Estimation of the statistical parameters from the traveltime fluctuations of refracted waves

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ABSTRACT

Traveltime fluctuations of refracted waves in random elastic media are studied in the framework of geometrical optics approach. Covariation function of traveltime fluctuations is derived for the case of a constant gradient of the average wave velocity. The random inhomogeneities are supposed to be strongly anisotropic (anisomeric): for example vertical correlation scale, l_z can be much smaller than horizontal, l_x, one. In this case, a surprising phenomenon has been found: unlike expectations traveltime variance starts decreasing at sufficiently large offsets for the case when velocity fluctuations are proportional to the average velocity. This new phenomenon is proved by the results of numerical modelling for traveltime fluctuations in random media. Theoretical consideration is in agreement with the results of numerical modelling. Numerical modelling demonstrates also an opportunity for extracting statistical parameters of random media from empirical data on traveltime fluctuations.

INTRODUCTION

The problem of wave propagation through randomly inhomogeneous media was very popular in 60th - 70th of last century mainly because of its wide applications in optics, radar and underwater acoustics (Chernov, 1960; Tatarskii, 1961, 1967 and 1971). Studies of statistical characteristics of elastic waves in rocks attracted somewhat less attention, probably, because destructive role of random inhomogeneities in seismic problems was not so evident as compared to other natural media. In the present time, statistical characteristics of elastic media are of much more interest as compared to period of 20-30 years before, which is motivated by several reasons.

Firstly, information on statistical properties of inhomogeneities in the elastic medium are neccessary for estimating uncertainities of seismic images. This is especially important for inhomogeneities of size on a limit of the seismic resolution. Secondly, small random inhomogeneities affect seismic amplitudes. This effect must be understood and described in order to allow interpretation of seismic attenuation. The amplitude effects of random inhomogeneities can be compensated, if their statistics is known. Thirdly, statistical properties of heterogeneities can be used in the seismic inversion combined with geostatistical approaches. Similar approach is applied quite often in the characterization of hydrocarbon reservoirs. Finally, statistics of heterogeneities might serve as a new seismic attribute useful for making a bridge between seismic and lithological rocks description.

Significant progress in solution of statistical inverse problems in seismics was achieved due to efforts of Touati (1996), Iooss (1998), Iooss et al. (2000), Gaerets et al. (2001), who suggested to extract statistical parameters of elastic media from traveltime fluctuations of signal, reflected from sufficiently contrast interface. Such an observation scheme is known as reflection seismics. In the framework of above mentioned approach the rays twice cross random inhomogeneities, so that elastic wave experiences double passage effect, which manifest itself also in other geophysical situations: under radio wave reflection from randomly inhomogeneous ionosphere (Denisov and Erukhimov 1962; Kravtsov 1965) and under light reflection from a mirror in a turbulent atmosphere (Kravtsov and Saichev 1982 and 1985). Detailed analysis of traveltime statistics of reflected seismic waves was performed recently by Kravtsov et al. (2003).

In this publication we intend to study the potential of another observational scheme - refraction geometry, which implies traveltime fluctuation measurement for signals which, return to the earth surface due to refraction. The main instrument for data interpretation both in reflection and in refraction seismics is geometrical optics (GO) method. The necessary information on the GO method is outlined and the general properties of covariation function for traveltime fluctuations along curved rays are presented. The variance for traveltime fluctuations is derived in a plane stratified elastic media with a constant gradient of wave velocity. This variance is shown to reduce at large offsets, what is an important new result of the study. Numerical simulations are performed in order to verify the theoretical consideration. Theoretical results are shown to be in good agreement with the results of numerical simulations.

GEOMETRICAL OPTICS RELATIONS FOR TRAVELTIME

A broad experience shows that the geometrical optics (GO) is a suitable approximation for computation of seismic traveltimes. In the framework of GO approximation properties of inhomogeneous elastic medium are characterized by the wave velocity $v(\boldsymbol{r})$ or by slowness $\mu(\boldsymbol{r}) = 1/v(\boldsymbol{r})$. For our purposes it is convenient to deal with refractive index

$$n(\mathbf{r}) = \frac{v_0}{v(\mathbf{r})} = v_0\mu(\mathbf{r}), \tag{1}$$

where v_0 is a typical velocity in a given area, say, a velocity near the earth surface.

For wave, propagating in time-independent (stationary) and dispersionless media GO provides preservation of pulse shape on the whole pulse path through inhomogeneous medium. According to Born and Wolf (1999), (see also Kravtsov and Orlov 1990; Červený 2001) the lowest (zeroth) approximation of GO suggests the following expression for the wave field:

$$u(\mathbf{r}, t) = A(\mathbf{r}) f\left(t - \frac{\psi(\mathbf{r})}{v_0}\right). \tag{2}$$

Here "optical path" or "eikonal" $\psi(\boldsymbol{r})$ obeys the eikonal equation

$$(\nabla\psi)^2 = n^2(\mathbf{r}), \tag{3}$$

whereas the amplitude $A(\boldsymbol{r})$ can be found from the energy flow conservation law in a ray tube.

According to eq. (2), the signal, received in the point of observation $\boldsymbol{r}$ reaches it maximum value at the time

$$t|_{|f(t)|=max} \equiv t = \frac{\psi(\mathbf{r})}{v_0}, \tag{4}$$

which corresponds to condition, that the argument $t - \frac{\psi}{v_0}$ of propagating pulse $f\left(t - \frac{\psi(\boldsymbol{r})}{v_0}\right)$ is zero. Relation (4) is basical for determination of time-delay between emitted and received pulses in the framework of GO method.

