Stress sensitivity of elastic moduli and electrical resistivity in porous rocks

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ABSTRACT

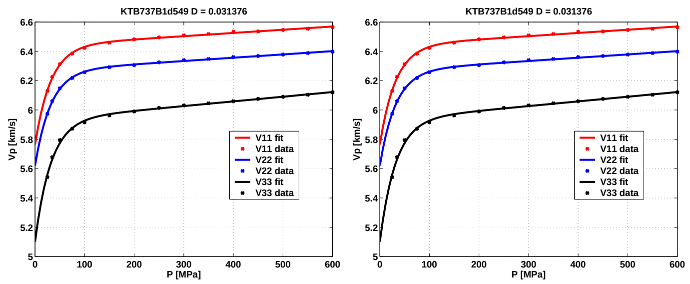
Stress dependencies of elastic moduli and velocities for anisotropic rocks and electrical resistivity are derived as functions of pore space deformation due to an applied arbitrary load. All dependencies have the form of a four parametric exponential equation $V(P) = A + KP - B\exp(-DP)$. The stress dependencies are mainly controlled by the tensor of stress sensitivity. One result of our derivations is that if this tensor is isotropic and the rock sample is loaded hydrostatically the argument D of the exponential term is a univers quantity for all mentioned rock characteristics. We show that laboratory derived velocity, dilatancy, and resistivity measurments as a function of effective pressure support this result.

INTRODUCTION

Understanding stress dependencies of elastic moduli and seismic velocities is important for interpretation of very different seismic and seismologic data. For example, it is necessary for studies of earthquakes and seismogenic processes, for exploring tectonic stress distributions in space and time, for borehole constructions and developments of hydrocarbon and geothermal reservoirs. Specifically, in the exploration seismology knowledges of velocity stress dependencies are required in different applications ranging from AVO and velocity analysis to overpressure prediction and 4D seismic monitoring of hydrocarbon or geothermal reservoirs.

Many studies have shown that seismic velocities are sensitive to changes of the in situ state of stress as induced by reservoir depletion or fluid injection. Due to the combination of this sensitivity with high spatial resolution seismic methods are frequently used for reservoir monitoring purposes. In order to interprete the signature of the current state of stress and induced changes on seismic waves theoretical approaches are required to relate elastic moduli to stress. Some of these approaches take into account that rocks behave like non-linear elastic bodies or even more complex. Several, quite successful attempts to use the formalism of non-linear elasticity theory for this goal are known from recent literature (e.g., Sarkar et al., 2003; Johnson and Rasolofosaon, 1996; Winkler and Liu, 1996; Rasolofosaon, 1998). However, these models are restricted to small ranges of stress variations only. As a consequence, the resulting stress dependencies of elastic properties are principally linear functions of stress only.

Several other approaches can be understood as attempts to specify models of pore space geometry in order to arrive at a more specific elastic non-linear rock characterization. These are spherical contacts models (Duffy and Mindlin, 1957) or crack contacts models (Gangi and Carlson, 1996; Mavko et al., 1995). These approaches are used in geophysical applications (see Merkel et al., 2001; Carcione and Tinivella, 2001). They lead to different quite complex stress dependencies of elastic properties. Moreover, some of them work in very limited ranges of pore pressure changes or under very restrictive geometrical or geomechanical conditions.



(a) **(b)**

Figure 1: Best fit of anisotropic velocities (1(a): P-waves; 1(b): S-waves) from metamorphic rock sample. A successful fit with a universe D = 0.031 1/MPa indicates an isotropic stress sensitvity tensor.

However, laboratory observations show that under hydrostatic load pore pressure, confining stress and differential pressure dependencies of seismic velocities or elastic moduli are phenomenologically described by the following simple relationship (Zimmerman et al., 1986; Eberhart-Phillips et al., 1989; Freund, 1992; Jones, 1995; Prasad and Manghnani, 1997; Khaksar et al., 1999; Carcione and Tinivella, 2001; Kirstetter and MacBeth, 2001)

$$V(P) = A + KP - B\exp(-PD), \tag{1}$$

where $P = P_c - P_{fl}$ is the differential pressure, $P_c = -\sigma_{ii}/3$ is a confining pressure, σ_{ij} is a component of the total stress tensor (here, the compression stress is negative and the summation over repeating indices is assumed) and P_{fl} is a pore pressure. The coefficients A, K, B and D of equation (1) are fitting parameters for a given set of measurements.

It is often observed that equation (1) or similar ones provide very good approximations for velocities and elastic moduli of dry as well as saturated rocks in a range of stress changes of several hundred Megapascal. Moreover, it is also observed that this equation provides a very good approximation for elastic properties even in the case of anisotropic rocks (see Figure 1) and Kaselow and Shapiro (2003).

On the other hand it is known that electrical resistivity is remarkably more sensitive to porosity, temperature, and fluid saturation than seismic velocities (e.g., Wilt and Alumbaugh, 1998). Numerous laboratory experiments have been conducted in the past to understand the electrical properties of very different rock types. A review of these studies can be found, e.g., in Wyllie (1963); Olhoeft (1980); Parkhomenko (1982). However, in high porosity reservoir rocks electrical resistivity is usually assumed to be independent from changes in the in situ stress field. For example, Daily and Lin (1985) found that the electrical conductivity primarily resulted from electrical volume conduction and that resistivity is not effected by changing elastic moduli through crack closure due to compression as long as the large aspect ratio pores remain open. Lockner and Byerlee (1985) compared the stress dependent complex resistivity of Westerly granite with the one of Berea sandstone. They found that the stress dependence of resistivity is much smaller in sandstones than in granites. For one granitic sample the real part of the low-frequency conductivity dropped of by 94 % at 200 MPa confining pressure whereas the conductivity of the sandstone decreased by only 24 %. Crystalline rocks seem to behave like sandstones when partially saturated (Brace and Orange, 1968).

