Image-wave remigration in elliptically isotropic media

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keywords: *remigration, anisotropy, image wave*

ABSTRACT

The image-wave equation for the problem of depth remigration in elliptically isotropic media is a second-order partial differential equation similar to the acoustic wave equation. The propagation variable is the medium ellipticity rather than time. In this work, we derive this differential equation from the kinematic properties of anisotropic remigration. The objective is to enable the construction of subsurface images that correspond to different degrees of medium anisotropy. In this way, "anisotropy panels" can be obtained in a completely analogous way to velocity panels for a migration velocity analysis.

INTRODUCTION

When a seismic migration is repeatedly carried out using different velocity models, the images of the seismic reflectors are positioned at different depth locations. To transform these migrated reflector images from one to another in a direct way, i.e., without going back to the original time section, is a seismic imaging task that can be achieved by a remigration, also known as velocity continuation or residual or cascaded migration. In this way, an improved seismic reflector image for an improved migration velocity is obtained by applying a migration operator to the already migrated rather then unmigrated section. Residual migration is based on the fact that the migrated image obtained from migrating a second time (with the migration velocity v_2) a seismic section that has already been migrated (with the migration velocity v_1) is identical to the one that would have been obtained from migrating the original time section once, with the effective migration velocity $v_{eff} = \sqrt{v_1^2 + v_2^2}$ Rocca and Salvador (1982). Given the first (wrong) migration velocity v_1 and the desired effective (true) migration velocity v_{eff}, a residual migration is nothing more than a conventional migration with the residual migration velocity $v_2 = \sqrt{v_{eff}^2 - v_1^2}$ Rothman et al. (1985). Cascaded migration involves an iterative procedure Larner and Beasley (1987). By performing n times a migration with a small velocity increment Δv, the desired effective migration velocity $v_{eff} = \sqrt{n\Delta v^2}$ is finally reached.

Is is not difficult to accept that by choosing a large number n of steps and a very small velocity increment Δv, a cascaded migration simulates a quasi-continuous change of the migration velocity. In this situation, the sequence of images of a certain reflector as subsequently migrated with varying migration velocities creates an impression of a propagating wavefront. This "propagating wavefront" was termed an "image wave" by Hubral et al. (1996). The propagation variable, however, is not time as is the case for conventional physical waves, but the migration velocity. Moreover, due to the different kinematic behaviour, this image-wave propagation is not described by a conventional (acoustic or elastic) wave equation.

The kinematic behaviour of image waves as a function of the (constant) migration velocity has been studied in time (Fomel, 1994; Hubral et al., 1996) and in depth (Hubral et al., 1996; Mann, 1998; Schleicher et al., 2004). By treating them in a similar way as conventional acoustic waves, they derived partial differential equations that describe the "propagation" of the reflector image as a function of migration velocity for both, time and depth remigration. Therefore, these partial differential equations have been termed "image-wave equations." Both image wave equations for time and depth remigration are equations similar to the acoustic wave equation (Fomel, 1994; Hubral et al., 1996; Mann, 1998). An independent earlier derivation of the time remigration equation by Claerbout (1986) has later also been made available to the public.

The image-wave equation for velocity continuation in *time* remigration has already been theoretically studied and implemented (Jaya et al., 1996; Jaya, 1997), as well as successfully applied to real data from ground-penetrating radar (Jaya, 1997; Jaya et al., 1999). Recently, its kinematics and dynamics have been thoroughly discussed (Fomel, 003b) and its use for time-migration velocity analysis in the prestack domain has been suggested (Fomel, 003a). The image-wave equation for velocity continuation in *depth* remigration has been recently investigated by Schleicher et al. (2004).

In this paper, we extend the idea of image waves to the remigration of images as a function of the medium anisotropy. For simplicity, we study the situation in media with elliptical isotropy, which can be described with one additional medium parameter. We choose the parameter describing the medium ellipticity to be the ratio between the squares of the vertical and horizontal velocities. We investigate the variation (or “propagation”) of the reflector image as a function of this parameter, which we denote φ. In other words, φ assumes the role of the propagation variable for the image wave in elliptically isotropic media.