The solution of eikonal equation (3) for "optical path" $\psi(\boldsymbol{r})$ can be presented in one of two equivalent forms (Kravtsov and Orlov 1990; Červený 2001):

$$\psi(\mathbf{r}) = \int n[\mathbf{r}(s)]ds \tag{5}$$

and

$$\psi(\mathbf{r}) = \int n^2[\mathbf{r}(\tau)]d\tau. \tag{6}$$

where $r = r(s)$ is the ray trajectory, s is the arclength and τ is an alternative papametere along ray which is connected with the arclength s by a relation $ds = n d\tau$.

According to formula (4), traveltime is proportional to the optical path $\psi(\boldsymbol{r})$:

$$t = \frac{\psi(\mathbf{r})}{v_0} = \frac{1}{v_0}\int n[\mathbf{r}(s)]ds = \frac{1}{v_0}\int n^2[\mathbf{r}(\tau)]d\tau. \tag{7}$$

Therefore all the results, obtained earlier for eikonal variations (Chernov 1960; Tatarskii 1961, 1967, and 1971; see also Ishimaru 1978, 1997; Rytov et al. 1989b), can be equally used for analysis of traveltime fluctuations, which are of significant interest in seismics.

FIRST ORDER TRAVELTIME VARIATIONS

In a smoothly inhomogeneous random elastic medium refractive index n can be presented as sums of average (regular), $\overline{n}(\boldsymbol{r})$, and random, $\tilde{n}(\boldsymbol{r})$, parts as follows:

$$n(r) = \overline{n}(\mathbf{r}) + \tilde{n}(\mathbf{r}). \tag{8}$$

Smallness of random medium parameters fluctuations allows to restrict analysis by consideration of only first order perturbations. In the framework of the first order perturbation theory the value $\tilde{v} = v - \overline{v}$ is connected with $\tilde{n}$ by linear relations

$$\tilde{v} = -\frac{\overline{v}^2 \tilde{n}}{v_0}. \tag{9}$$

All other values of interest also can be presented in the form like (8):

$$\mathbf{r} = \overline{\mathbf{r}} + \tilde{\mathbf{r}}, \qquad \psi = \overline{\psi} + \tilde{\psi}, \qquad t = \overline{t} + \tilde{t}. \tag{10}$$

Substitution of $n = \overline{n} + \tilde{n}$ and $\psi = \overline{\psi} + \tilde{\psi}$ into eikonal eq. (3) brings about the zeroth approximation equation

$$(\nabla\overline{\psi})^2 = (\overline{n})^2, \tag{11}$$

which solution can be written as integral

$$\overline{\psi} = \int (\overline{n})^2 d\overline{\tau} = \int \overline{n} d\overline{s}, \tag{12}$$

along unperturbed ray $\overline{\boldsymbol{r}} = \overline{\boldsymbol{r}}(\overline{\tau})$ or $\overline{\boldsymbol{r}} = \overline{\boldsymbol{r}}(\overline{s})$. The first order eikonal equation

$$\nabla\overline{\psi} \cdot \nabla\tilde{\psi} = \overline{n}\tilde{n}, \tag{13}$$

lead to well known expression (Chernov 1960; Tatarskii 1961, 1967, 1971; Ishimaru, 1978, 1997; Rytov et al. 1989b; Kravtsov and Orlov 1990)

$$\tilde{\psi} = \int \tilde{n} d\overline{s}, \tag{14}$$

which implies integration of refraction index fluctuations along the unperturbed ray. Correspondingly the first order theory of perturbation for traveltime fluctuations $\tilde{t}$ gives

$$\tilde{t} = \frac{\tilde{\psi}}{v_0} = \frac{1}{v_0} \int \tilde{n} d\overline{s}, \tag{15}$$

which also deals with integration of fluctuations $\tilde{n}$ over unperturbed ray. A detailed analysis of the ray perturbation theory series was performed also by Snieder and Sambridge (1992) and Witte et al. (1996). Relation (15) arises in their analysis as the first order term. For brevity we shall omit below the upper bar over regular ray trajectory $\overline{\boldsymbol{r}}$ and ray parameters $\overline{\tau}$ and $\overline{s}$.

TRAVELTIME COVARIANCE FUNCTION IN A MEDIUM WITH QUASI-HOMOGENEOUS STATISTICS

The covariance of traveltime registered by two receivers, placed at points $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ on the Earth surface, as shown in Figure 1, has a form of averaged product of fluctuations $\tilde{t}(\boldsymbol{R}_1)$ and $\tilde{t}(\boldsymbol{R}_2)$, given by eq. (15):

$$C_t(\mathbf{R}_1, \mathbf{R}_2) = \langle \tilde{t}(\mathbf{R}_1)\tilde{t}(\mathbf{R}_2)\rangle = \frac{1}{v_0^2} \int_0^{S(\mathbf{R}_1)} ds_1 \int_0^{S(\mathbf{R}_2)} ds_2 C_n[\mathbf{r}_1(s_1), \mathbf{r}_2(s_2)]. \tag{16}$$

Here $S_1(\boldsymbol{R}_1)$ and $S_2(\boldsymbol{R}_2)$ are arclengths of the rays, arriving correspondingly to the receivers $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$.