This work attempts to explain observations referred to above. We follow the concept of stress sensitivity, introduced by Shapiro (2003). We understand the stress dependence of elastic moduli in isotropic as well as anisotropic porous rocks as the result of pore space deformation. The porosity is assumed to consist of a stiff and compliant part (fig. 2). The stiff porosity consists of more or less isometric pores, the compliant porosity represents cracks and grain contact vicinities. A general relation is derived which defines the stress dependence of the pore space deformation. Then, we use the separation of the pore space into two different porosity domains with a distinct deformation behaviour to relate the dry rock compliance to porosity. We derive an equation for the dependence of porosity on an arbitrary load. This result is used to obtain a first order relation between the compliances of an anisotropic rock and applied stress. Then, we restrict

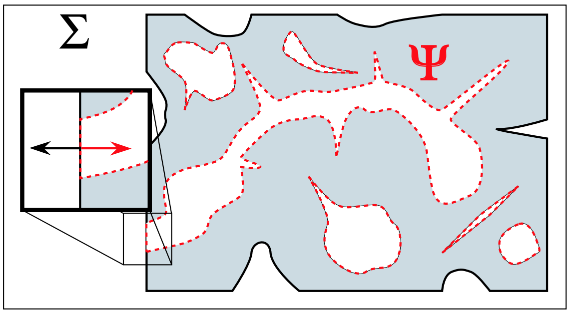


Figure 2: Sketch of complex pore structure consisting of crack-like compliant voids and stiff more or less isometric pores. Σ denotes the outer surface of the sample, indicated by the thick black line, while the dashed red line illustrates the inner surface of the rock, encasing the pore space (denoted as Ψ). The zoom square shows a situation where the external surface cuts a pore and thus coincides with the inner surface. Note, their normals are defined positive in opposite directions. In 3D all pores build an interconnected space effective for fluid flow.

our considerations to isotropic rocks and derive the dependence of bulk and shear modulus, P and S wave velocity, and electrical resistivity on the applied load. Our results for elastic properties are applicable to a broad range of rocks.

However, the dependence of electrical resistivity and hydraulic permeability on porosity is more sophisticated. In order to obtain a stress dependence of electrical resistivity using the load dependent deformation of the pore space, we have to restrict our derivations to rocks where only electrolytic charge transport through an interconnected pore space occurs.

STRESS DEPENDENCE OF POROSITY

Two load components can act on a porous rock, an externally applied confining stress $\sigma^{\mathrm{c}}_{\mathrm{ij}}$ and an internally applied stress $\sigma^{\mathrm{f}}_{\mathrm{ij}}$. In most realistic geological situations $\sigma^{\mathrm{c}}_{\mathrm{ij}}$ corresponds to the overburden stress and -$\sigma^{\mathrm{f}}_{\mathrm{ij}}$ to the pore pressure P_{fl}. Moreover, in hydrostatically conducted laboratory experiments -$\sigma^{\mathrm{c}}_{\mathrm{ij}}$ becomes the confining pressure P_{c}. Both load components acting compressionally with respect to the rock material are negative per definition.

In poromechanics it is common to combine both load components. Hereby, the differential pressure P_{diff} is usually defined as

$$P_{\mathrm{diff}} = P_{\mathrm{c}} - P_{\mathrm{fl}}. \tag{2}$$

In the case of a non-hydrostatic confining load, we define the difference between confining stress σ^{c} and the internally applied stress σ^{f} as the effective stress $\sigma^{\mathrm{e}}_{\mathrm{ij}}$, since the term 'differential stress' would lead to confusion with the corresponding definition used in tectonophysics where it denotes the difference between the minimum and maximum principal stresses. Thus,

$$\sigma^{\mathrm{e}}_{\mathrm{ij}} = \sigma^{\mathrm{c}}_{\mathrm{ij}} + \delta_{\mathrm{ij}} P_{\mathrm{fl}}, \tag{3}$$

where δ_{ij} is the Kronecker delta function. Moreover, we will show in the following under which conditions only the difference between confining stress and internal stress is effective for porosity changes and elastic and transport properties as well. Thus, in the case of hydrostatic load differential pressure equals effective pressure:

$$P_{\mathrm{eff}} = P_{\mathrm{diff}}$$

In order to describe the deformation of the pore space resulting from the application of a load, we define two surfaces of the porous rock. The surface Σ is the external surface of the rock (fig. 2). Where Σ cuts a pore it simultaneously seals the latter. The second surface Ψ is the internal surface of the rock. In this way, we can represent the bulk and pore space volume of the rock sample in terms of the encasing surfaces

Σ and Ψ, respectively. Thus, it is possible to describe changes of both volumes by the displacement of the surface points of Σ and Ψ.

Let us assume that the confining and/or pore pressure have changed from an initial state of stress $(P^0_{\text{eff}}, P^0_{\text{fl}})$ to the current state (P_{eff}, P_{fl}). As a result, points of the external surface have been displaced by $u_i(\hat{x})$. The displacement is assumed to be very small in comparison to the size of the rock volume under consideration.

Following Brown and Korringa (1975) we introduce a symmetric tensor

$$\eta_{ij} = \int_\Sigma \frac{1}{2}(u_i n_j + u_j n_i) d^2\hat{x}. \tag{4}$$

In the case of a continuous elastic body replacing the porous rock (i.e., a differentiable displacement is given at all its points) the Gauss' theorem gives

$$\eta_{ij} = \int_V \frac{1}{2}(\partial_j u_i + \partial_i u_j) d^3 x. \tag{5}$$

The integrand here is the strain tensor. Thus, $\epsilon_{ij} = \eta_{ij}/V$ is the volume averaged strain.

