Pore pressure dependency of elastic anisotropy in rocks.

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ABSTRACT

In this work we present the extension of the piezosensitivity approach to anisotropic media. The theoretical considerations show that the stress dependence of the seismic velocities and of all elastic parameters depends mainly on one parameter. This parameter is equal for all velocities and elastic parameters of a rock in all directions and independent from the differential stress.
We present first results from the application of the piezosensitivity approach for anisotropic media to a set of ten metamorphic rock samples from the pilot hole of the German Continental Deep Drilling Project. The laboratory data, three P- and six corresponding S-wave velocity measurements per sample, cover a differential pressure range up to 600 MPa. All velocities can be fitted well with our model. As derived from our theoretical considerations the parameter D is constant for all observed velocities of a sample in any direction. Its magnitude ranges from approx. 0.01 to 0.05 per MPa. A comparison with data from the literature shows that D ranges in sandstones in general from 0.1 to 0.3 per MPa. Furthermore, we suggest a strategy for the non-linear least squares fitting process.

INTRODUCTION

Stress dependences of seismic velocities are important for interpretation of very different seismic data, ranging from AVO and velocity analysis to overpressure prediction and 4D seismic monitoring of reservoirs. Some times, rather complex forms of these dependences based on specific models of porous space geometry are used. For example, spherical contact models (Duffy and Mindlin, 1957 and Merkel et al, 2001) or crack contacts models (Gangi and Carlson, 1996) have been used in different studies. However, usually, the pore pressure velocity dependence along with the velocity dependence on the differential stress is phenomenologically described by the following simple relationship (Zimmerman et al, 1986; Eberhart-Phillips et al., 1989; Freund, 1992; Jones, 1992; Prasad and Manghnani, 1997; Kirstetter and MacBeth, 2001):

$$V(P) = A + KP - B\exp(-PD), \tag{1}$$

where $P = P_c - P_p$ is the differential stress, $P_c = -\sigma_{ii}/3$ is a confining pressure, σ_{ij} is a component of the total stress tensor (here, the compression stress is negative and the summation over repeating indices is assumed) and P_p is a pore pressure. The coefficients A, K, B and D of equation (1) are fitting parameters for a given set of measurements.

It is often observed that equation (1) or similar equations describing an exponential saturation to a linear trend provide very good approximations for velocities and elastic moduli of dry as well as saturated rocks. Moreover, it is also observed that this equation provides a very good approximation for elastic properties of anisotropic rocks.

In our previous publication we considered this equation for isotropic rocks. In this paper we show how equation (1) can be derived from a rather general consideration even in the case of anisotropic rocks. Under several, quite natural assumptions the stress dependences of the stiff and compliant porosities can be found from the theory of poroelasticity. These results can then be used to derive the seismic velocities as functions of the differential stress. Our derivation will clarify the physical meaning of quantities A, K, B and D which is quite similar in the isotropic and anisotropic cases.

DIFFERENTIAL STRESS AS A CONTROLLING FACTOR

For simplicity we consider a hydrostatic change of a stress state in a porous anisotropic rock. This means that the pore pressure as well as the confining stress acting on the outer boundaries of the rock can be changed. However, changes of the confining stress can be hydrostatic only. In a reservoir, such a hydrostatic change of the state of stress could be induced by a pumping test or an injection test.

Let us introduce compressibilities of an anisotropic porous body. Following the classical paper of Brown and Korringa (1975), there are 3 independent compressibilities characterizing changes of the complete body volume and of the volume of the pore space in this body:

$$C_{dr} = -\frac{1}{V}(\frac{\partial V}{\partial P})_{P_p}, \tag{2}$$

$$C_{mt} = -\frac{1}{V}(\frac{\partial V}{\partial P_p})_{P}, \tag{3}$$

$$C_{p} = -\frac{1}{V_p}(\frac{\partial V_p}{\partial P_p})_{P}, \tag{4}$$

where V is the volume of the porous body and V_p is the volume of all its connected pores.