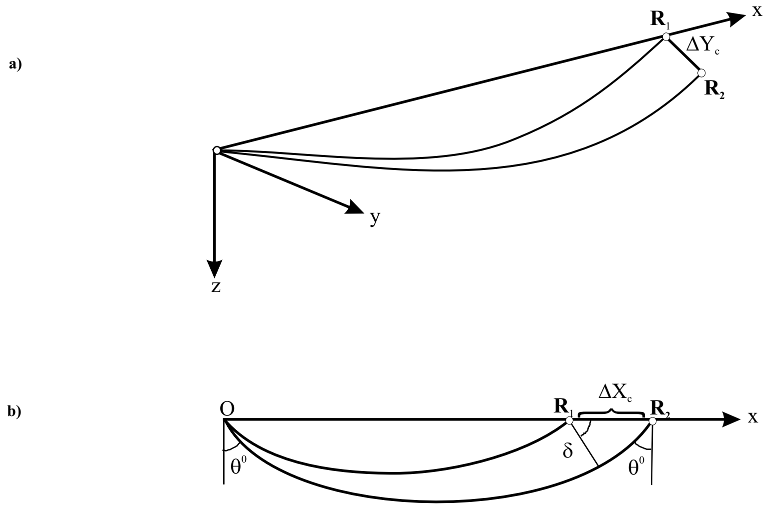

Figure 1: Transverse (a) and longitudinal (b) displacement of observation points $\mathbf{R}_1$ and $\mathbf{R}_2$.

For a covariance function

$$C_n(\mathbf{r}_1, \mathbf{r}_2) = \langle \tilde{n}(\mathbf{r}_1)\tilde{n}(\mathbf{r}_2)\rangle \tag{17}$$

of refractive index fluctuations we make use of the model of quasi-homogeneous fluctuations, (QHF), (Rytov et al. 1989a; 1989b):

$$C_n(\mathbf{r}_1, \mathbf{r}_2) = \sigma_n^2(\mathbf{r}_+)K_n(\mathbf{r}_1 - \mathbf{r}_2;\ \mathbf{r}_+). \tag{18}$$

Here $\boldsymbol{r}_+ = (\boldsymbol{r}_1 + \boldsymbol{r}_2)/2$ is the radius vector of center of gravity of vectors $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, and K_n is a normalized correlation function (correlation coefficient) which equals to unit at $\boldsymbol{r}_1 - \boldsymbol{r}_2 = 0$,

$$K_n(0; \mathbf{r}_+) = 1,$$

and is supposed to be reduced significantly, when distance $\mid \boldsymbol{r}_1 - \boldsymbol{r}_2 \mid$ exceeds a characteristic (correlation) length l_c.

Model of QHF allows to describe anisomeric (statistically anisotropic) fluctuations with different correlation lengths $l_x,\ l_y$ and l_z in x, y and z directions, respectively. By introducing new variables into equation (16):

$$s_- = s_1 - s_2, \qquad s_+ = \frac{s_1 + s_2}{2} \tag{19}$$

and expand trajectores $\boldsymbol{r}_1(s_1)$ and $\boldsymbol{r}_2(s_2)$ into power series in difference variable s_-, saving only the zeroth and first order terms in a difference $\boldsymbol{r}(s_1; \boldsymbol{R}_1) - \boldsymbol{r}(s_2; \boldsymbol{R}_2)$ and only the zeroth-order term in $\boldsymbol{r}_+ = (\boldsymbol{r}_1 + \boldsymbol{r}_2)/2 = \boldsymbol{r}_+(s_+)$ the difference $\boldsymbol{r}_1 - \boldsymbol{r}_2$ in (18) becomes

$$\mathbf{r}_1(s_1) - \mathbf{r}_2(s_2) \cong \boldsymbol{\kappa}(s_+)s_- + \boldsymbol{\delta}(s_+), \tag{20}$$

where $\boldsymbol{\kappa}(s_+) = \frac{d\boldsymbol{r}_+}{ds}$ is a unit vector, tangent to the "middle" ray $\boldsymbol{r} = \boldsymbol{r}(s_+)$ (Figure 2) and $\boldsymbol{\delta}(s_+)$ is a vector, connecting the nearest points at the neighbouring rays. Taking into account that $ds_1 ds_2 = ds_- ds_+$, one can transform eq. (16) as follows

$$C_t(\mathbf{R}_1, \mathbf{R}_2) = \frac{2}{v_0^2}\int_0^{S_<} \sigma_n^2[\mathbf{r}_+(s_+)]ds_+ \int_0^\infty ds_- K_n[\boldsymbol{\kappa}(s_+)s_- + \boldsymbol{\delta}(s_+)]. \tag{21}$$

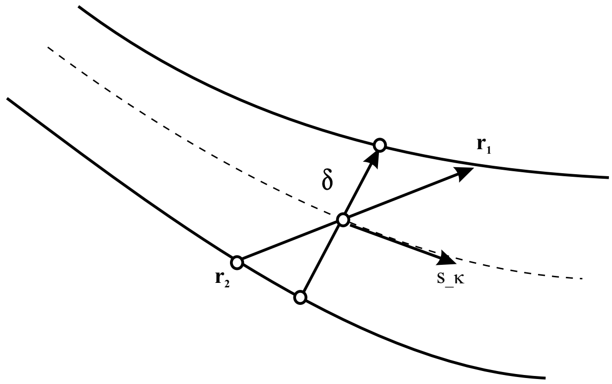

Figure 2: Expansion of difference $\mathbf{r}_1 - \mathbf{r}_2$ into sum of transverse $\boldsymbol{\delta}$ and longitudinal $s_{-}\kappa$ components (illustration to derivation of (20)).