We see that the tensor η_{ij} is related to the deformation of the rock sample. In the same way, we introduce also a second symmetric tensor related to the deformation of the pore space.

$$\zeta_{ij} = \int_\Psi \frac{1}{2}(u_i n_j + u_j n_i) d^2\hat{x}, \tag{6}$$

where Ψ is the surface of the pore space, $\hat{x}$ is a point of this surface, u_i is a component of the displacement of points $\hat{x}$ of this surface slightly deformed by changing of the load, and n_i is a component of the outward normal to this surface (the normal is directed into the space of pores). In points, where surface Σ seals the pores it coincides with surface Ψ. However, their normals are opposite (fig. 2).

In analogy to the paper of Brown and Korringa (1975), we introduce three fundamental compliances of an anisotropic porous body:

$$S^{dr}_{ij} = -\frac{1}{V}(\frac{\partial \eta_{ij}}{\partial P_{\text{eff}}})_{P_{\text{fl}}}, \tag{7}$$

$$S^{mt}_{ij} = -\frac{1}{V}(\frac{\partial \eta_{ij}}{\partial P_{\text{fl}}})_{P_{\text{eff}}}, \tag{8}$$

$$S^{p}_{ij} = \frac{1}{V_p}(\frac{\partial \zeta_{ij}}{\partial P_{\text{fl}}})_{P_{\text{eff}}}, \tag{9}$$

Using the reciprocity theorem we can define a fourth, but not independent compliances S'_{ijkl}:

$$S'_{ij} = S^{dry}_{ij} - S^{mt}_{ij}. \tag{10}$$

S^{dry}_{ij} is the compliances of the dry (drained) rock matrix, S^{mt}_{ij} is the compliances of the grain material, and S^p_{ij} is the compliance of the pore space. The compliances S^x_{ij} defined above have the following relation to the 4th rank tensors of elastic compliances:

$$S^x_{ij} = S^x_{ijkk} = S^x_{ij11} + S^x_{ij22} + S^x_{ij33}, \tag{11}$$

where x represents mt, dry, and p.

In the special case of an isotropic rock the corresponding bulk moduli are:

$$\frac{1}{K_{dry,mt,p}} = S^{dry,mt,p}_{11} + S^{dry,mt,p}_{22} + S^{dry,mt,p}_{33}. \tag{12}$$

Note, in the literature (e.g., Mavko et al., 1998) bulk moduli K_{mt} and K_p are usually denoted as K_0 and K_ϕ, respectively. In the following, we limit our consideration to isotropic rocks and use the latter notation.

If the applied load hydrostatically changes, the quantity ζ_{ij} will change due to δP_{eff} by keeping a constant pressure P_{fl} plus an effect of applying δP_{fl} from inside and outside while keeping $P_{\text{eff}} = const.$:

$$\delta\zeta_{ij} = -(\frac{\partial\zeta_{ij}}{\partial P_{\text{eff}}})_{P_{\text{fl}}}\delta P_{\text{eff}} - (\frac{\partial\zeta_{ij}}{\partial P_{\text{fl}}})_{P_{\text{eff}}}\delta P_{\text{fl}}. \tag{13}$$

Also for the quantity η_{ij} an analogous equation can be written:

$$\delta\eta_{ij} = -(\frac{\partial\eta_{ij}}{\partial P_{\text{eff}}})_{P_{\text{fl}}}\delta P_{\text{eff}} - (\frac{\partial\eta_{ij}}{\partial P_{\text{fl}}})_{P_{\text{eff}}}\delta P_{\text{fl}}. \tag{14}$$

Note that $\delta V = \delta\eta_{ii}$:

$$\delta V = -(\frac{\partial\eta_{ii}}{\partial P_{\text{eff}}})_{P_{\text{fl}}}\delta P_{\text{eff}} - (\frac{\partial\eta_{ii}}{\partial P_{\text{fl}}})_{P_{\text{eff}}}\delta P_{\text{eff}}. \tag{15}$$

Note also:

$$\delta\phi = \delta(\frac{V_p}{V}) = \frac{\delta V_p}{V} - \phi\frac{\delta V}{V}. \tag{16}$$

Thus, with definition (7) - (9) using equation (12) and (13) - (16), we can now formulate a general pressure dependence of porosity changes for isotropic rocks as:

$$\delta\phi = \left(\frac{1}{K_{\text{dry}}} - \frac{1}{K_0} - \frac{\phi}{K_{\text{dry}}}\right)\delta P_{\text{eff}} - \phi\left(\frac{1}{K_\phi} - \frac{1}{K_0}\right)\delta P_{\text{fl}}. \tag{17}$$

Equation (17) shows that, in general, porosity is a function of the difference between confining pressure and pore pressure as well as the pore pressure itself. However, if the grain matrix is homogeneous and linear (K_ϕ= K_0), i.e., if the rock is in the Gassmann limit, and/or porosity is small, then porosity is a function of the pure difference between confining stress and pore pressure alone. This coincides with the results from Zimmerman et al. (1986). By substituting the bulk moduli in equation (17) with the corresponding compliances and the pressure components by corresponding stresses this equation can be extended to arbitrary anisotropic rocks under arbitrary load changes.

In equation (17) only two quantities are significantly stress dependent: K_{dry} and ϕ. However, ϕ obviously depends on K_{dry} and, the other side, K_{dry} depends on ϕ in a complex manner. A necessary equation relating them to each other can not be found directly, since K_{dry} depends especially on the geometry of the pore space rather than on the magnitude of ϕ alone.

STRESS DEPENDENCE OF ELASTIC MODULI

From numerous observations of elastic velocities as a function of applied effective stress/pressure it is known that velocities increase remarkably up to an effective stress of approx. 100 to 150 MPa, dependent on the rock type under consideration. For higher stresses the slope of velocities stress relation decreases significantly and tapers out to a flat linear increase.