DERIVATION OF THE IMAGE-WAVE EQUATION

In this section, we describe the variation of the position of a reflector image when the medium anisotropy changes. This variation will become the kinematics of the image-wave propagation of the image wave as a function of the medium anisotropy. For this purpose, we study the behaviour of a single point on the image of a seismic reflector when the medium ellipticity varies. This situation can be understood in analogy to the propagation of a Huygens wave emanating from a secondary source. The Huygens wave describes the behaviour of a single point on the wave front when time varies.

The procedure follows the lines applied by Hubral et al. (1996) to derive the velocity-dependent image-wave equation. It starts by the construction of the “Huygens image wave”, that is, the set of points that describes the possible location of the original point on the reflector after a variation of the propagation variable. In a second step, the coordinates of the original image point are replace by derivatives, in this way constructing an image eikonal equation the solution of which is the Huygens image wave. In a last step, the most simple of all second-order partial differential equations that generate this image eikonal equation is identified as the searched-for image-wave equation.

Elliptically isotropic medium

An elliptically isotropic medium is characterized by possessing a vertical symmetry. Its density-normalized elastic tensor, i.e., $A_{ik} = C_{ik}/\rho$, can be written as a 6×6-matrix of the form

$$\boldsymbol{A} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & 0 & 0 & 0 \\ A_{12} & A_{11} & A_{13} & 0 & 0 & 0 \\ A_{13} & A_{13} & A_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{66} \end{pmatrix}, \tag{1}$$

with the additional restrictions that

$$\begin{aligned} A_{12} &= A_{11} - 2A_{66}, \\ (A_{13} + A_{44})^2 &= (A_{11} - A_{44})(A_{33} - A_{44}). \end{aligned} \tag{2}$$

In this way, an elliptically isotropic medium is described by four independent elastic parameters.

Propagation velocity. For seismic imaging purposes, the most important medium parameter is the velocity of seismic wave propagation. Here, we need expressions for this this parameter in elliptically isotropic media. For more information on elliptically isotropic media, the reader is referred to Helbig (1983) or Vanelle (2002).

The group velocity vector of the quasi-P wave, $\vec{v}$, depends only on two of the components of the elasticity tensor. It is given by

$$\vec{v} = \left(\frac{A_{11}}{V} \sin\phi, 0, \frac{A_{33}}{V} \cos\phi \right), \tag{3}$$

where A_{11} and A_{33} are components of the elastic tensor and ϕ is the angle between the normal to the wavefront and the vertical z-axis. Moreover, quantity

$$V = \sqrt{A_{11} \sin^2\phi + A_{33} \cos^2\phi} \tag{4}$$

denotes the phase velocity of the quasi-P wave.

In a homogeneous elliptically isotropic medium, the propagation of a quasi-P wave takes place in a plane. For simplicity, we assume here and in the following that this plane is the (x, z)-plane. Therefore, the y-component in vector (3) is zero, and so are all y-components below. We can therefore treat the problem as a two-dimensional one. All formulas below can readily be extended to 3D by adding analogous y-components.

From equations (3) and (4), we conclude that the modulus of the group velocity can be represented as

$$|\vec{v}| = v(\phi) = \frac{\sqrt{A_{11}^2 \sin^2\phi + A_{33}^2 \cos^2\phi}}{V} \tag{5}$$

However, in anisotropic media, the wavefront normal does not generally point into the propagation direction of the wave. For our purposes, we need the propagation velocity as a function of the propagation direction. Therefore, we need to introduce the propagation angle θ, i.e., the angle between the group velocity vector $\vec{v}$ (which points into the propagation direction) and the vertical z-axis. The relationship between ϕ and θ is given by (Vanelle, 2002)

$$\tan\theta = \frac{A_{11}}{A_{33}} \tan\phi. \tag{6}$$

Introducing this relationship in equation (5), we find that the modulus of the group velocity depends on the propagation direction according to

$$v(\theta) = \left[\frac{\sin^2\theta}{A_{11}} + \frac{\cos^2\theta}{A_{33}} \right]^{-1/2}. \tag{7}$$

As a consequence of the medium anisotropy, the propagation velocities of the quasi-P wave depend on the propagation direction. In particular, there are different wave velocities in the vertical and horizontal directions. From equation (7), we recognize that the vertical ($\theta = 0$) and horizontal ($\theta = \pi/2$) velocities are given by

$$v_v = \sqrt{A_{33}} \qquad \text{and} \qquad v_h = \sqrt{A_{11}}, \tag{8}$$

respectively.