Taking in eq. (20) $\boldsymbol{R}_1 = \boldsymbol{R}_2 = \boldsymbol{R}$, what brings about $\boldsymbol{\delta} = 0$, one can obtain a traveltime variance:

$$\sigma_{\tilde{t}}^2 = Var[\tilde{t}(\tilde{\mathbf{R}})] = \frac{2}{v_0^2} \int_0^{S(\mathbf{R})} ds_+ \sigma_n^2[\mathbf{r}_+(s_+)] l_{eff}(\boldsymbol{\kappa}; s_+), \tag{22}$$

where $l_{eff}(\boldsymbol{\kappa}; s_+)$ is an effective correlation length in the point $\boldsymbol{r}_+(s_+)$ along the ray direction $\boldsymbol{\kappa}(s_+) = \frac{d\boldsymbol{r}_+}{ds}$:

$$l_{eff}(\boldsymbol{\kappa}; s_+) = \int_0^\infty K_n(\boldsymbol{\kappa} s_-) ds_-. \tag{23}$$

For an anisomeric normalized correlation function $K_n(\triangle \boldsymbol{r})$ is given as

$$K_n(\triangle \mathbf{r}) = \beta[g(\Delta \mathbf{r})], \qquad \Delta \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \tag{24}$$

where (Iooss, 1998)

$$g(\Delta \mathbf{r}) = \left[\frac{(\Delta x)^2}{l_x^2} + \frac{(\Delta y)^2}{l_y^2} + \frac{(\Delta z)^2}{l_z^2} \right]^{1/2}. \tag{25}$$

We suppose for definiteness that the main axes of correlation function (24) are oriented along $x,\ y$ and z coordinate axis, and that $l_x \geq l_y \geq l_z$. Let $\Delta \boldsymbol{r} = \boldsymbol{\kappa} s_-$ and unit vector $\boldsymbol{\kappa}$ has components $\sin\theta\cos\phi$, $\sin\theta\sin\phi$, $\cos\theta$, θ being polar and ϕ azimuthal angles. Then

$$l_{eff} = \int_0^\infty K_n(\Delta \mathbf{r}) ds_- = \int_0^\infty \beta[s_-/l_\kappa(\theta,\phi)] ds_- = l_\kappa(\theta,\phi) \int \beta(g) dg = \Gamma l_\kappa(\theta,\phi), \tag{26}$$

where

$$l_\kappa(\theta,\phi) = \left(\frac{\kappa_x^2}{l_x^2} + \frac{\kappa_y^2}{l_y^2} + \frac{\kappa_z^2}{l_z^2} \right)^{-1/2} = \left[\frac{(\sin\theta\cos\phi)^2}{l_x^2} + \frac{(\sin\theta\sin\phi)^2}{l_y^2} + \frac{\cos^2\theta}{l_z^2} \right]^{-1/2} \tag{27}$$

is a characteristic scale of function $g(\Delta \boldsymbol{r})$ in the ray direction $\boldsymbol{\kappa}$ and Γ is a formfactor, which depends only on the profile of function $\beta(g)$:

$$\Gamma = \int_0^\infty \beta(g) dg. \tag{28}$$

For instance, the value Γ for Gaussian coefficient of correlation $\beta(g) = \exp(-g^2)$ is equal

$$\Gamma = \int_0^\infty \exp(-g^2) dg = \frac{\sqrt{\pi}}{2}, \tag{29}$$

whereas exponential correlation coefficient $\beta(g) = \exp(-g)$ gives

$$\Gamma = 1. \tag{30}$$

In the case of horizontaly isotropic (isomeric) fluctuations, for which $l_x = l_y = l_h$, the value l_{eff} and l_κ do not depend on azymuthal angle ϕ:

$$l_{eff} = \Gamma l_\kappa = \Gamma \left[\frac{\sin^2\theta}{l_h^2} + \frac{\cos^2\theta}{l_z^2} \right]^{-1/2} . \tag{31}$$

TRAVELTIME VARIANCES IN A LAYERED MEDIUM

We consider a regular ray trajectory in a plane layered medium with refractive index n and a constant velocity gradient depending on vertical coordinate (depth) z. The depth dependent velocity is represented by

$$v(z) = v_0(1 + z/H) = v_0 + kz, \tag{32}$$

where depth $z = H$ corresponds to doubling of velocity value, $k = v_0/H$ is a velocity gradient. In this case refraction index $n(z)$ takes a form

$$n(z) = \frac{v_0}{v(z)} = \frac{1}{1 + z/H}, \tag{33}$$

and the ray trajectory which has a depth dependence on x is given as

$$z = \sqrt{H^2 + xX - x^2} - H, \ z > 0, \tag{34}$$

where X is a final point of the ray. The regular (mean) traveltime $\bar{t}$ along the ray trajectory can be calculated as

$$\bar{t} = \frac{2H}{v_0} \ln \left[\frac{X}{2H} + \sqrt{1 + \frac{X^2}{4H^2}} \right] . \tag{35}$$

We suppose that velocity fluctuations $\tilde{v}$ are proportional to average velocity $\overline{v}(z)$:

$$\tilde{v} = \overline{v}\xi \tag{36}$$

where ξ is statistically homogeneous random field with variance σ_ξ^2 and a given normalized correlation function $K_\xi(\mathbf{r}_1 - \mathbf{r}_2)$:

$$< \xi(\mathbf{r}_1)\xi(\mathbf{r}_2) >= \sigma_\xi^2 K_\xi(\mathbf{r}_1 - \mathbf{r}_2). \tag{37}$$

According to (9), in the framework of the model (36) one has

$$\tilde{n} = -\frac{v_0\xi}{\overline{v}} = -\overline{n}\xi, \tag{38}$$

and

$$\sigma_n^2 = (\overline{n})^2\sigma_\xi^2, \qquad \sigma_v^2 = (\overline{v})^2\sigma_\xi^2 . \tag{39}$$