A quite reasonable and common explanation of the increase in velocity with increasing stress is the closure of porosity. From many observations it is known that velocities change by approx. 10 % over the mentioned pressure range. In contrast, porosities do not change at all or only by less than 1 %. Although the interpretation that porosity changes are responsible for velocity variations is still reasonable this illustrates that treating the porosity as a single scalar quantity is not sufficient. A common interpretation for this behaviour is the progressive closure of two mechanically distinct porosity domains with increasing effective stress. The front part of the velocity stress relation is controlled by the closure of cracks and grain contact vicinities (denoted as compliant porosity) which deform much easier under stress than spherical-like voids (representing the so-called stiff porosity). The closure of the latter controls the flat back part of the velocity stress relation when the compliant porosity is assumed to be completely closed.

In analogy to Shapiro (2003) we formulate this separation of porosity as

$$\phi = \phi_{\text{c}} + [\phi_{\text{s0}} + \phi_{\text{s}}] \tag{18}$$

Here, ϕ is the bulk interconnected porosity and ϕ_{c} is the compliant porosity. The stiff porosity is further separated into a part ϕ_{s0}, defined at P_{eff} = 0, and a part ϕ_{s} induced by an applied pressure. If effective pressure is positive ϕ_{s} is negative, if P_{eff} < 0 ϕ_{s} is positive and if no load is applied ϕ_{s} = 0. As a rule of thumb the following inequation is valid for many rocks:

$$|\phi_{\mathrm{s0}}| \gg |\phi_{\mathrm{s}}| \gg |\phi_{\mathrm{c}}| \tag{19}$$

In the following we consider only isotropic rocks. We also assume that the rocks remain approximately isotropic under hydrostatic load. Under this approximation and taking into account that the changes of ϕ_{s} and ϕ_{c} are in the order of strain and thus small, it is reasonable to approximate the matrix bulk modulus as a linear function of these quantities:

$$K_{\mathrm{dry}}([\phi_{\mathrm{s0}} + \phi_{\mathrm{s}}], \phi_{\mathrm{c}}) = K_{\mathrm{dryS}}[1 + \theta_s \phi_{\mathrm{s}} + \theta_c \phi_{\mathrm{c}}] \tag{20}$$

Here, K_{dryS} is a hypothetical matrix bulk modulus, since it is defined at $P_{\mathrm{eff}} = 0$ with ϕ_{s}= ϕ_{c}= 0. As shown by Shapiro (2003) the order of magnitudes of quantities θ_s and θ_c are 1 and $> 10^2$, respectively. Moreover, θ_c is approx. proportional to the inverse of the effective crack aspect ratio. We call θ_c in the following stress sensitivity.

In its most general form θ_c is a tensor of rank 6 (Shapiro and Kaselow, 2003). If this tensor is isotropic only one entry is independent and thus the tensor of stress sensitivity reduces to one single scalar value. In the following we consider only rocks with isotropic stress sensitivity tensor.

If we assume, that the changes in stiff and compliant porosity are independent we obtain for the effective pressure dependence of $\phi_{\mathrm{s}}$and ϕ_{c}(Shapiro, 2003)

$$\phi_{\mathrm{s}}(P_{\mathrm{eff}}) = -P_{\mathrm{eff}} \left(\frac{1}{K_{\mathrm{dryS}}} - \frac{1}{K_0} \right) \tag{21}$$

$$\phi_{\mathrm{c}}(P_{\mathrm{eff}}) = \phi_{\mathrm{c0}} \exp\left(-\theta_c \frac{1}{K_{\mathrm{dryS}}} P_{\mathrm{eff}} \right). \tag{22}$$

Using equation (21) and (22) with equation (20) we arrive at the dependence of the dry matrix bulk modulus on stress, that reads:

$$K_{\mathrm{dry}}(P_{\mathrm{eff}}) = K_{\mathrm{dryS}} \left[1 + \theta_s \left(\frac{1}{K_{\mathrm{dryS}}} - \frac{1}{K_0} \right) P_{\mathrm{eff}} - \phi_{\mathrm{c0}} \theta_c \exp\left(-\theta_c \frac{1}{K_{\mathrm{dryS}}} P_{\mathrm{eff}} \right) \right] \tag{23}$$

A similar equation can be arrived for the pressure dependence of the dry matrix shear modulus μ_{dry} as well as for P- and S-wave velocities (see Shapiro, 2003, for details).

We can summarize an alayzis of the derived velocity dependencies as followed:

1. All mentioned stress dependencies have the form of a four parametric exponential equation

$$\Gamma(P_{\mathrm{eff}}) = A_\Gamma + K_\Gamma P_{\mathrm{eff}} - B_\Gamma \exp(-D_\Gamma P_{\mathrm{eff}}) \tag{24}$$

   where Γ is the property under consideration.

2. Parameter $D = \frac{\theta_c}{K_{\mathrm{dryS}}}$ is a universal quantity for all properties under consideration.

Empirical relations of the form of equation (24) or similar were used in many studies to investigate the relationship between effective pressure, porosity and velocities, and other properties. For example, Eberhart-Phillips et al. (1989); Jones (1995); Freund (1992) used empirical relations of this form to fit velocity vs. pressure relations successfully. Moreover, Eberhart-Phillips et al. (1989) found no correlation between stress dependence of velocities and porosity or clay content. In fact, they conclude that the most useful form of pressure dependence is that changes of velocities are proportional to the exponential term. This result coincides with our derivatives.

An additional important result is that the exponential argument is identical for all elastic moduli and velocities of a given sample, if a hydrostatic load is applied and the rocks remain isotropic under the load.