One more but not-independent compressibility can be introduced:

$$C' = -\frac{1}{V}(\frac{\partial V_p}{\partial P})_{P_p}. \tag{5}$$

Using the reciprocity theorem Brown and Korringa (1975) showed that

$$C' = C_{dr} - C_{mt}. \tag{6}$$

A hydrostatic load introduces changes of the confining and pore pressures, δP_c and δP_p respectively. Also the differential pressure will be changed: $\delta P = \delta P_c - \delta P_p$. The volume change of a porous body will result from a volume change due to δP_d by keeping a constant pore pressure plus an effect of applying δP_p from inside and outside (i.e., $P = const.$):

$$\delta V = (\frac{\partial V}{\partial P})_{P_p}\delta P + (\frac{\partial V}{\partial P_p})_{P}\delta P_p. \tag{7}$$

An analogous equation is valid for the volume of the connected porosity:

$$\delta V_p = (\frac{\partial V_p}{\partial P})_{P_p}\delta P + (\frac{\partial V_p}{\partial P_p})_{P}\delta P_p. \tag{8}$$

The porosity changes correspond to the following rules:

$$\delta\phi \equiv \delta(\frac{V_p}{V}) = \frac{\delta V_p}{V} - \phi\frac{\delta V}{V}. \tag{9}$$

Taking into account these three equations and the above definitions of the compressibilities we obtain the following differential equation for porosity changes:

$$d\phi = (C_{mt} + (\phi - 1)C_{dr})dP + \phi(C_{mt} - C_p)dP_p. \tag{10}$$

We see that if $C_{mt} = C_p$ (this assumption is consistent with the Gassman's equation) and/or the connected porosity is very small then the porosity depends on the differential pressure only (see Zimmerman, et al., 1986; Detournay and Cheng, 1993; Goulty, 1998; and Gurevich 2002).

The differential equation for the porosity is then reduced to those one derived by Zimmerman, et al., (1986); and Detournay and Cheng, (1993):

$$\frac{d\phi}{dP} = C_{mt} - (1-\phi)C_{dr}. \tag{11}$$

The compressibilities C_{mt} and C_p are practically independent of P. Thus, in the equations (10) and (11) two quantities are significantly stress dependent only: ϕ and C_{dr}. Therefore, in order to obtain stress dependencies of these two quantities at least one more equation relating them to the stress or just an equation mutually relating ϕ and C_{dr} is required.

COMPRESSIBILITIES OF AN ANISOTROPIC MEDIUM

The compressibility C_{dr} characterizes the drained skeleton of the rock. Let us assume that the skeleton under a reference stress state is a generally anisotropic medium characterized by the compliances tensor with components S_{ijkl}. Taking into account the Hook's law,

$$e_{ij} = S_{ijkl}\sigma_{kl}, \tag{12}$$

and applying it to the dilatation

$$\frac{\delta V}{V} = e_{ii} \tag{13}$$

occurring due to the confining stress $P_c\delta_{ij}$ we arrive at the following equation for the skeleton's compressibility:

$$C_{dr} = S_{1111} + S_{2222} + S_{3333} + 2(S_{1122} + S_{1133} + S_{2233}) \equiv S_{iikk}. \tag{14}$$

COMPLIANCES VERSUS STIFF AND COMPLIANT POROSITIES

We separate the total porosity ϕ into two parts

$$\phi = \phi_c + [\phi_{s0} + \phi_s], \tag{15}$$

where the first part, ϕ_c, is a compliant porosity supported by thin cracks and grain contacts vicinities. According to laboratory observations we expect that the compliant porosity will close up by a differential stress of a few hundred megapascals. This corresponds to voids with an aspect ratio γ (a relationship between the minimal and maximal dimensions of a pore) less than $0,01$ (see Zimmerman et al., 1986). The second part, $[\phi_{s0} + \phi_s]$ is a stiff porosity supported by more or less isometric pores (i.e., equidimensional or equant pores, see also Hudson et al. (2001); Thomsen (1995)). The aspect ratio of such pores is typically larger than 0.1. Such a subdivision of the porosity to a compliant and stiff parts is very similar to the definitions of stiff and soft porosity by Mavko and Jizba (1991).

In turn, we separate the stiff porosity into a part ϕ_{s0}, which is equal to the stiff porosity in the case of $P = 0$, and to a part ϕ_s which is a change of the stiff porosity due to a deviation of the differential stress from zero. We assume that the relative changes of the stiff porosity, ϕ_s/ϕ_{s0}, are small. In contrast, the relative changes of the compliant porosity $(\phi_c - \phi_{c0})/\phi_{c0}$ can be very large, i.e., of the order of 1 (ϕ_{c0} denotes the compliant porosity in the case of $P = 0$). Note, however, that ϕ_c and ϕ_{c0} are usually very small quantities. As a rule, (e.g., in porous sandstones) they are much smaller than ϕ_{s0} and even than the absolute value of ϕ_s. Thus, the following inequality is usually valid:

$$\phi_{s0} \gg |\phi_s| \gg \phi_c. \tag{16}$$

For example, in porous sandstones typical orders of magnitude of these quantities are $\phi_{s0} = 0.1$, $|\phi_s| = 0.01$ and $\phi_c = 0.001$.