Zero-offset configuration

We assume that the migrated section to be remigrated was obtained from zero-offset (or stacked) data under application of a zero-offset migration. The coincident source-receiver pairs where localized at a planar horizontal surface ($z = 0$) at points $S = (\xi, 0)$ (Figure 1).

We denote by x and z the coordinates of a certain point P within the medium under consideration. Moreover, we denote by ℓ its distance from a source S, such that $\ell^2 = (x - \xi)^2 + z^2$. The propagation angle of a wave that propagates from $S = (\xi, 0)$ to $P = (x, z)$ thus satisfies

$$\cos\theta = \frac{z}{\ell} \qquad \text{and} \qquad \sin\theta = \frac{x - \xi}{\ell}. \tag{9}$$

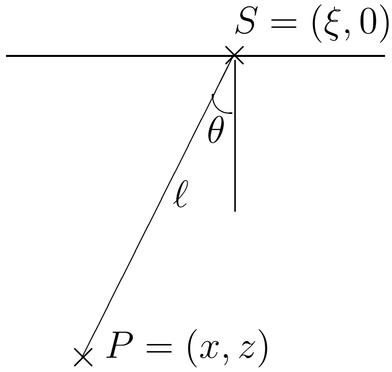

Figure 1: Geometry of the zero-offset ray connecting a source at $S = (\xi, 0)$ to a point $P = (x, z)$ on the seismic reflector image.

For the geometry, see again Figure 1.

Using equations (8) and (9) in expression (7), we find the following alternative representation for the modulus of the group velocity vector in explicit dependence of the coordinates of point P rather than the propagation angle θ,

$$v(x,z) = \ell \left[\frac{(x-\xi)^2}{A_{11}} + \frac{z^2}{A_{33}}\right]^{-1/2} = \ell v_v \left[\varphi(x-\xi)^2 + z^2\right]^{-1/2}. \tag{10}$$

Here, we have used the first of equations (8). Moreover, we have introduced the medium parameter

$$\varphi = \frac{A_{33}}{A_{11}} = \frac{v_v^2}{v_h^2}, \tag{11}$$

which we term the medium ellipticity.

Traveltime. With these results on the propagation velocity, we are now ready to describe the traveltime T of a wave that was emitted and registered at S and reflected at P. From formula (10) for the propagation velocity as a function of the coordinates of P, we obtain for the desired traveltime

$$T(\xi; x, z) = \frac{2\ell}{v(x,z)} = \frac{2}{v_v}\left[\varphi(x-\xi)^2 + z^2\right]^{1/2}. \tag{12}$$

The factor 2 is due to the fact that in equation (10), we have $v(\theta) = v(\theta + \pi)$. Therefore, the traveltime for the wave to arrive at the depth point P is the same as the time it takes from there back to the receiver at S.

Remigration

Seismic remigration tries to establish a relationship between to media of wave propagation in such a way that identical seismic surveys on their respective surfaces would yield the same seismic data. One of these media is the wrong velocity model used for the original migration. The other medium represents the updated model within which a new image of the subsurface needs to be constructed.

Let us suppose that the original migration has been realized with a model that is characterized by the same vertical velocity v_v as in the updated model, but a different ellipticity φ_0. In this old model, the same diffraction traveltime T of equation (12) is consumed by a different wave, reflected at a different point $P_0 = (x_0, z_0)$. It is therefore given by the modified equation

$$T(\xi; x_0, z_0) = \frac{2\ell_0}{v(x_0,z_0)} = \frac{2}{v_v}\left[\varphi_0(x_0-\xi)^2 + z_0^2\right]^{1/2}. \tag{13}$$

The top part of Figure 2 shows the diffraction traveltime described by equation (13) for a set of parameters $(x_0, z_0, \varphi_0, v_v)$ with realistic values.

We choose the convention of referring to the old model with ellipticity φ_0 as model M_0 and to the updated model with ellipticity φ as model M.