STRESS DEPENDENCE OF ELECTRICAL RESISTIVITY

Formulating the stress dependence of electrical resistivity in terms of stress sensitivity as presented above for elastic moduli requires the limitation to rocks where only electrolytic charge transport is assumed to take place, i.e., surface conductivity should be neglectable and highly conducting mineral phases should not be present. In other words, we restrict our considerations to rocks where electrical resitivity can be described by the well known Archie law (Archie, 1942):

$$\frac{1}{\Omega} = \frac{1}{F\Omega_{\text{fl}}} = \frac{1}{\Omega_{\text{fl}}}\phi^m. \tag{25}$$

Here, Ω_{fl} is the resistivity of the pore fluid, F is the formation factor, ϕ is the porosity, and m is Archie's cementation exponent. In general, m is in the range $1 \le m \le 2$ but occasionally reaches 2.3 (Berryman, 1992). Archie's law shows that electrical resistivity is in general not a linear function of porosity. In fact, only in the special case $m = 1$ electrical resistivity linearly depends on porosity. However, equation (25) shows that the logarithm of resistivity depends linearly on porosity.

Rearranging equation (25), using equation (18) and taking the logarithm gives:

$$\log\frac{\Omega}{\Omega_{\text{fl}}} = -\text{m}\cdot\log\phi = -\text{m}\cdot\log(\phi_{\text{s0}} + \phi_{\text{s}} + \phi_{\text{c}}). \tag{26}$$

Obviously, the logarithm of the $F = \Omega/\Omega_{\text{fl}}$ is a linear function of porosity.

Using a Taylor expansion gives:

$$\log\frac{\Omega}{\Omega_{\text{fl}}} = -\text{m}\log\phi_{\text{s0}} - \frac{\text{m}}{\phi_{\text{s0}}}\phi_{\text{s}} - \frac{\text{m}}{\phi_{\text{s0}}}\phi_{\text{c}}. \tag{27}$$

If we now use the stress dependent formulations for ϕ_{s} and ϕ_{c} as given by equation (21) and (22), we finally obtain:

$$\log\frac{\Omega}{\Omega_{\text{fl}}} = -\text{m}\log\phi_{\text{s0}} - \frac{\text{m}}{\phi_{\text{s0}}}\left(\frac{1}{K_{\text{dryS}}} - \frac{1}{K_0}\right)P - \frac{\text{m}}{\phi_{\text{s0}}}\phi_{\text{c0}}\exp\left(-\frac{\theta_{\text{c}}}{K_{\text{dryS}}}P\right) \tag{28}$$

Comparing equation (28) with (24) illustrates the physical meaning of the fit parameters A, K, B, and D in the case of stress dependence of logarithmic formation factor:

$$A = -\text{m}\log\phi_{\text{s0}} \tag{29}$$

$$K = -\frac{\text{m}}{\phi_{\text{s0}}}\left(\frac{1}{K_{\text{dryS}}} - \frac{1}{K_0}\right) \tag{30}$$

$$B = \frac{\text{m}}{\phi_{\text{s0}}}\phi_{\text{c0}} \tag{31}$$

$$D = \frac{\theta_{\text{c}}}{K_{\text{dryS}}}. \tag{32}$$

Fit parameter A corresponds exactly to Archie's Law if ϕ in equation (25) is equal to the stress independent part ϕ_{s0} of the bulk porosity. In the case of fit parameter D we obtain the same expression as for elastic moduli and seismic velocities.

Here, however, the fit parameters K and B are significantly different in comparison to their corresponding formulation in the case of elastic moduli and velocities. The magnitudes of K and B are proportional to $1/\phi_{\text{s}}$ and $\phi_{\text{c}}/\phi_{\text{s0}}$, respectively, while they are proportional to $\theta_{\text{c}}\phi_{\text{s}}$ and $\theta_{\text{c}}\phi_{\text{c}}$ in the case of the other elastic moduli and velocities. This has an important consequence. In most reservoir rocks stiff porosity and even the stress induced change in stiff porosity is much larger than compliant porosity (see eq. (19)). Despite this fact the closure of the crack porosity is dominant for the stress dependence of the elastic moduli and velocities. These properties are rather sensitive to the relative change of the different porosity domains, expressed in the θ terms ($|\theta_{\text{s}}| \ll |\theta_{\text{c}}|$), than to the absolute change (Shapiro, 2003).

In contrast, equation (28) states that the stress dependence of electrical resistivity is controlled by the absolute change of the porosities. Consequently, the change in stiff porosity controls electrical resistivity

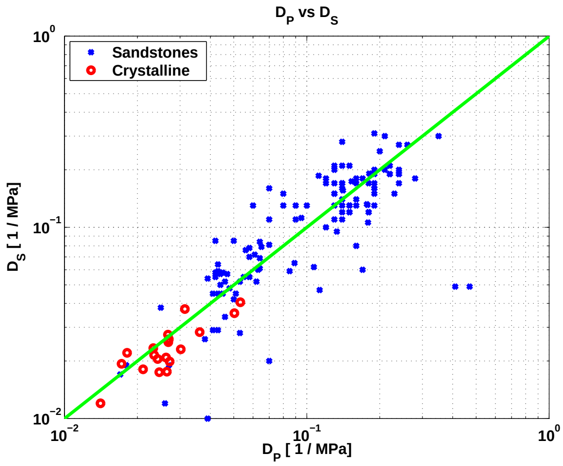


Figure 3: This figure illustrates parameter D for P- and S- wave data, indicated respectively, by subscribts P and S, as obtained from observations on sandstones (blue) published by Eberhart-Phillips et al. (1989), Jones (1995), and Freund (1992). Red symbols illustrate Ds vs. Dp as obtained from ten data sets from metamorphic core samples of the German Continental Deep Drilling site (Kern et al., 1991; Kern et al., 1994). The green line denotes Dp = Ds.

as a function of effective pressure in reservoir rocks and not the change in compliant porosity. In turn, the stress dependence of stiff porosity can be neglected over the effective stress range of interest (up to 200 MPa). This might be the reason that electrical resistivity is usually assumed to be independent from stress.

