Multiple Attenuation by Combining WHLP and CRS

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ABSTRACT

In the sedimentary basins of the amazonic region, the generation and accumulation of hydrocarbons are related to the presence of diabase sills. These rocks have a large contrast of impedance with the host formations, which results in the generation of external and internal multiples with amplitudes similar to the primaries. These multiples can predominate over the information originated at the deeper interfaces, making difficult the processing, the interpretation and the imaging of the seismic section. In the present work, we have performed multiple attenuation in synthetic common-shot (CS) time sections, through the combination of the Wiener-Hopf-Levinson for prediction (WHLP) and the common-reflection-surface stack (CRS) methods, here denominated WHLP-CRS. The deconvolution operator is calculated from the real amplitudes of the seismic signal trace-by-trace, in order to give efficiency in the multiple attenuation. The multiple identification is performed in zero-offset (ZO) section simulated from the CRS-stack using the periodicity criteria between the elected primary and its possible repeated multiple. The wavefront attributes, obtained by the CRS-stack, are used to shift the windows in the time-space domain, and to calculate the WHLP-CRS operator for the multiple attenuation in CS section.

INTRODUCTION

The present paper is a computational exercise related to seismic multiple attenuation by combining two theories: the classical Wiener-Hopf-Levinson for prediction (WHLP), and the common-reflection-surface (CRS) stack, here denoted as WHLP-CRS. The study aims at geological structures of the Amazon sedimentary basins, where a fundamental problem related to the presence of diabase sills in processing, interpretation and imaging of seismic reflection sections has to be contemplated. Multiples related to these high velocity layers can dominate or complicate information from deeper layers, and make more difficult the processing alternatives for petroleum exploration (Eiras (1998); Eiras and Wanderley (2003)). Classically, the study of multiples is related to marine ambients, and a special publication of SEGt's TLE (1999) is dedicated to questions of multiples, where it is naturally emphasized the non existence of a single technique for multiple recognizing and attenuating that can be applied to all possible cases due to diversity of the geology responsible for the generation of multiples. The problem of multiple attenuation in the zero-offset configuration can be approached by the original classical methodology, where the prediction operator is calculated after processing, and it tends to contain undesirable effects, as pulse stretching and deformation due to stack, that can reduce the operator's performance in multiple attenuation. Searching for operators, in the sense of better resolution, there is a tendency to calculate attenuation operators for the survey configuration using the real amplitudes of the signal what, in principle, can result in a better performance. Through the development of the present subject, we circumvented the inconvenience of the processed section to obtain a zero-offset panel, and performed multiple attenuation in common-source configuration. To accomplish this aim, we reorganized the WHLP theory to include the CRS theory, in order to design a deconvolution operator for the survey configuration, where the WHLP-CRS operator is calculated with the real amplitudes of the signal.

THE WHLP-CRS OPERATOR

We continue with a short revision of the conventional WHL method, in order to understand the introduction of the CRS attributes into the WHL operator, and to see the implications of the basic and fundamental concepts behind its concepts (Robinson and Treitel (1969); Makhoul (1978)). The WHL filter coefficients are obtained as a result of function fit between z_k (the desired output) and y_k (the real output) in the least-square sense. The object function is the expectancy of the deviation expressed as:

$$e(h_j) = E(z_k - y_k)^2, \tag{1}$$

to be minimized with respect to the filter coefficients h_j, which means search for a variance minimum, and $Ez_k - y_k$. The real filter output is represented by the convolution of the filter operator with the observed input series according to the equation:

$$y_k = \sum_{i=0}^{P-1} = h_i \; g_{k-i}, \tag{2}$$

The theoretical operation E. is supposed to make the randomness over the variance estimation disappear and, consequently, the function $e(h_j)$ becomes non-random, and differential and integral operations are applicable. The problem is of linear estimation, and the minimization criterion is that the partial derivatives with respect to the coefficients h_j be null:

$$\frac{\partial e(h_j)}{\partial h_j} = 0 \tag{3}$$

The above mathematical operation results in the linear system of normal equations:

$$\sum_{i=0}^{P} -1 h_i \phi_{gg}(j-i) = \phi_{gg}(l+T). \tag{4}$$

This is the WHL equation in discrete form, and the solution determines the h_i coefficients that minimizes the error-function, whose value $e(h_j)$ can be calculated. $\phi_{z_g}(i)$ is the unilateral positive part of the hypothetical stochastic crosscorrelation function between the input and desired output series. The principles behind the WHL theory allows establish several single-channel and multi-channel operations based on a priori information over z_k. The present case corresponds to prediction, where the desired output is defined as $z_k = g_{k+T}$, which means that z_k is a prediction of g_k at a distance T, and as a result:

$$\sum_{k=0}^{N-1} h_k \phi_{gg}(l-k) = \phi_{gg}(l+T). \tag{5}$$

h_k is denominated as error operator, and h_k* as prediction-error operator expressed by:

$$h* = 1, 0, 0, ..., 0, 0, -h_0, -h_1, -h_2, ..., -h_{N-1}. \tag{6}$$

The WHLP filter is to be applied under the principle of temporal periodicity (period T) between the primary and its multiples. This is the case of the zero offset sections (horizontal-, dipping-plane and curved interfaces), but not for any other configuration (common-source, common-receiver, mid-point). The new extension of the conventional WHLP method allows models with plane-dipping and curved interfaces. The multiple attenuation is performed in common-data section, and the operator is calculated with the real amplitudes of the signal, what makes the operator independent of dimensions and units of the observation. In the present strategy, the normal WHL equations are modified for the prediction operator be calculated and applied to the information limited to a time window, with lower border given by $W_1(x_m, h; T_{hyp})$ and upper border given by $W_2(x_m, h; T_{hyp})$, and calculated by the model function at $T_{hyp}(t = T_1)$ and $T_{hyp}(t = T_2)$, for the multiple coverage cube. $W_1(x_m, h; T_{hyp})$ and $W_2(x_m, h; T_{hyp})$ shift together in time-space domain, based on a travel-time law, which is a function here expressed as:

$$T_{hyp} = T_{hyp}(x_m, h; T_0, K_n, K_{nip}, \beta_0, V_0) \tag{7}$$

where (x_m, h) are the independent space variables, (K_n, K_{nip}, β_0) are a priori conditions, and are the wave attributes. The function T_{hyp} serves to introduce the necessary periodicity between the primary and its multiple. Since the attenuation is performed in a common-source configuration, with help of the wave front attributes outputted by the CRS stack, the WHL equation for prediction is modified such that for each pair of positions (x_m, h) a prediction operator is calculated with the information windowed by W_1 and W_2 by the equation:

$$\sum_{k=0}^{N-1} h_k \phi_{gg}(l-k; x_m, h, T_{hyp}) = \phi(l+T; x_m, h, T_{hyp}), W_1 < l < W_2 \tag{8}$$

and the computed operator is applied in the same window. Under the conditions of periodicity between the primary and its multiple, we adopted the relation:

$$W_2(x_m, h; T_{hyp}) - W_1(x_m, h; T_{hyp}) = 2T + 2C \tag{9}$$

to define the width of the time window over the correlation functions, where *T* is the periodicity, is the pulse length. The form of application of the WHLP-CRS filter is explained with the help of the flow chart of following Figure 1.

T_{HYP} FROM PARAXIAL RAY THEORY

Continuing the above description, we now present the equation for the adopted travel time T_{hyp}. The basic formula for the CRS-stack rests on the paraxial ray theory, and it is attractive because it does not present a strong restriction with respect to curved interfaces, and with respect to the velocity model. (Mann (2002); H. Trappe and Pruessmann (2001)). The 2D model used considers a flat observation surface, and a subsurface model formed by homogeneous, isotropic layers bounded by curved interfaces. The result for the double travel time around a central ray, three cinematic attributes for the hypothetical NIP and N waves are used in the CRS stack process, they are related to the normal ray emergent at point $x_m = x_0$, and they are: the emergence angle (β_0) of the observed wave; the curvature K_{NIP} (radius of curvature $R_{NIP} = 1/KNIP$) of the NIP wave, the curvature K_N (radius of curvature $R_N = 1/KN$) of the N wave. Therefore, these waves are interpreted as circular wave fronts on 2D models. For a complex medium (heterogeneous, but still isotropic), the interpretation of the wave front attributes of the NIP and N waves are not direct and intuitive as for a simple medium, but continues associated to an integrated effect of the stratified medium over the emergence and curvatures. The adopted expression for the primary reflection double travel time approximation was the hyperbolic, instead of the parabolic approximation. For an arbitrary configuration:

$$t^2(x_m, h) = (t_0 + \frac{2\sin\beta 0}{v_0}(x_m - x_0)^2 + \frac{2t_0 cos^2\beta_0}{v_0}(\frac{(x_m - x_0)^2}{R_N} + \frac{h^2}{R_{NIP}}) \tag{10}$$

where t_0 is the double travel time along the reference central ray in the zero offset, and v_0 is a velocity around the observation point x_0 (but, it is the velocity for the model layer above the actual reflector). The relation between the coordinates of the survey geometry is given by $x_m = (x_G + x_S)/2$ and $x_m = (x_G - x_S)/2$ for the half-offset (h) and mid-point (x_m). The quantity and are the horizontal coordinates of the source and receptor, respectively. The time function has as a priori knowledge, and it is taken as independent of the macro- or iso-velocity model. The image point is denoted by $P_0 = (x_0, t_0)$ to simulate the zero offset section, once the three attributes (β_0, R_{NIP}, R_N) are known. For each point $P_0 = (t_0, x_0)$, the CRS operator is to stack the events contained in the stack surface of the multi-coverage data. The process of obtaining the three attributes can be described as curve fitting in the least-squares sense, where the curves are represented by the observed data and the forward model (stack operator), the object function is the semblance cube, and methods of non-linear optimization are applied for local and global search. For non-linear curve fitting, it is necessary a starting point, in order to proceed on the parameter update. For CRS stack, the starting point is obtained by the simplification of the stack operator expression, where a first condition is $R_N = R_{NIP}$, which constrains the attributes to a diffraction point in subsurface, and the operator is simplified to:

$$t^2(x_m,h) = (t_0 + \frac{2sin\beta_0}{v_0}(x_m - x_0))^2 + \frac{2t_0cos^2\beta_0}{v_0R_{NIP}}((x_m - x_0)^2 + h^2) \tag{11}$$

A second condition is defined by $x_m = x_0$, the mid-point coincides with the reference point, and the operator is limited to two attributes β_0 e R_{NIP}:

$$t^2(h) = t_0^2 + \frac{2t_0cos^2\beta_0}{v_0}\frac{h^2}{R_{NIP}} \tag{12}$$

Setting $q = \frac{cos^2\beta_0}{R_{NIP}}$, $v_{MNO} = \frac{v_0}{cos\beta_0}$, the hyperbolic approximation corresponds to the NMO case.

NUMERICAL RESULTS

Figure 2 shows a schematic representation section of the Solimões sedimentarybasin (Amazon area), where diabase sills have an important presence. The geological knowledge of this basin serves to justify the construction of a synthetic model formed by 4 homogeneous and isotropic layers over a half-space, with velocities that vary from 1750 m/s to 4150 m/s (Figure 3). A high velocity layer (6300 m/s) is used to represent a diabase sill with high contrast with respect to the neighbor layers. The common-source sections were generated by the program SEIS88 (Psencisk and Molokov (2088)) to obtain seismograms and rays. There were generated 201 common-source sections, each with 72 traces and 50m interval between receptors and consecutive shots. The seismic pulse was represented by the Gabor function with sampling interval of 4 ms. The data contains primary reflections associated with each interface, and a multiple reflection for the high velocity layer (Alves (2003)). To estimate the wave front attributes (β, R_{NIP}, R_N) (panels of Figure 6), and simulate a zero offset section (Figure 7), we used the computer program of (Garabito, 2001).

The efficiency of the WHLP-CRS operator is directly related to the resolution of the CRS attributes, to the autocorrelation of the event selected for attenuation (primary and its multiple), and to the space-time window of validity of the operator. Figure 4 shows the correspondent common-source section number 40 without additive noise, before and after the application of the WHLP-CRS operator, where the blue line indicates the primary and the red line the multiple. Figure 5 shows the same Figure 4 with additive noise. We observe that the primary and the multiple have amplitude values close to the additive noise, and even this way the WHLP-CRS operator efficiently attenuates the multiple, as shows the zero offset section of Figure 7.

