traveltime value could not be assigned (see Figure 1). This happens because the ray method fails in the vicinity of surfaces along which the ray field is not regular. Examples of such surfaces are caustic surfaces and boundaries of shadow zones (Cerveny, 2001, p.608). As a consequence, WFC algorithms, or, more generally techniques based on the ray method, can encounter difficulties of handling first arrivals in convergence (i.e. caustic) areas, where a first-arrival wavefront shrinks into a point (Vinje et al., 1993).

In this paper, we suggest a very simple technique to mend the gaps in affected traveltime maps. It is based on a hyperbolic interpolation of traveltimes and should be carried out as a pre-process to the intended application, if the latter is expected to be sensitive to the blank spaces. One example for such an application is the traveltime-based strategy for true-amplitude migration (Vanelle et al., 2004). After giving a description of the method, we investigate its accuracy.

METHOD

For simplicity we will consider only the 2-D case here; the extension to 3-D is straightforward.

To fill isolated gaps in traveltime maps generated with WFC methods we apply the second-order interpolation technique introduced by Vanelle and Gajewski (2002). It is based on a Taylor expansion of the squared traveltime and leads to a hyperbolic expression. Following Vanelle and Gajewski (2002), the squared traveltime T^2 to a point at $\mathbf{r} = \mathbf{r}_0 + \Delta\mathbf{r} = (x, y, z)^\top$ is approximated by

$$T^2(\mathbf{r}) = (T_0 + \mathbf{q}\,\Delta\mathbf{r})^2 \;+\; T_0\,\Delta\mathbf{r}^\top \underline{\mathbf{G}}\,\Delta\mathbf{r} \quad , \tag{1}$$

introducing the slowness vector $\mathbf{q}$ and the second-order derivative matrix $\underline{\mathbf{G}}$,

$$q_i = \frac{\partial T}{\partial r_i} \qquad \text{and} \qquad G_{ij} = \frac{\partial^2 T}{\partial r_i\,\partial r_j} \quad .$$

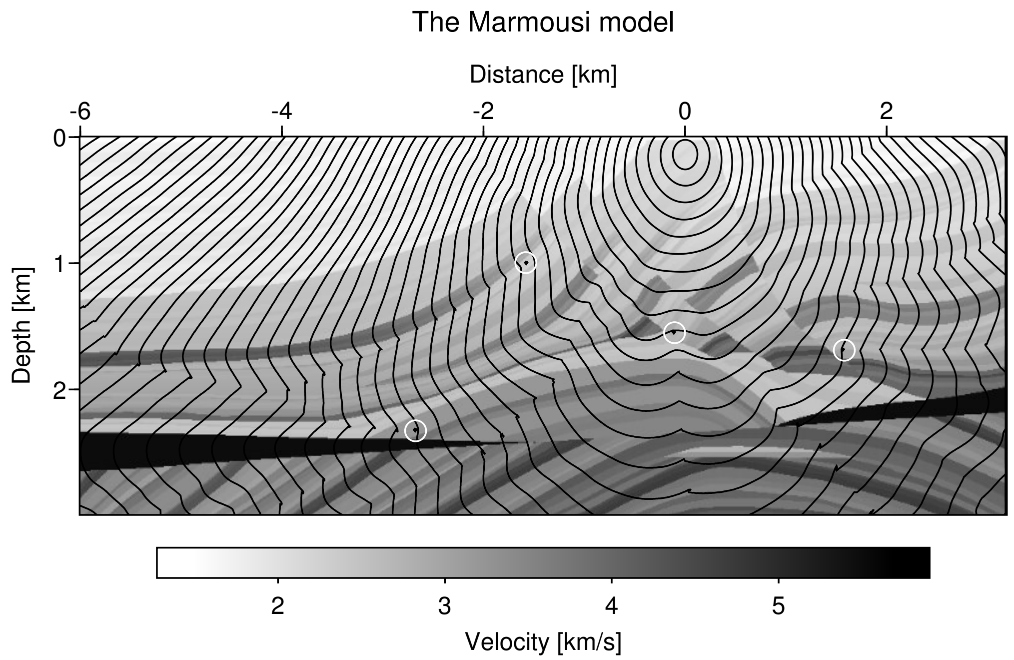

Figure 1: The complex Marmousi velocity model. The black lines are isochrones of first-arrival traveltimes generated with a wavefront construction (WFC) algorithm. White circles indicate grid points in the traveltime maps, where no traveltime value could be assigned. Instead, the value at these points is an initial default value to which the traveltime array was set by the WFC routine. In our WFC implementation, this value was –1 s. Correspondingly, the gaps show as black dots caused by the clustering of isochrones down to –1 s, the initial value. As we can see, the gaps occur at the beginning of a triplication of the wavefront, i.e. at the onset of later arrivals.

Futhermore, T_0 is the traveltime in the expansion point $\mathbf{r}_0$, where the position $\mathbf{r}_0$ is assumed to coincide with a node on the (coarse) grid on which the traveltime map is sampled.

Figure 2 shows a traveltime curve as a function of x at a fixed depth. First- and later-arrival traveltimes are displayed. At the node with index $i = 5$ the two branches of first arrivals (left and right of $i = 5$) intersect. At the intersection point we find a gap indicated by the white circle, a node to which no traveltime value could be assigned. This can occur at the onset of later arrivals, if the traveltimes were computed by ray methods. In the following we will show how the hyperbolic traveltime formula (1) can be applied to provide a suitable estimate for the traveltime value at the affected grid point.