EXAMPLES

In order to check our theoretical considerations we collected data from literature (these are Eberhart-Phillips et al., 1989; Jones, 1995; Freund, 1992) where a four parametric exponential equation as given by equation (24) was used to fit successfully observed velocity vs. stress data from sandstone samples. In the mentioned publications the best fit parameters A, K, B, and D are given for both P- and S-wave observations. Figure 3 shows a one-to-one catch line where the D values for S-wave velocities (Ds) were plotted as blue crosses against the corresponding D values of P-wave velocities (Dp). Although the D values scatter around this line the data follow the expected trend at least statistically. On the one hand side the scatter could be caused by possible measurement errors, on the other side, it can be understood as a measure for the (an)isotropy of the stress sensitivity tensor.

We have enlarged the used data base by applying the stress sensitivity approach to velocity vs. stress observations from 10 core samples from the pilot hole of the German Continental Deep Drilling Project. On every anisotropic dry low-porosity metamorphic rock sample three P- and six corresponding S-wave velocities were measured in a orthogonal coordinate system over an hydrostatic effective stress range up to 600 MPa (for details, Kern and Schmidt, 1990; Kern et al., 1991, 1994). Independent from the elastic anisotropy, a fit of equation (24) to all velocities of every single sample should deliver the same parameter D if the stress sensitivity of the rock under consideration is isotropic. In figure 1 the successful fit of the observations is shown as an example, representative for all investigated samples with respect to the fit quality. A successful fit with a universe D = 0.031 1/MPa indicates an isotropic stress sensitivity tensor for the sample.

We have added the obtained D values for all investigated metamorphic samples as opened red circles to figure 3. They scatter remarkably less than the sandstone data. We understand the successful fit of the velocity versus stress observations and figure 3 in such a way that there are, in fact, rocks, that show an isotropic stress sensitivity tensor. However, in the following we want to investigate if parameter D is a universe property even for electrical resistivity.

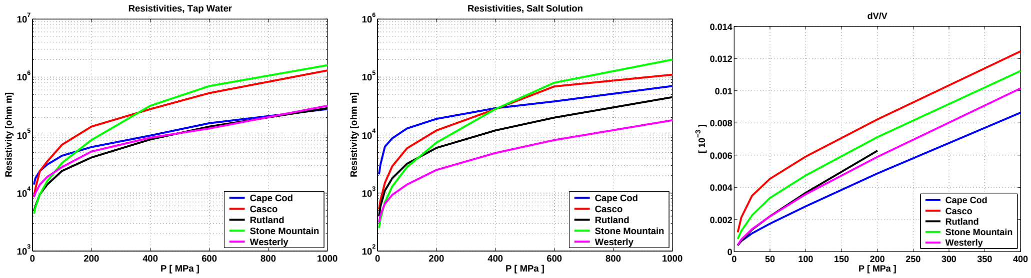


Figure 4: Resistivity of rock samples saturated with tap water (a), NaCl solution (b), and bulk volume deformation (c) of rock samples (data from Brace et al., 1965).

Parameter	Cape Cod			Casco		
	Ω_T	Ω_S	K	Ω_T	Ω_S	K
A	2.924	4.525	44.6	3.126	4.184	49.5
K	0.001	0.001	0.029	0.002	0.002	-0.01
B	0.472	0.861	37.8	0.928	1.070	49.2
D	0.028	0.048	0.059	0.024	0.027	0.017
χ^2	0.002	0.004	0.273	0.010	0.002	8.90

Table 1: Best fit parameter as obtained from fitting resistivity and dilatancy data obtained from Cape Cod granodiorite and Casco granite. In the case of resistivity Ω units are [ohm m] for parameter A and B, [(ohm m) / MPa] for K, and [1/MPa] for D. Subscribes T and S denote saturation with tape water and salt solution, respectively. In the case of the bulk modulus K units are similar with [GPa] instead of [ohm m]. χ^2 is the sum of the squared deviations between the best fit model and the observations.

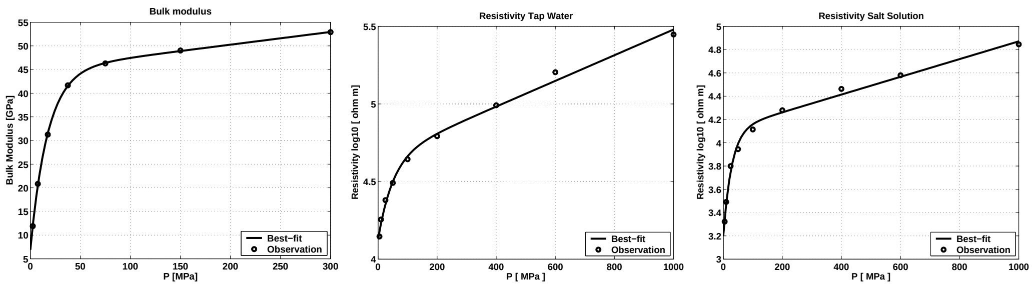
Brace et al. (1965) have published data from stress dependent deformation and electrical resistivity measurements simultaneously conducted on low porosity crystalline rocks. The samples were isostatically loaded up to 1 GPa (10 kbar), whereby a constant pore fluid pressure of approx. P_{fl}= 0 was maintained during the experiments. Although all measurements were conducted on saturated samples the pore pressure during compression was maintained approx. zero. Tap water, with a resistivity $\Omega_t = 45 - 50 ohmm$ and a NaCl solution ($\Omega_s = 0.3 ohmm$) were used as saturating fluids. The suite of samples generally shows bulk porosities below 1%. In this study, we used five rocks, namely Casco, Stone Mountain, and Westerly Granite as well as Rutland Quartzite, and Cape Cod Granodiorite.