As a result eq.(22) converts into

$$\sigma_t^2 = \frac{2\sigma_\xi^2}{v_0^2} \int_0^S \overline{n}^2 ds_+ l_{eff} . \tag{40}$$

This integral can be transformed to the form, convenient for numerical analysis. For this, we shall use offset x as a variable along the ray:

$$ds_+ = \frac{dx}{\sin\theta} . \tag{41}$$

Refractive index $\overline{n}$ for a medium with constant velocity gradient can be written, according to eq. (33), as

$$\overline{n}\left[z(x)\right] = \frac{1}{1 + \frac{z}{H}} = \frac{1}{\sqrt{1 + \frac{\alpha(x)}{H^2}}} = \frac{1}{\sqrt{1 + \nu(\gamma - \nu)}} , \tag{42}$$

where $\nu = x/H$, $\gamma = X/H$. Thirdly, involving the ratio $\rho = l_z/l_x$ one can rewrite effective correlation length (31) as

$$l_{eff} = \Gamma l_z \left[\rho^2 \sin^2(\theta) + \cos^2(\theta)\right]^{-1/2}, \qquad \rho = l_z/l_x. \tag{43}$$

Considering the ray going along the x axis and using the relation

$$\frac{dz}{dx} = \frac{\pm\sqrt{n^2(z) - \sin^2\theta^0}}{\sin\theta^0} = \cot\theta, \tag{44}$$

where θ^0 is the polar angle of incidence counted from the vertical axis z one can readily show that

$$\sin\theta = \sqrt{B^2 - \eta^2}/B, \quad \cos\theta = -\eta/B, \quad \overline{n} = (B^2 - \eta^2)^{1/2}, \tag{45}$$

where

$$\eta = \nu - \frac{\gamma}{2} = \frac{x - X/2}{H}, \quad B = \sqrt{1 + \left(\frac{\gamma}{2}\right)^2}. \tag{46}$$

Substituting eqs. (41)-(46) into eq. (40), and using variable η instead of x, we have

$$\sigma_t^2 = DJ(\rho, \gamma), \tag{47}$$

where

$$D = \frac{\sqrt{\pi} l_z \sigma_\xi^2 H}{v_0^2}, \tag{48}$$

and

$$J(\rho, \gamma) = B^2 \int_{-\gamma/2}^{\gamma/2} \frac{d\eta}{(B^2 - \eta^2)^{3/2}\sqrt{\rho^2(B^2 - \eta^2) + \eta^2}}. \tag{49}$$

Integral (49), calculated numerically for $\rho = 0.1$ is presented on Figure 3 by a continious line. At small distances, $\gamma = X/H$, the integral J rapidly increases as

$$J(\rho, \gamma) \approx \frac{\gamma}{\rho},$$

reaching the value $J \approx 2$ at $\gamma \approx 2\rho$. Then J approaches the maximum value $J_{max} = 4.57$ at $\gamma = 1.9$ and then slowly reduces, tending to asymptotic value $J_\infty = 2$ when $\gamma \to \infty$. Reducing J at large distances looks unusual: this phenomenon was never met before neither in acoustics and optics, nor in radio wave propagation. Traveltime variance reducing with a distance results due to fast "diving" of the ray into deeper layers where refractive index fluctuates weaker and due to strong anisotropy (anisomery) of random inhomogeneities: at $\rho = l_z/l_x = 1$ (isomeric inhomogeneities) and $\rho = l_z/l_x = 10$ (anisomeric inhomogeneties, $l_z \gg l_x$) reducing of J does not occure (Figure 3, dashed and dotted lines, respectively).

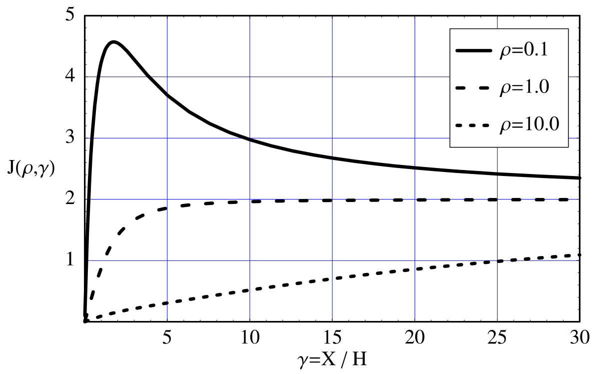

Figure 3: Dependence of factor $J(\rho, \gamma)$ for determining traveltime variance σ_t^2 on dimensionless distance $\gamma = X/H$. The curves corresponds to the model $\sigma_v^2 = \overline{v}^2\sigma_\xi^2$: for $\rho = 0.1$ (continious curve), $\rho = 1.0$ (dashed curve) and $\rho = 10.0$ (dotted curve).

TRAVELTIME TRANSVERSE AND LONGITUDINAL CORRELATION SCALES IN A LAYERED MEDIUM

Involving variable $dx = \sin\theta ds$ like in eq. (40), one can rewrite the basic formula (22) for a layered medium in a form:

$$C_t(\mathbf{R_1}, \mathbf{R_2}) = \frac{2}{v_0^2}\int_0^{X_<} \frac{dx_+}{\sin\theta^0}\sigma_n^2[\mathbf{r}_+]\int_0^\infty ds_- K_n(\kappa s_- + \delta). \tag{50}$$

This formula allows to estimate both transverse and longitudinal correlation scales.