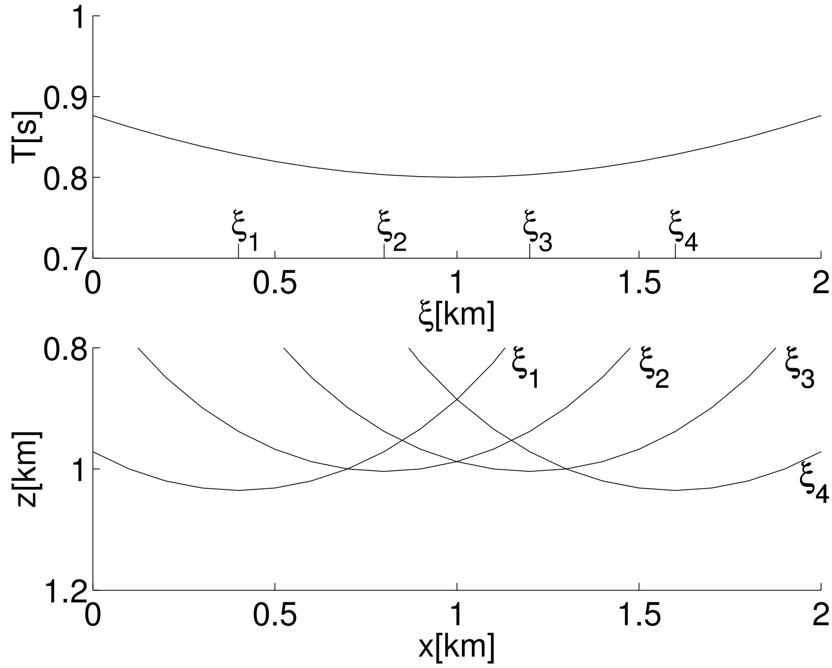

Figure 2: Top: Diffraction traveltime as described by equation (13) for a point P_0 with coordinates $x_0 =$ 1 km, $z_0 = 1$ km in model M_0 with ellipticity $\varphi_0 = 0.2$ and vertical velocity $v_v = 2.5$ km/s. Bottom: Family of isochrons for the same point P_0 as above, calculated by equation (14) in a model M with the same vertical velocity but a different ellipticity $\varphi = 0.8$, at the four points $\xi_1 = 0.4$ km, $\xi_2 = 0.8$ km, $\xi_3 = 1.2$ km, and $\xi_4 = 1.6$ km.

Huygens image wave. To derive the desired image-wave equation, we follow the lines of Hubral et al. (1996). Firstly, we need to find the set of all points $P = (x, z)$ in medium M for which the diffraction traveltime of equation (12) is equal to the diffraction traveltime (13) of point $P_0 = (x_0, z_0)$ in medium M_0. In other words, we are interested in localizing the so-called Huygens wave for this kind of image-wave propagation. This Huygens image wave then describes the position $z(x)$ of the image at the "instant" φ that "originated" at the "instant" φ_0 at point P_0. For this purpose, we equal the times T of equations (12) and (13), resulting in

$$F(x, z, \xi, \varphi) = \varphi(x - \xi)^2 + z^2 - \varphi_0(x_0 - \xi)^2 - z_0^2 = 0. \tag{14}$$

This equation represents a family of curves $z(x; \xi)$ that, for a fixed ξ, connect all points P in model M that possess the same diffraction traveltime $T(\xi; x, z)$ as P_0 in model M_0 for the same ξ. The bottom part of Figure 2 depicts four of these curves as obtained from equation (14) for four different values of ξ.

The set of points P such that $T(\xi; x, z)$ is equal to $T(\xi; x_0, z_0)$ for *all* values of x and z is given by the envelope of this family of curves described by $F(x, z, \xi, \varphi)$. This envelope is the mentioned Huygens image wave that represents the image in model M of point P_0 in model M_0. Application of the envelope condition

$$\frac{\partial F}{\partial \xi} = 0, \tag{15}$$

to equation (14) yields the desired curve. Taking the derivative of equation (14), we obtain

$$\varphi(x - \xi) - \varphi_0(x_0 - \xi) = 0 \quad : \quad (\varphi - \varphi_0)\xi = \varphi x - \varphi_0 x_0. \tag{16}$$

This expression can be solved for the stationary value of ξ, yielding

$$\xi = \frac{\varphi x - \varphi_0 x_0}{(\varphi - \varphi_0)} = \frac{\alpha x - x_0}{\alpha - 1} = \frac{x_0 - \alpha x}{1 - \alpha}, \tag{17}$$

where α denote the ratio between the medium ellipticities, i.e., $\alpha = \varphi/\varphi_0$.