Pressure dependent resistivity for the mentioned rocks saturated with tap water and salt solution are shown in fig. II and II, respectively, as well as strain data (fig. II).

Equation (24) was fitted to both, logarithmic formation factor as well as bulk moduli data for every sample in a two step process, using a Levenberg-Marquardt algorithm. In the first step resistivity and bulk modulus data were fitted separately with all four fit parameters A, K, B, and D. The necessity of a second fit arises from the theoretical result that parameter D should be the same for the resistivity as well as bulk modulus data of a certain sample (compare the argument of the exponential terms in equation (23) and (28/32). Therefore, we calculate the average of both D values obtained from the first fit and fit the data again, now with a fixed D and A, K, and B as remaining fit parameters. Hereby, an averaged D is separately calculated for any of both bulk modulus vs. formation factor pairs of a given sample. This means we calculated a mean D from bulk modulus vs. formation factor of tap water saturated rock and vs. formation factor of salt solution saturated rock.

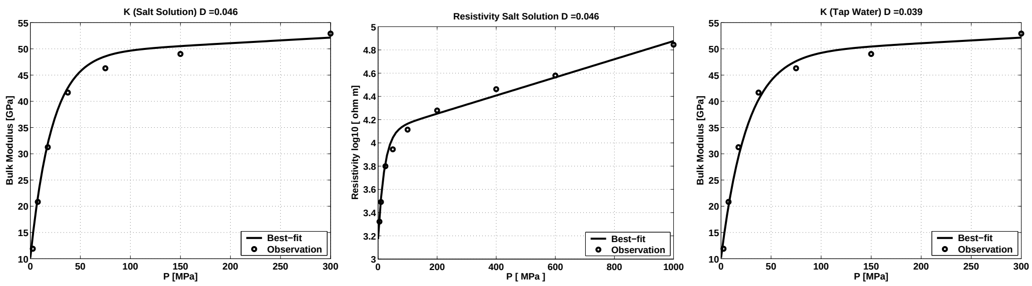
Figure 5 and 6 show a comparison between the observations and the fit results for Cape Cod granodiorite and Casco granite (fig. 7 and 8). It was possible to fit the bulk modulus (fig. 5(a) and 7(a)) as well as the resistivity data - sample saturated with tap water (fig. 5(b), 7(b)) and salt solution (fig. 5(c), 7(c)) - quite well. The best fit parameters for the initial fits are listed in tab. 1.

As representative examples the best fit parameters for Cape Cod granodiorite and Casco granite are

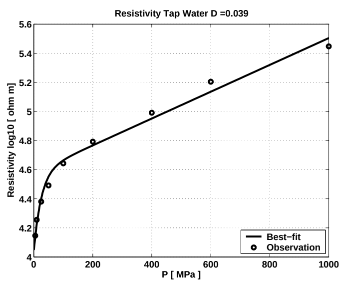


(a) (b) (c)

Figure 5: Best fit of Cape Cod granodiorite data. Bulk modulus (fig. 5(a)), logarithmic bulk resistivity of rock saturated with tap water (fig. 5(b)) and salt solution (fig. 5(c)) were fitted separately with A, K, B, and D as fit parameters. Circles denote observations, lines the best fit.

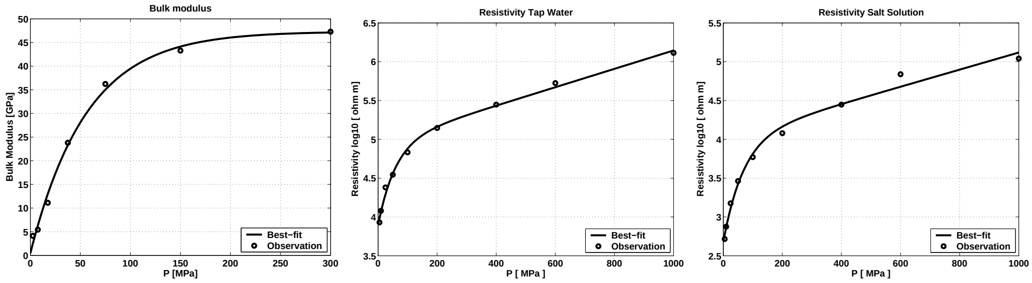


(a) (b) (c)



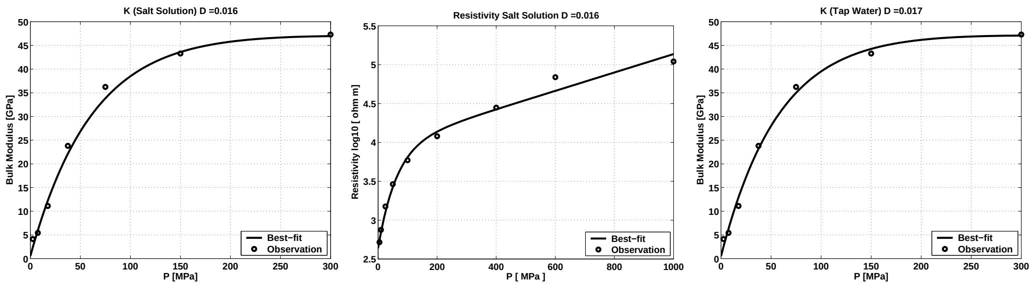
(d)

Figure 6: Second best fit of Cape Cod granodiorite data. Circles denote observations, lines best fit. Figure (6(a)) and (6(b)) show the repeated fit (see text for details) of bulk modulus and formation factor (salt solution saturated), respectively, with an averaged and fixed parameter D=0.054 MPa^{-1}. Figure (6(c)) and (6(d)) illustrate the result of the repeated fit with a fixed D=0.043 MPa^{-1} for bulk modulus and formation factor, respectively, where rock was saturated with tap water.