CONCLUSIONS

The WHL prediction operator is only applicable in time domain zero offset sections, where normal incidence allows echoes to have periodicity between primaries and multiples, which is particularly simple for models with horizontal plane interfaces. Time-space moving windows allows the extension of the WHL prediction operator to other configurations, as long as there is a mathematical law that relates the primary and its multiple. For the present case, we simply extended the WHL to common source configuration, based on paraxial ray laws by using the CRS attributes. Therefore, the WHL method is extended to handle plane-dipping and curved interfaces. The extended WHLP-CRS operator is typically data driven, and depend on the capacity to find a mathematical relation between primaries and multiples. Difficulties are related to conflicts between primaries and multiples, and to the capacity to isolate the desired event in the autocorrelation function.

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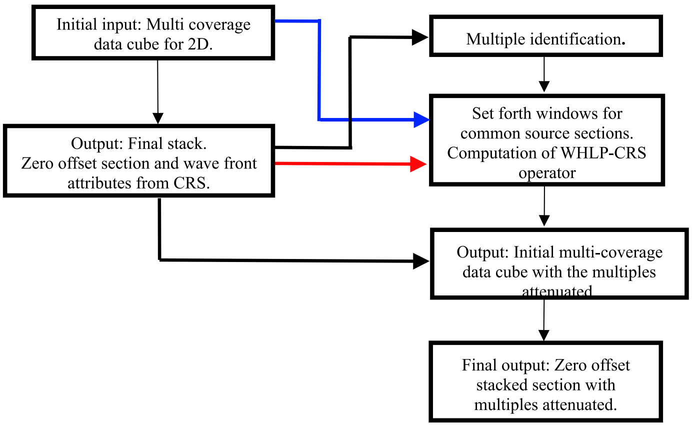

Figure 1: Flowchart for processing multiple attenuation with the WHLP-CRS operator.

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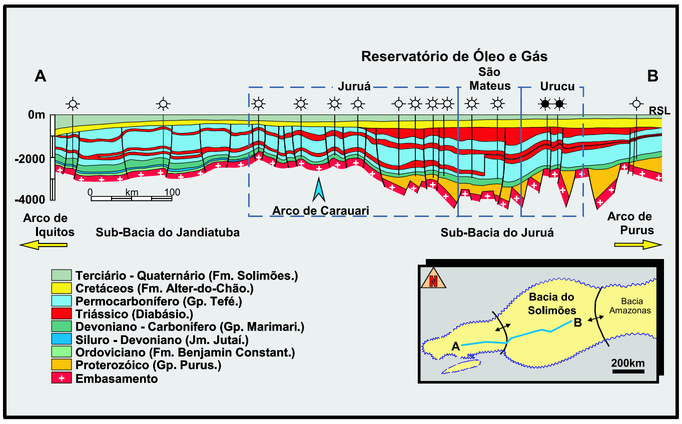

After Eiras, 1996

Figure 2: Geological section of the Solimôes sedimentary basin used for seismic simulations. The red zones represent mapped diabase sills.

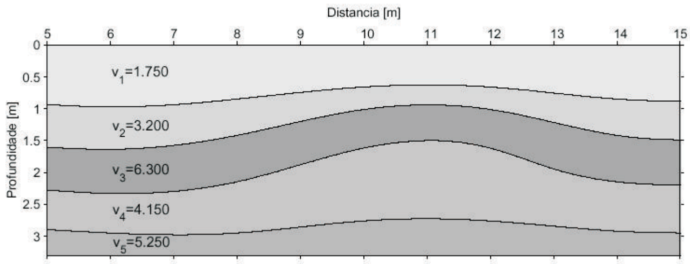

Figure 3: Synthetic subsurface model formed by 4 layers over a half-space, with velocities that vary from 1750 to 4150 m/s. The high velocity layer (6300 m/s) represents a diabase sill. The synthetic data were generated by the computer program SEIS88. There were computed 201 common-source sections, each with 72 traces, and 50m interval between receptors and consecutive shots. The effective seismic source signal used was a Gabor function with 4 ms sampling interval. The data contains primary reflections associated to each interface, and a multiple reflection relative to the high velocity layer.