Expression (14) can be recast into the moere convenient form

$$z^2 = \varphi_0(x_0-\xi)^2 + z_0^2 - \varphi(x-\xi)^2, \tag{18}$$

the terms of which can be expressed using equation (17) as

$$\begin{aligned}
(x_0-\xi)^2 = \left[x_0 - \frac{x_0-\alpha x}{1-\alpha}\right]^2 &= \frac{\alpha^2}{(1-\alpha)^2}(x-x_0)^2, \\
(x-\xi)^2 = \left[x - \frac{x_0-\alpha x}{1-\alpha}\right]^2 &= \frac{1}{(1-\alpha)^2}(x-x_0)^2.
\end{aligned} \tag{19}$$

Substitution of these two relationships back in equation (18) yields

$$\begin{aligned}
z^2 &= z_0^2 + \varphi_0\frac{\alpha^2}{(1-\alpha)^2}(x-x_0)^2 - \varphi\frac{1}{(1-\alpha)^2}(x-x_0)^2 \\
&= z_0^2 + \frac{\varphi\alpha}{(1-\alpha)^2}(x-x_0)^2 - \frac{\varphi}{(1-\alpha)^2}(x-x_0)^2 \\
&= z_0^2 - \frac{\varphi(x-x_0)^2}{1-\alpha},
\end{aligned} \tag{20}$$

which can be rewritten as

$$z = \sqrt{z_0^2 + \varphi\varphi_0\frac{(x-x_0)^2}{\varphi-\varphi_0}}. \tag{21}$$

Equation (21) describes the position of the Huygens image wave that was excited with the initial conditions $(x_0, z_0; \varphi_0)$. In the top part of Figure 3, this Huygens image wave described by equation (21) is added to the four isochrons of the bottom part of Figure 2. Figure 3 nicely demonstrates the characteristic property of the Huygens image wave (21), i.e., being the envelope of the set of isochrons described by equation (14). The bottom part of Figure 3 depicts a set of these Huygens image waves for different values of the medium ellipticity φ.

Eikonal equation. The Huygens image wave of equation (21) describes the variation of a single point P_0 on a reflector image under variation of the medium ellipticity φ, starting at an initial ellipticity φ_0. To transform this expression into one that describes the variation of any abritrarily shaped reflector image for arbitrary ellipticity variations, we need to eliminate these initial conditions from equation (21). In other words, need to replace the constants x_0, z_0, and φ_0 in equation (21) by derivatives, so as to describe image-wave propagation for any set of initial conditions.

For this purpose, we introduce the image-wave eikonal $\varphi = \Phi(x, z)$. An explicit expression for $\Phi(x, z)$ can be found by solving equation (21) for φ. By replacing φ by $\Phi(x, z)$ in equation (21) and taking the derivatives with respect to x and z of the resulting expression, we find a differential equation for Φ, the solution of which for initial conditions $(x_0, z_0; \varphi_0)$ is equation (21) solved for φ. This differential equation is the image-wave eikonal equation. It describes the kinematics of image-wave propagation for any arbitrary set of initial conditions, not only of that of a single initial point.

Taking the implicit derivative of equation (21) with respect to z, we find

$$\begin{aligned}
1 &= \frac{-1}{2z}\left[\Phi_0\Phi_z\frac{(x-x_0)^2}{\Phi-\Phi_0} - \Phi_0\Phi\frac{(x-x_0)^2}{(\Phi-\Phi_0)^2}\Phi_z\right] \\
&= \frac{-\Phi_z}{2z}\left[\frac{(\Phi-\Phi_0)\Phi_0}{(\Phi-\Phi_0)^2}(x-x_0)^2 - \Phi_0\Phi\frac{(x-x_0)^2}{(\Phi-\Phi_0)^2}\right] \\
&= \frac{\Phi_z}{2z}\frac{\Phi_0^2(x-x_0)^2}{(\Phi-\Phi_0)^2}.
\end{aligned} \tag{22}$$