Let the point $\boldsymbol{R}_2 = (X, \Delta Y, 0)$ is shifted in y direction relatively the point $\boldsymbol{R}_1 = (X, 0, 0)$ (Figure 1a). In this case $\delta_z = \delta_x = 0$, whereas current distance δ_y between the rays, arriving to points $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$, is proportional to x:

$$\delta_y = \frac{x}{X}\Delta Y. \tag{51}$$

The argument g of anisomeric correlation function (24) in this case is equal to

$$g = \left(\frac{\kappa_x^2}{l_x^2} + \frac{\kappa_z^2}{l_z^2}\right)s_-^2 + \frac{(\Delta Y)^2}{l_y^2}\left(\frac{x}{X}\right)^2. \tag{52}$$

One can readily conclude from eqs. (50) and (52) that transverse correlation scale ΔY_c is comparable with l_y or, if $l_x = l_y = l_h$, with horizontal correalation scale l_h. Similar properties are characteristic for spherical and cylindirical waves in statistically homogeneous random media (Rytov et al., 1989b).

Estimates for longitudinal correlation scale ΔX_c happen to be somewhat troublesome. Let the points of observation $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are seperated by the interval ΔX in x direction (Figure 1b):

$$\mathbf{R}_1 = (X, 0, 0), \qquad \mathbf{R}_2 = (X + \Delta X, 0, 0). \tag{53}$$

Under these conditions distance between the rays can be calculated from the relation

$$\delta \equiv \left(\delta_z^2 + \delta_x^2\right)^{1/2} = |\Delta Z|\sin\theta, \tag{54}$$

where $|\Delta Z|$ is a current vertical distance between the rays, arriving to the points X and $X + \Delta X$:

$$\Delta Z(x) = \sqrt{H^2 + x(X + \Delta X) - x^2} - \sqrt{H^2 + xX - x^2}. \tag{55}$$

Considering $x + \Delta X$ to be small as compared to H^2, one can expand eq. (55) into Taylor series in ΔX. Restricting ourselves by linear term in ΔX and taking into account that $\sin\theta = \frac{\sqrt{H^2+xX-x^2}}{\sqrt{H^2+(X/2)^2}}$ one has

$$\delta(x) = \frac{x\Delta X}{2\sqrt{H^2 + (X/2)^2}}. \tag{56}$$

Taking into account that $\boldsymbol{\kappa} = (\sin\theta, \cos\theta)$ and $\boldsymbol{\delta} = (\delta\cos\theta, -\delta\sin\theta)$ one can present the argument g, eq.(25) of correlation function (24) as

$$g = \left[\frac{(s_-\sin\theta + \delta\cos\theta)^2}{l_x^2} + \frac{(s_-\cos\theta - \delta\sin\theta)^2}{l_z^2}\right]^{1/2}. \tag{57}$$

Dealing with Gaussian correlation function $K_n(g) = exp(-g^2)$ one can show that internal integral in eq. (50) equals

$$\int K_n(\kappa s_- + \delta)ds_- = l_{eff}\,exp\left(-\frac{\delta^2}{l_\delta^2}\right). \tag{58}$$

where effective correlation length l_{eff} is given by eq.(31) with $\Gamma = \sqrt{\pi}/2$ and characteristic transverse length l_δ is

$$l_\delta = \left[l_z^2\sin^2\theta + l_x^2\cos^2\theta\right]^{1/2} = l_z\left[\sin^2\theta + \frac{1}{\rho^2}\cos^2\theta\right]^{1/2}. \tag{59}$$

As a result longitudinal covariance function takes a form

$$C_t(X, X+\Delta X) = \frac{2\Gamma\sigma_\xi^2 l_z}{v_0^2}\int_0^{X_<}\frac{\overline{n}^2 dx}{\sqrt{\rho^2\sin^2\theta + \cos^2\theta}}exp\left(-\frac{(x\Delta X)^2}{(4H^2+X^2)l_\delta^2}\right). \tag{60}$$

In dimensionless variables η and γ, used in eq. (46), longitudinal covariance (60) takes a form

$$C_t(X, X+\Delta X) = DP(\rho,\gamma,\Lambda), \tag{61}$$

where

$$P(\rho,\gamma,\Lambda) = \int_{-\gamma/2}^{\gamma/2}\frac{Bd\eta exp(-Q)}{(B^2-\eta^2)^{3/2}\sqrt{\rho^2(B^2-\eta^2)+\eta^2}}, \tag{62}$$

where factor D is given by (48),

$$Q = \frac{\left(\eta+\frac{\gamma}{2}\right)^2\Lambda^2}{B^2\left[\rho^2(B^2-\eta^2)+\eta^2\right]}, \tag{63}$$

and

$$\Lambda = \frac{\Delta X}{l_x}. \tag{64}$$

Note that at $\Lambda = 0$ integral (62) turns out to be equal J:

$$P(\rho,\gamma,0) = J(\rho,\gamma). \tag{65}$$

Dimensionless longitudinal correlation radius $\Lambda_c = \Delta X_c/l_x$ can be determined then at given values ρ and γ, from the equation:

$$P(\rho,\gamma,\Lambda_c) = \frac{1}{2}P(\rho,\gamma,0) = \frac{1}{2}J(\rho,\gamma). \tag{66}$$

Numerical solution of this equation for $\rho = 0.1$ is presented in Figure 4. According to this plot, longitudinal correlation radius Λ_c exceeds a unit, $\Lambda > 1$ ($\Delta X_c > l_x$) at small distances $\gamma < 0.5$, takes a minimum value $\Lambda_{min} = 0.53$ at $\gamma = 2.19$ and slowly increases at $\gamma > 2.19$. It is worth noticing that on the most part of the plot, presented on Figure 4, longitudinal correlation radius ΔX_c is less than l_x, $\Delta X_c < l_x$, but at least five times as larger than l_z.

