A comparison of seismic attributes

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ABSTRACT

In the last two decades, many approximations for the P-P reflection coefficient have been proposed in
the literature. Basically, all of them are derived from the classical approximation of Aki & Richards,
using additional assumptions on the medium parameters. The aim of constructing such approxima-
tions is to establish reliable attributes that can be capable to indicate the presence of oil or gas. In this
work we review some well known approximations and their respective attributes, We also introduce a
new indicator based on a impedance-type of approximation for the reflection coefficient. Numerical
examples are also provided.

INTRODUCTION

The variation of amplitude with offset (AVO) is a powerful tool to discriminate rocks containing gas and
oil. Several approximations of the P-P reflection coefficient (R) have been proposed and different AVO
indicators can be extracted from them. However, there is no agreement about which is the best attribute
and in which situation it would be better applied. The aim of this work is to present a general approach of
the well-known approximations of the reflection coefficient and its respective attributes. The starting point
for all the approximations is the classical approximation of Aki and Richards (2002), which is based on a weak
contrast in the media parameters and a small angle of incidence. Recently, impedance-type approximations
for the reflection coefficient have been introduced. See, e.g., Connolly (1999) and Santos and Tygel (2004).
Based on this kind of approximation we introduce a new indicator and numerical examples demonstrate
the ability of the attributes to discriminate between gas and oil.

APPROXIMATIONS FOR $R$ AND THE ASSOCIATED SEISMIC ATTRIBUTES

Let us consider two isotropic homogeneous elastic media separated by a smooth interface. Each medium
has a P-wave velocity $c_1$, a S-wave velocity $c_2$ and a density $\rho$. Further, let us consider an incident com-
pressional plane wave impinging upon this interface. The P-P reflection coefficient $R$ for a compressional
reflected wave has an exact expression knowing as the Zoeppritz-Knott formula. This formula is very hard
to handle and it is difficult to extract the physical sense of their terms.

For a small contrast between the properties of the two media and a small angle of incidence, the well
known first-order approximation of Aki and Richards (2002) is given by

$$R \approx \frac{1}{2} \left[ 1 - \frac{\beta^2}{\alpha^2} \sin^2 \theta \right] \frac{\Delta \rho}{\rho} + \frac{\sec^2 \theta \Delta \alpha}{\alpha} - \frac{\beta^2}{\alpha} \sin^2 \theta \frac{\Delta \beta}{\beta},$$

(1)

where $\theta$ is the incident angle, $u = (u_2 + u_1)/2$ and $\Delta u = u_2 - u_1$ for $u = \alpha, \beta,$ and $\rho$.

Shuey (1985) rewrote the expression (1) as a function of $\theta$:

$$R \approx A + B \sin^2 \theta + C [\tan^2 \theta - \sin^2 \theta],$$

(2)
where the parameters $A$ (Intercept), $B$ (Gradient) and $C$ are given by

$$A = \frac{1}{2} \left[ \frac{\Delta \rho}{\rho} + \frac{\Delta \alpha}{\alpha} \right], \quad B = \frac{1}{2} \frac{\Delta \alpha}{\alpha} - \frac{2 \beta^2}{\alpha^2} \left[ \frac{\Delta \rho}{\rho} + 2 \frac{\Delta \beta}{\beta} \right], \quad \text{and} \quad C = \frac{1}{2} \frac{\Delta \alpha}{\alpha}. \quad (3)$$

Shuey was further on: for incidence angles smaller than 30 degrees, $\tan^2 \theta \approx \sin^2 \theta$ and then, equation (2) turns to be

$$R \approx A + B \sin^2 \theta. \quad (4)$$

Equation (4) is the most used AVO formula. Castagna and Smith (1994) presented a large study using $A$ and $B$, $A \times B$ and $(A + B)/2$ as AVO indicators. In that work they have shown that the difference between the normal incidence P-P and S-S reflection coefficients can be well approximated by $(A + B)/2$. Moreover, it is also a robust indicator for clastic section to separate brine sands and gas sands, as shown on the right of Figure 1. However, for some of their suite of 25 measurements of the Gulf of Mexico and Gulf Coast, this indicator has failed. On the left of Figure 1 we observe that $A \times B$ attribute is not a good discriminator either.

![Figure 1](dataset:image)

**Figure 1:** Shuey based attributes: shale over gas sand (+) and shale over brine sand (o).

Smith and Gidlow (1987) used Gardner’s relationship for water-saturated rocks (Gardner et al., 1974), $\rho = \alpha^{1/4}$, to obtain the following approximation for $R$,

$$R \approx \left[ \frac{5}{8} - \frac{1}{2} \frac{\beta^2}{\alpha^2} \sin^2 \theta + \frac{1}{2} \tan^2 \theta \right] \frac{\Delta \alpha}{\alpha} - \frac{4 \beta^2}{\alpha^2} \frac{\Delta \beta}{\beta}. \quad (5)$$