This can be conveniently rewritten as

$$\frac{\Phi_0^2(x-x_0)^2}{(\Phi-\Phi_0)^2} = \frac{2z}{\Phi_z}. \tag{23}$$

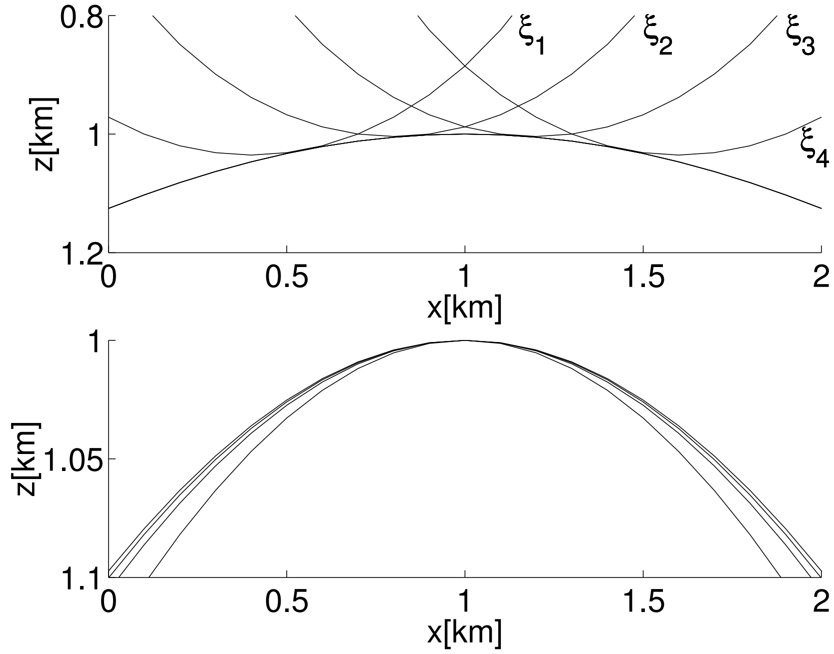

Figure 3: Top: The family of isochrons of Figure 2 with their envelope as determined by equation (21). This envelope is the desired position of the Huygens image wave. The medium parameters are the same as in Figure 2. Bottom: Different Huygens image waves for the same point P_0, indicating the propagation of the Huygens image wave. The curves are depicted for $\varphi = 0.8$, 2.0, 4.0, and 10.

Correspondingly, taking the derivative of equation (21) with respect to x, we arrive at

$$0 = \frac{-1}{2z}\left[-\Phi_x\frac{\Phi_0^2(x-x_0)^2}{(\Phi-\Phi_0)^2} + \Phi\Phi_0\frac{2(x-x_0)}{\Phi-\Phi_0}\right] = \frac{-1}{2z}\left[\frac{\Phi_x}{\Phi_z}2z + 2\Phi\frac{\Phi_0(x-x_0)}{\Phi-\Phi_0}\right], \tag{24}$$

which leads to

$$\frac{\Phi_x}{\Phi_z} = \frac{\Phi}{z}\left[\frac{\Phi_0(x-x_0)}{\Phi-\Phi_0}\right]. \tag{25}$$

Substitution of equation (23) in the squared equation (25) yields

$$\frac{\Phi_x^2}{\Phi_z^2} = \frac{\Phi^2}{z^2}\frac{2z}{\Phi_z} \quad : \quad \frac{\Phi_x^2}{\Phi_z^2} = \frac{2\Phi^2}{z\Phi_z}. \tag{26}$$

By simplifying the last expression, we finally arrive at the image-wave eikonal equation

$$\Phi_x^2 - \frac{2\Phi^2}{z}\Phi_z = 0, \tag{27}$$

which describes the kinematics of the propagation of a reflector image as a function of the medium ellipticity.

Image-wave equation. Now we want to find a partial differential equation such that equation (27) is its associated eikonal equation. In other words, upon substitution of the ray-theory ansatz

$$p(x, z, \varphi) = p_0(x, z)f[\varphi - \Phi(x, z)] \tag{28}$$

into our desired differential equation, the leading-order terms need to provide equation (27).