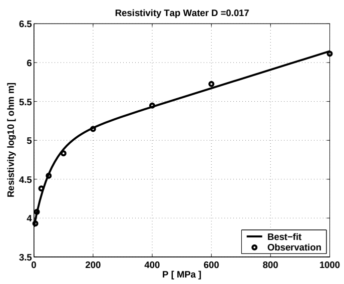


(a) (b) (c)

Figure 7: Casco granite data. Bulk modulus (fig. 7(a)), logarithmic bulk resistivity of rock saturated with tap water (fig. 7(b)) and salt solution (fig. 7(c)) were fitted separately with A, K, B, and D as fit parameters. Circles denote observations, lines the best fit.



(a) (b) (c)



(d)

Figure 8: Second best fit of Casco granite data. Circles denote observations, lines best fit. Figure (8(a)) and (8(b)) show the repeated fit (see text for details) of bulk modulus and resistivity (salt solution saturated), respectively, with an averaged and fixed parameter D=0.022 MPa^{-1}. Figure (8(c)) and (8(d)) illustrate the result of the second fit for bulk modulus and resistivity (rock saturated with tap water), respectively, with D=0.021 MPa^{-1}

	Cape Cod		Casco	
Parameter	Ω_T	K	Ω_T	K
D	0.043		0.021	
A	2.862	49.80	3.178	42.3
K	0.001	-0.01	0.002	0.0218
B	0.453	41.00	0.957	42.3
χ^2	0.003	4.481	0.011	10.70
Parameter	Ω_S	K	Ω_S	K
D	0.054		0.022	
A	4.549	49.811	4.291	42.3
K	0.001	-0.008	0.002	0.022
B	0.856	40.996	1.134	42.31
χ^2	0.004	4.550	0.004	10.7

Table 2: Best fit parameter from second fit of Cape Cod and Casco data. Units are as mentioned in tab. 1.

listed in tab. 2. Cape Cod stands for the worst agreement between theory and observation concerning a common D and Casco for the best (compare the D values in tab. 1). For Cape Cod we obtain a mean D of 0.043 $1/MPa$ (bulk modulus vs. resistivity of tap water saturated rock) and of 0.054 $1/MPa$ (bulk modulus vs. resistivity of rock saturated with salt solution). For Casco granite we found corresponding D values of 0.021 and 0.022.

The quality of best fits for the remaining three samples was as good as for the mentioned two examples. In general, all logarithmic resistivity and dilatancy data could be fit well. The best agreement between the different D values was obtained for Casco granite, as shown in tab. 2. Here, the first fit of the dilatancy data delivered negative values for fit parameter K. However, a negative K does not seem to be a physically meaningful result. Since the absolute magnitudes of all K values for all samples are very small in comparison to the other parameters we assume that negative K values result rather from numerics of the fitting process than from any physical process occuring during compression. This might be interpreted in that way, that the closure of stiff porosity does not effect the mentioned stress dependencies at all and K could be eliminated from the fit equation when fitting bulk moduli data.

The good agreement between the parameter D for logarithmic formation factor and bulk modulus data seem to support our assumption that it is a universal characteristic for a given rock sample, not only for elastic moduli but also for transport properties like the electrical resistivity. This is, of course, limited to rocks where only electrolytic charge transport occurs. If these results are valid then the stress dependence of electrical resistivity on low porosity rocks is also mainly controlled by the elastic stress sensitivity and thus proportional to the inverse of an effective crack aspect ratio. In sediments, where the amount of stiff porosity is in general 2 orders of magnitudes higher than compliant porosity the pressure dependence of resistivity is controlled by the pressure dependence of stiff porosity. However, the universality of parameter D should also be valid in such rocks as long as the stress sensitivity tensor is isotropic.

CONCLUSIONS

We have shown for arbitrary anisotropic rocks that the deformation of the pore space due to an arbitrary applied load is always a function of the difference between confining stress and pore pressure and the pore pressure itself. In terms of the porosity this load dependence reduces to the difference between confining stress and pore pressure alone, if the porosity is low and/or the rock is in the Gassmann limit.

Taking the mechanically different deformation behaviour of the stiff and compliant porosity into account the stress dependence of compliances of anisotropic rocks can be derived in a first order approximation as a function of the stress induced deformation of the pore space. In the same manner we have derived the stress dependence of electrical resistivity for isotropic rocks where electrical resistivity can be described in terms of Archie's law. All obtained stress dependencies of compliances and elastic moduli as well as electrical resistivity have the same simple form of a four parametric exponential equation. In this context the most important rock characteristics controlling these stress dependencies of a porous rock is the rank-6 tensor of stress sensitivity.

We have shown that there are rocks where this tensor of stress sensitivity is isotropic and can be reduced to one single scalar value. In this case and if the rocks are subjected to hydrostatic load the exponential argument D of the stress dependence is a universal quantity for all elastic moduli and velocities, even in the case of anisotropic rocks.

Moreover, the universality of parameter D is also valid for electrical resistivity, at least in rocks where only electrolytic charge transport occurs. This might be understood as a link between elastic moduli and transport properties. Since seismic velocities are sensitive to changes in compliant porosity and electrical resistivity is sensitive to absolute changes in stiff porosity a simultaneous analyzis of resistivity and velocity changes due to artificial pore pressure variations might help to understand how bulk porosity is changed by reservoir depletion or fluid injection.

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