Consider the traveltime curve in Figure 2 with the gap at node $i = 5$, and the nodes $i = 6, 7, 8$. If the x-component x_0 of the expansion point $\mathbf{r}_0$ is chosen at $i = 6$ with the traveltime $T_0 = T(i = 6) \equiv T_6$, the traveltimes at $i = 7$ and $i = 8$ can be expressed with Equation (1):

$$\begin{aligned} T^2(i=7) \equiv T_7^2 &= (T_6 + \quad q_x\, d)^2 + \quad T_6\, G_{xx}\, d^2 \quad , \\ T^2(i=8) \equiv T_8^2 &= (T_6 + 2\, q_x\, d)^2 + 4\, T_6\, G_{xx}\, d^2 \quad . \end{aligned} \tag{2}$$

With the known traveltimes T_6, T_7, T_8, and the grid spacing $d = \triangle x$, the system (2) can be solved for the coefficients q_x and G_{xx} in the expansion point $i = 6$:

$$\begin{aligned} q_x &= \frac{-3\, T_6^2 + 4\, T_7^2 - T_8^2}{4\, T_6\, d} \quad , \\ G_{xx} &= \frac{T_8^2 + T_7^2 - 2\, T_6^2}{5\, T_6\, d^2} - \frac{6\, q_x}{5\, d} - \frac{q_x^2}{T_6} \quad . \end{aligned} \tag{3}$$

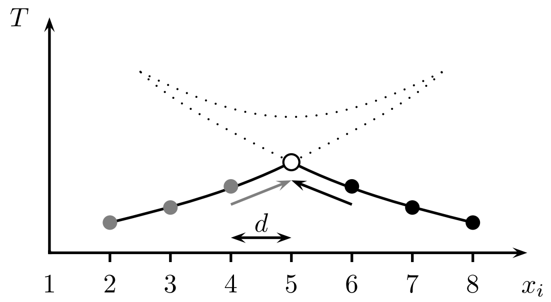

Figure 2: Traveltime curve with first (solid line) and later arrivals (dotted lines) along the x direction. Traveltime values are given on nodes of a grid with the spacing $d = \triangle x$. The node positions corresponding to x_i are given by the index i. At the node with $i = 5$ we find a gap (white circle), a grid point where no traveltime value has been assigned. This can occur at the onset of later arrivals, if the traveltimes were generated with a ray-based algorithm like wavefront construction. If at least three grid points to the left (gray nodes) or the right (black nodes) of such a node exist with traveltime values, these can be used to fill the gap with a hyperbolic extrapolation of the traveltimes from the adjoining nodes (gray and black arrows).

Extrapolation from $i = 6$ leads to a simple expression for the (squared) traveltime at $i = 5$,

$$T^2_{6\to 5} = 3\,T^2_6 - 3\,T^2_7 + T^2_8 \quad . \tag{4}$$

Here, the subscript $6 \to 5$ indicates the extrapolation from node 6 to node 5. Similarly, extrapolation from $i = 4$ yields

$$T^2_{4\to 5} = 3\,T^2_4 - 3\,T^2_3 + T^2_2 \quad . \tag{5}$$

The traveltime at $i = 5$ should then be taken as the mean of $T_{6\to 5}$ and $T_{4\to 5}$. If only one of the traveltime values $T_{6\to 5}$ or $T_{6\to 5}$ can be determined, e.g. at a node with less than three grid points to the model boundary, that single value should be taken. However, in most cases where a gap exists, it will be an isolated point at the onset of the later arrivals, where the extrapolation results from both sides should coincide.

Although the extrapolation could be carried out also in depth, we suggest to extrapolate in x (and y in 3D) rather, as in most geological situations the velocity varies stronger with depth than laterally.

APPLICATION

For obvious reasons, a direct comparison between the extrapolated and correct traveltime at gaps could not be carried out, as the correct traveltime could not be determined at the gaps. We have, therefore, simulated gaps in traveltime maps by assuming for each node of the gridded traveltime map that it corresponds to a gap. The traveltime at each point was thus computed by extrapolation following (4) and (5), and that value was compared to its original value. Our first example has a constant velocity gradient with V=3 km/s at the source, $\partial V/\partial z$= 0.5/s and $\partial V/\partial x$=0.1/s. Extrapolation of the traveltimes yields a mean relative error of 0.0031 %.

We have also investigated the accuracy of the extrapolation for the Marmousi model (Figure 1). The mean of the relative error resulting from the extrapolation is 0.31 %. This value is, however, dominated by high maximum errors of up to 15.8 %. These occur for those grid points in the vicinity of the onset of later arrivals. Although this appears to be a contradiction, it does not concern the intended application to fill gaps: strictly, the extrapolation is only allowed when it is ensured that the expansion point and gap lie on the same branch of the traveltime curve, as it is the case for the gaps in ray traveltime maps (see also Figure 2). In assuming a gap at each point of the Marmousi model, the extrapolation was applied from both sides, regardless of the traveltime branch. As a consequence, those grid points have been assigned traveltimes with higher errors due to the wrong extrapolation. If gaps occur, they are located at the onset of later-arrivals, where the two traveltime branches which form the triplication meet. This means that they have the same traveltime value at that point, and therefore extrapolation from both sides of the gap can be applied.

CONCLUSIONS

The hyperbolic traveltime equation by Vanelle and Gajewski (2002) can be applied to fill gaps in ray traveltime maps. These gaps can occur at the onset of later arrivals, if first-arrival traveltime maps are computed with ray techniques like wavefront construction. Traveltimes in a close vicinity of a gap are extrapolated until the gap with the hyperbolic coefficients. Owing to the high accuracy of the hyperbolic method, the result is a very simple but reliable formula. It can be applied as a pre-process to ray traveltime maps with gaps, whenever they are subject to a process like migration that is sensitive with regards to the blank grid nodes.

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