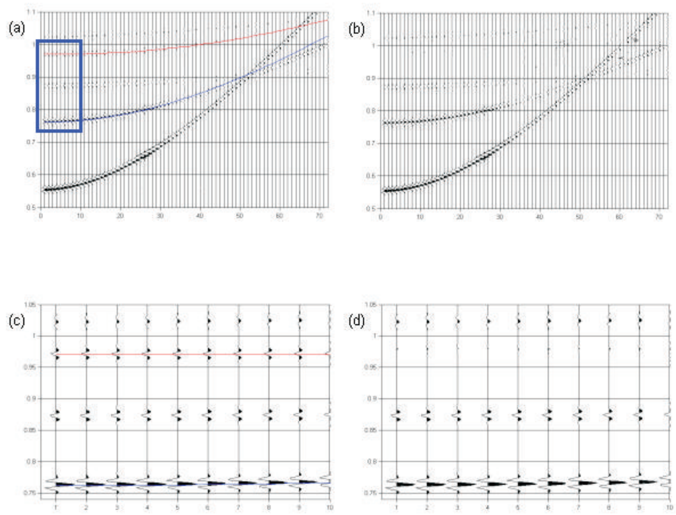

Figure 4: (a) Common-source section number 40 without additive noise, and a multiple present. The blue line marks the primary, and the red line the multiple. (b) Section 40 with the multiple attenuated. (c) Zoom over the area marked by the blue line in 'a', where the red line indicates the multiple. (d) Zoom over the area to show details of the WHLP-CRS operator output, where the multiple has been attenuated.

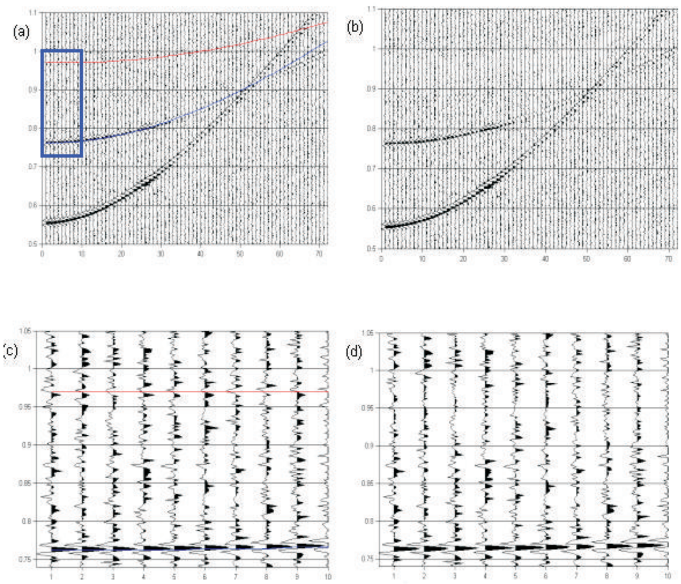

Figure 5: (a) Common-source section number 40 with gain, additive noise, and a multiple present. The blue line marks the primary, and the red line the multiple. (b) Section 40 with the multiple attenuated. The multiple has not a good visualization in this panel, but shows up clearly in the zero offset CRS simulated. (c) Zoom over the area marked by the blue line in 'a', where the red line indicates the multiple. (d) Zoom over the area to show details of the WHLP-CRS operator output, where the multiple is attenuated.