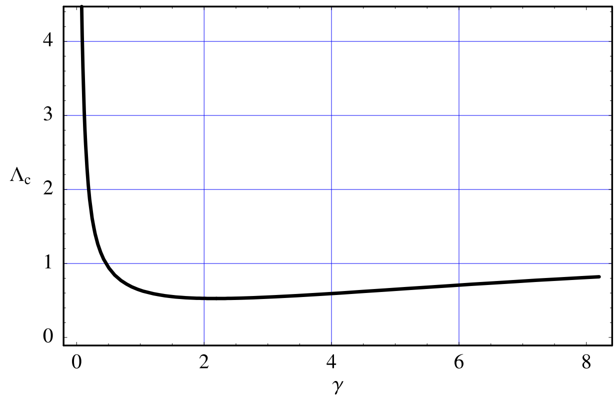

Figure 4: Dependence of dimensionless longitudinal scale $\Lambda_c = \Delta X_c/H$ on distance $\gamma = X/H$.

NUMERICAL MODELLING

In order to verify the theoretical results, numerical calculations were performed by using ray tracing method in 2D elastic random media. The random field ξ was generated with a Gaussian probability density function with zero mean and unit variance and multiplied by the square root of the Gaussian correlation function in the wave number domain. The random field in real space which represents the velocity fluctuations was obtained by taking the inverse Fourier transform. The random media represented with the refractive index fluctuations are calculated by using eq.(9). The average value of velocity $\overline{v}$ is supposed to grow with a depth z as $\overline{v} = v_0 + az$, whereas the random part of the velocity $\tilde{v}$ is taken proportional to the average velocity: $\tilde{v} = (v_0 + az)\xi$. The grid points for the model were selected with an increment of 10m in both directions ($\Delta x = \Delta z$). Total number of the grid points was $n_z = 1024$ in z direction and $n_x = 2048$ in x direction. The inhomogeneity scale lengths are selected as $l_z = 50m$ and $l_x = 500m$ in z and x directions respectively so that $(l_z \gg l_x)$ what gives a ratio of $\rho = 0.1$. The initial velocity and the velocity gradient were selected as $v_0 = 2000m/s$, $a = 0.8s^{-1}$ respectively. Standart deviation σ_ξ of dimensionless velocity fluctuations $\xi = \tilde{v}/\overline{v}$ was $\sigma_\xi = 0.01$.

Traveltime fluctuations $\tilde{t}$ were calculated by using eq.(15) which was transformed to the form

$$\tilde{t} = \frac{1}{v_0}\sqrt{H^2 + \frac{X^2}{4}} \int_0^X \frac{\tilde{n}\left[x, z\left(x\right)\right] dx}{\sqrt{H^2 + xX - x^2}}. \tag{67}$$

which is more convenient for numerical calculations. The refractive index fluctuation field $\tilde{n}\left[x, z(x)\right] = \xi v_0/\overline{v}$ was calculated in each grid point of the model. The rays were traced by using eq. (34). Linear interpolation was performed in all the cases, when the ray trajectories did not match the grid points.

The numerical calculations were performed for 100 realization of the medium. An example of 10 realizations of travel time fluctuations versus distance are plotted in Figure 5. The variance of the traveltime fluctuations was calculated as a function of distance and shown in Figure 6 as a thin wavy curve. Significant variations of “empirical” curve $\sigma^2_{t_{num}}$ are caused by limited number of realizations N=100: expected value of relative variations can be estimated as $1/\sqrt{N} = 0.1$, that is about $\pm 10\%$.

$$\sigma^2_{t_{num}} =< \tilde{t}^2 >= \frac{1}{N}\sum_{i=1}^{N}\left[\tilde{t}_i(x)\right]^2, \quad N = 100. \tag{68}$$

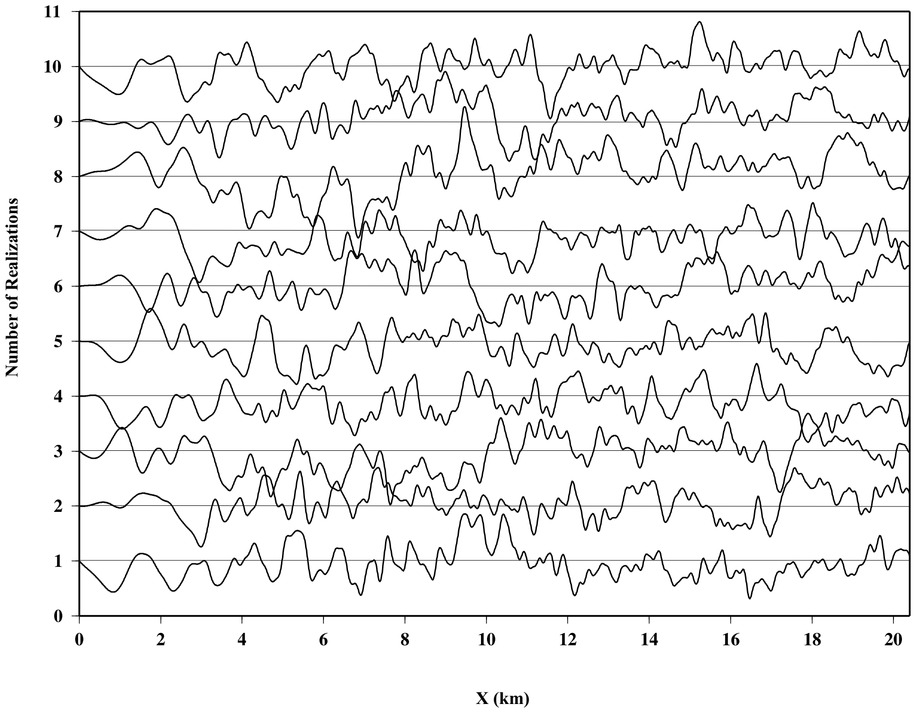

Figure 5: Examples of traveltime fluctuations along x axis, obtained by numerical simulations. Each trace has a dimension of time in seconds versus x(km) distance.