From the fact that equation (27) contains the square of Φ_x, we recognize that our desired partial differential equation must be of second order. The second derivative of the ansatz (28) with respect to x is given by

$$\begin{aligned} p_{xx} &= \frac{\partial^2 p_0}{\partial x^2}(x,z) f(\varphi - \Phi(x,z)) - 2\frac{\partial p_0}{\partial x}(x,z) f'(\varphi - \Phi(x,z)) \frac{\partial \Phi}{\partial x}(x,z) \\ &+ p_0(x,z) f''(\varphi - \Phi(x,z)) \left(\frac{\partial \Phi}{\partial x}(x,z)\right)^2 - p_0(x,z) f'(\varphi - \Phi(x,z)) \frac{\partial^2 \Phi}{\partial x^2}(x,z). \end{aligned} \tag{29}$$

Correspondingly, the mixed derivative of ansatz (28) with respect to z and φ is

$$p_{z\varphi} = \frac{\partial p_0}{\partial z}(x,z) f'(\varphi - \Phi(x,z)) - p_0(x,z) f''(\varphi - \Phi(x,z)) \frac{\partial \Phi}{\partial z}(x,z). \tag{30}$$

Combining the leading-order terms, i.e., those involving f'', of these two expressions we recognize that the second-order partial differential equation

$$p_{xx} + \frac{2\varphi^2}{z} p_{z\varphi} = 0 \tag{31}$$

fulfils the condition that its associated eikonal equation is given by equation (27).

Note that equation (31) is the simplest one with the desired property. Any additional terms involving arbitrary combinations of the first derivatives of p with respect to x, z, or φ, and/or involving p or the independent variables x, z, or φ themselves, do not alter the associated eikonal equation and therefore, neither the kinematic behaviour of the solution of equation (31). Since at this moment, we are interested only in this kinematic behaviour of the image-wave propagation, we can choose this simplest form. We refer to equation (31) as the image-wave equation for remigration in elliptically isotropic media.

It is important to observe that the image-wave equation (31) can be transformed into a partial differential equation with constant coefficients. Upon the introduction of the new variables $\gamma = 1/\varphi$, and $\zeta = z^2/4$, the mixed derivative becomes

$$p_{z\varphi} = p_{\zeta\gamma}\, \zeta_z\, \gamma_\varphi = p_{\zeta\gamma}\, \frac{z}{2}\, \frac{-1}{\varphi^2}. = -\frac{z}{2\varphi^2}\, p_{\zeta\gamma}. \tag{32}$$

Under this variable transformation, the image-wave equation (31) thus takes the form

$$p_{xx} - p_{\gamma\zeta} \ . \tag{33}$$

The transformation into equation (33 is meaningful from an implementational point of view, since for differential equations with constant coefficients, it is generally much easier to find stable FD implementations.

As a final word on the image-wave equation (31) or its constant-coefficient version (33), let us mention that both equations do not depend on the vertical velocity v_v but only on the medium ellipticity φ. Thus, it can be expected that depth image-wave remigration in elliptically isotropic media should be relatively insensitive to the actual value of the vertical velocity. This, in turn, points towards a potentially broad applicability of the image-wave concept for elliptically isotropic remigration even in inhomogeneous media.

CONCLUSIONS

The changing position of a seismic reflector image under variation of the migration velocity model can be understood in an analogous way to the propagation of a physical wave (Fomel, 1994; Hubral et al., 1996).

In this work, we have derived a second-order partial differential equation that works as an image-wave equation for remigration in elliptically isotropic media. It describes this reflector propagation as a function of the medium ellipticity. We have studied the kinematics of the image wave in such media to derive the corresponding eikonal equation. From an inverted ray procedure, we have then inferred the desired image-wave equation the solutions of which exhibit this correct kinematic behaviour.

The description of the position of the reflector image as a function of the medium ellipticity can be very useful for the detection of this parameter. A set of migrated images for different medium ellipticities can be obtained from a single migrated image without the need for multiple anisotropic migrations. From additional information on the correct reflector position, focusing analysis, or the like, the best fitting value of the medium ellipticity can then be determined.

The probably most interesting application of this procedure would start with an initial condition of an isotropic medium, described by unit ellipticity, i.e., $\varphi_0 = 1$. Since isotropic migration is a very well understood field, the image-wave equation could then be used to transform an isotropically migrated image, which can be obtained with one of the highly sophisticated migration methods that are nowadays available, into an image that corresponds to an elliptically isotropic medium. Future investigations will have to show whether the concept can be extended to transversely isotropic media which are generally better suited to describe the kind of anisotropy that is observed in practice.

ACKNOWLEDGMENTS

This work was kindly supported by the Brazilian research agencies CNPq, CAPES, and FAPESP, as well as the sponsors of the *Wave Inversion Technology (WIT) Consortium*.

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