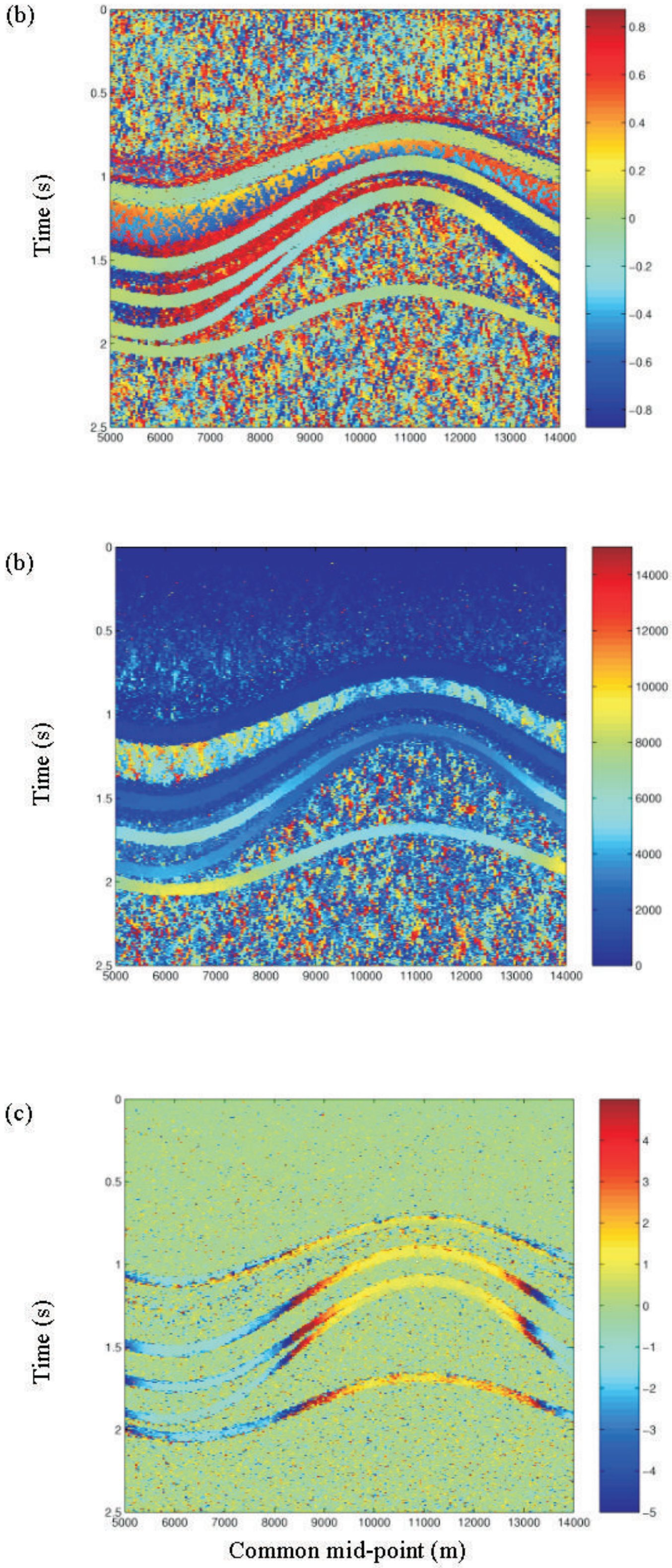

Figure 6: Panels of wavefront attributes (a) β_0 , (b) R_{NIP}, and (c) R_N used by the CRS stack operator CRS to simulate the zero offset sections to follow.

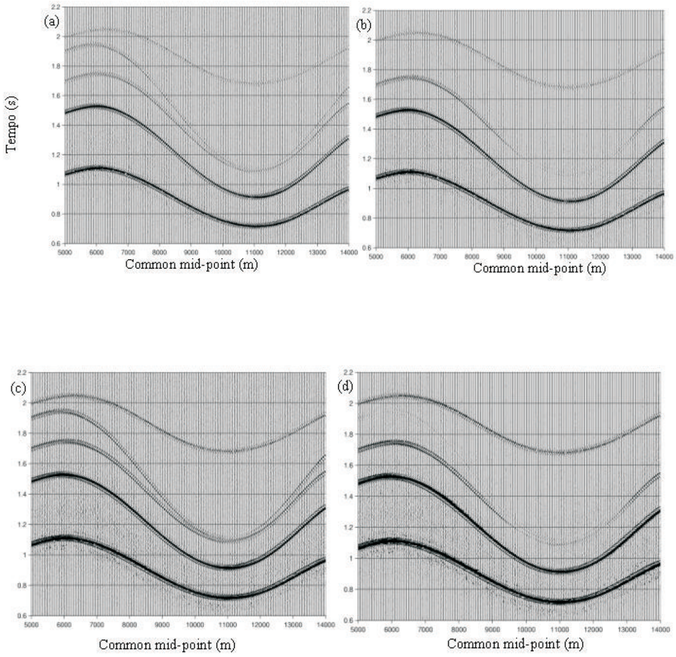

Figure 7: CRS stack zero offset time sections with additive noise. (a) Section without gain and multiple present. (b) Section without gain and the multiple attenuated. (c) Section with gain and multiple present. (d) Section with gain and multiple attenuated. We observe the good attenuation result, and that the residue left in the process is small, as shown in the section with gain. In the area where the primary and its multiple are too close, the WHLP-CRS operator attenuates the primary and its multiple together.