Considering the values $\sigma^2_{t_{num}}$ as "empirical data", one can fit the theoretical curve (47) to the "empirical data" and thereby extract the medium statistical parameter $\sigma^2_\xi l_z$ (fluctuation variance multiplied by the inhomogeneity scale length in z direction) and $\rho = l_z/l_x$ (correlation scales ratio). Performing nonlinear fit procedure, "empirical" $\sigma^2_\xi l_z$ was estimated as 0.005, in agreement with model value $\sigma^2_\xi l_z = 0.005$. Empirical estimate of $\rho_{emp} = 0.12$ is also sufficiently close to the model value $\rho = l_z/l_x = 0.1$. It is seen that numerical results satisfactorily agree with the theoretical derivations.

The correlation radius ΔX_c was calculated from the equation:

$$Cov\left[\tilde{t}(X), \tilde{t}(X + \Delta X_c)\right]_{num} \equiv \frac{1}{N}\sum_{i=1}^{N} \tilde{t}_i(X)\tilde{t}_i(X + \Delta X) = \frac{1}{2}\sigma^2_{t_{num}} \tag{69}$$

where $\tilde{t}_i(X)$ are data, obtained by numerical simulation. Resulting correlation radius $(\Delta X_c)_{num}$ is presented in Figure 7 by a thick curve.

The plot in Figure 7 allows to retrieve the horizontal correlation length l_x by fitting theoretical dependence

$$(\Delta X_c)_{theo} = l_x\,(\Lambda_c)_{theo}$$

(thin curve in Figure 7) to the results of numerical modelling. Here dimensionless correlation radius $(\Lambda_c)_{theo}$ is the root of the eq. (66), (plot of this value versus dimensionless distance $\gamma = X/H$ is shown in Figure 4).

The fitting procedure provides the best result at $(l_x)_{num} = 448m$, which is a sufficiently good approximation to the original correlation length $l_x = 500m$, corresponding to the medium model.

Having revealed the ratio $\rho = l_z/l_x = 0.12$ from previous calculations (Figure 6) one can estimate the vertical correlation scale l_z as $l_z = \rho l_x = 0.12 * 448m = 54m$ and fluctuation intensity σ_ξ from $\sigma^2_\xi l_z = 0.005$ as $\sigma_\xi = \sqrt{0.0050/54} = 0.01$.

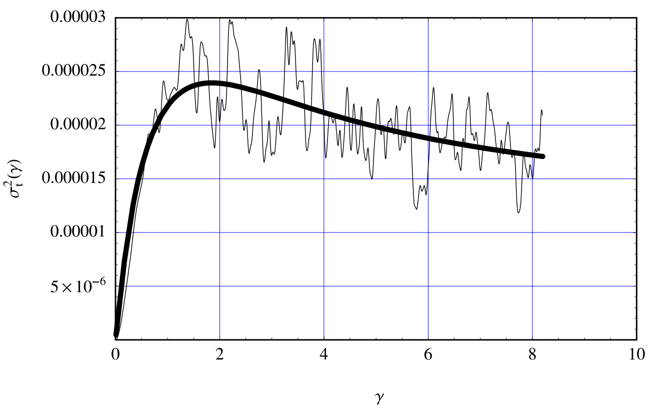

Figure 6: Variance of traveltime fluctuations obtained by numerical simulations (continious curve). Dashed curve is theoretical dependence of (47), fitted to "empirical" data.

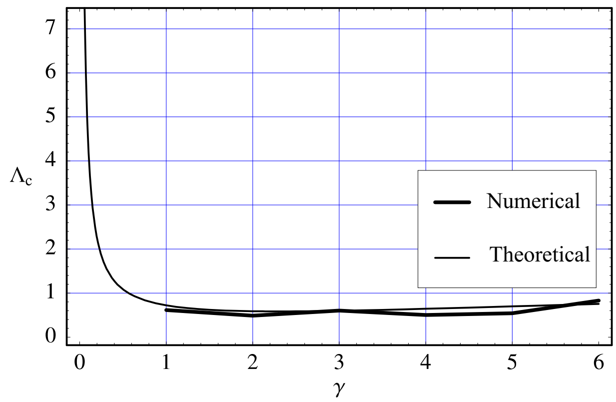

Figure 7: Longitudinal correlation scale ΔX_c versus distance $\gamma = X/H$: Numerical calculation (thick curve) and theoretical dependence (thin line) fitted to the "empirical" data.

CONCLUSIONS

In this paper the statistics of the traveltime fluctuations is studied for refraction geometry. The most important result of the study is that in a realistic medium with linear gradient of velocity the travel time variance decreases at larger distances and approaches a constant value. The reason for such an unusual behaviour is the fast rays diving into the deeper layers where refractive index fluctuates weaker in combination with the strong anisomery of the random inhomogeneities.

Calculations, performed in this paper, showed that refraction seismics is in position to deliver valuable information on random inhomogeneities of the elastic media. Though interpretation of experimental data in refraction geometry, is a little bit cumbersome as compared to reflection geometry, refraction seismics allows to estimate the main statistical parameters of random media: variance of refractive index, horizontal and vertical scales of inhomogeneities.

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