Using the mudrock line of Castagna et al. (1985),

$$\alpha = 1.36 + 1.16 \beta \quad \text{(in km/s)}, \quad (6)$$

which relates the P- and S-wave velocities for water-saturated sandstones, siltstones and shales, Smith and Gidlow (1987) define the “fluid factor” indicator $\Delta F$ as

$$\Delta F = \frac{\Delta \alpha}{\alpha} - 1.16 \frac{\beta}{\alpha} \frac{\Delta \beta}{\beta}, \quad (7)$$

where the $\alpha$ and $\beta$ contrast can be estimated from equation (5). The second term in the fluid factor is the value of $\Delta \alpha/\alpha$ predicted from $\Delta \beta/\beta$ using the mudrock line. Therefore, $\Delta F$ will be close to zero for water-bearing and shales rocks and nonzero for other type of rocks or fillings. Figure 2 depicts the behavior of the fluid factor for the same type of interfaces used previously.
Fundamental rock properties such as compressibility and shear rigidity are easier to understand than acoustic velocities and impedances. In this direction, several authors gave their contribution. For small angles of incidence and $\beta/\alpha$ ratio in the range $[1.5, 2.0]$, Fatti et al. (1994) rewrote equation (1) in terms of $P$ and $S$ impedance contrasts,

$$R \approx \frac{1}{2} \left[ 1 + \tan^2 \theta \right] \frac{\Delta I_P}{I_P} - 4 \left[ \frac{\beta^2}{\alpha^2 \sin^2 \theta} \right] \frac{\Delta I_S}{I_S}, \quad (8)$$

where $I_P = \rho \alpha$ and $I_S = \rho \beta$. From this equation it is possible, through inversion procedures, to extract the impedance contrasts and as Goodway et al. (1999) proposed, compute $\lambda \rho$ and $\mu \rho$ as AVO indicators from the relationships

$$\lambda \rho = I_P^2 - 2I_S^2 \quad \text{and} \quad \mu \rho = I_S^2, \quad (9)$$

where $\lambda$ and $\mu$ are the Lamé parameters.

Assuming that the ratio $\beta/\alpha$ is known, Gray et al. (1999) have used the contrasts in $\lambda$, $\mu$ and $\rho$ as parameters for the inversion of $R$, i.e.,

$$R \approx \left[ \frac{1}{4} - \frac{1}{2} \frac{\beta^2}{\alpha^2} \right] \sec^2 \theta \frac{\Delta \lambda}{\lambda} + \frac{\beta^2}{\alpha^2} \left[ \frac{1}{2} \sec^2 \theta - 2 \sin^2 \theta \right] \frac{\Delta \mu}{\mu} + \left[ \frac{1}{2} - \frac{1}{4} \sec^2 \theta \right] \frac{\Delta \rho}{\rho}. \quad (10)$$

The use of Lamé parameters as AVO indicators can be observed in Figure 3.

**IMPEDANCE-TYPE APPROXIMATIONS AND NEW ATTRIBUTES**

Following the simple cases of normal incidence in elastic media and general oblique incidence in acoustic media, two new approaches for approximating the reflection coefficient appear recently in the literature. The main idea is to write the reflection coefficient as a function of a “angular” impedances,

$$R \approx \frac{I_2 - I_1}{I_2 + I_1}, \quad (11)$$

where $I_1$ refers to the incident side and $I_2$ to the transmission side.

Connolly (1999) introduced the elastic impedance, $I = EI$,

$$EI = N_0 \alpha \sec^2 \theta \beta^{-8K} \sin^2 \theta \rho^{1-4K \sin^2 \theta}, \quad (12)$$

where $K = \beta^2/\alpha^2$ is assumed constant, and $N_0$ is a normalization constant (Whitcombe, 2002). In the derivation of $EI$, the angle $\theta$ was also considered constant in both sides of the interface, which is not true in the physical sense.

**Figure 2:** Fluid Factor $\Delta F$: shale over gas sand (+) and shale over brine sand (o).
Santos and Tygel (2004) has shown that no exact closed-form solution for equation (11) exists. However, under suitable restrictions in the medium parameters $\rho$ and $\beta$ (e.g., $\rho = b \beta^\gamma$), they introduce the reflection impedance, $I = RI$, 

$$RI = M_0 \frac{\rho \alpha}{\sqrt{1 - \alpha^2 p^2}} \exp\{-4 p^2[\beta^2 + f(\beta)]\}, \quad (13)$$

where $M_0$ is a normalization constant, $p$ is the ray parameter, and $f$ is a function that depends on the considered functional relation between $\rho$ and $\beta$. For the case $\rho = b \beta^\gamma$, $f(\beta) = \gamma \beta^2/2$. It is important to mention that in the derivation of $RI$, it is $p$ that is considered constant on both sides of the interface, following the actual physics of the ray.

For any choice of the impedance, $I = EI$ or $I = RI$, from equation (11), it is possible to define a new attribute $J$

$$J = J = \frac{I_2}{I_1} \approx \frac{1 + R}{1 - R}. \quad (14)$$

Clearly, this indicator depends on the angle of incidence. Figure 4 shows the behavior of $J$ for elastic and reflection impedances, taking $\theta = 30^\circ$. We can observe that this new attribute separates well gas sand from brine sand for all the 25 models presented previously.

CONCLUSIONS

Different approximations for the P-P reflection coefficient provide different indicators to discriminate gas and oil. When applied to a set of data, some of them are able to separate gas sand from brine sand. We introduced a new attribute which was more efficient in discriminate gas and oil for the same data. Further investigation is being carried to test the potential of the new attribute for well-log analysis.

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REFERENCES

Figure 4: Attribute J for elastic (JE) and reflection (JR) impedances and $\theta = 30^\circ$: shale over gas sand (+) and shale over brine sand (o).