Under such circumstances it is logic to assume the first, linear approximations of the compliances as functions of the porosities. The Taylor expansion gives:

$$S_{ijkl}(\phi_{s0} + \phi_s, \phi_c) = S_{0ijkl} + \theta_{sijkl}\phi_s + \theta_{cijkl}\phi_c, \tag{17}$$

where S_{0ijkl} is the drained compliancy of a hypothetical rock with a closed compliant porosity (i.e., $\phi_c = 0$) and the stiff porosity equal to ϕ_{s0}. Further,

$$\theta_{sijkl} = \frac{\partial S_{ijkl}}{\partial \phi_s}, \quad \theta_{cijkl} = \frac{\partial S_{ijkl}}{\partial \phi_c}, \tag{18}$$

where the derivatives are taken in points $\phi_s = 0$ and $\phi_c = 0$, respectively.

Approximation (17) implies that the both quantities $\theta_{sijkl}\phi_s/S_{0ijkl}$ and $\theta_{cijkl}\phi_c/S_{0ijkl}$ are smaller than 1. Numerous laboratory experiments and practical experience show that the drained compressibility depends strongly on changes in the compliant porosity, and it depends much weaker on changes in the stiff porosity. We will express this empirical observation by the restriction

$$|\theta_{sijkl}\phi_s| \ll |\theta_{cijkl}\phi_c|. \tag{19}$$

In spite of a very small porosity ϕ_c the quantity $\theta_{cijkl}\phi_c/S_{0ijkl}$ can be of the order of 0.1 or even larger. If so, approximation (17) is further simplified as follows:

$$C_{dr}(\phi_{s0} + \phi_s, \phi_c) = C_{drs}\left[1 + \theta_c\phi_c\right]. \tag{20}$$

Here, we introduced

$$C_{drs} \equiv S_{0iikk} \tag{21}$$

and

$$\theta_c \equiv \theta_{ciikk}/S_{0iikk} \tag{22}$$

Using approximation 20 and neglecting ϕ in comparison with 1 we obtain the following relationship instead of equation (11):

$$d\phi_s + d\phi_c = (C_{mt} - C_{drs} - \theta_c\phi_c C_{drs})dP + (\phi_c + \phi_{s0} + \phi_s)(C_{mt} - C_p)dP_p. \tag{23}$$

Again, if the porosity and/or pore pressure are small, the last term in this equation can be neglected.

STRESS DEPENDENCES OF THE STIFF AND COMPLIANT POROSITIES

We assume that stiff porosity changes with stress are independent of the changes of the compliant porosity. This means also, that changes of the stiff porosity are independent of the fact if the compliant porosity is closed or not. If the compliant porosity is closed then $\phi_c = 0$ and we obtain from (23)

$$d\phi_s = (C_{mt} - C_{drs})dP + (\phi_{s0} + \phi_s)(C_{mt} - C_p)dP_p. \tag{24}$$

However, if the assumption above is valid then this relationship will be valid also for an arbitrary (however, because of other assumptions, small) ϕ_c. Therefore,

$$d\phi_c = -\theta_c\phi_c C_{drs}dP + \phi_c(C_{mt} - C_p)dP_p. \tag{25}$$

These two equations immediately provide us with the following approximations of the stress dependences of the stiff and compliant porosities:

$$\phi_s = (C_{mt} - C_{drs})P + \phi_{s0}(C_{mt} - C_p)P_p \tag{26}$$

(here we neglected ϕ_s in comparison with ϕ_{s0}),

$$\phi_c = \phi_{c0}\exp\left(-\theta_c P C_{drs} + (C_{mt} - C_p)dP_p\right). \tag{27}$$

Note that equation (26) is not valid for very large P because in equation (20) we neglected the stiff-porosity dependence of the compressibility C_{drs}, which becomes equal to C_{mt} if $P \longrightarrow \infty$. The validity of such a simplification as well as the validity of equation (26) are restricted by the condition (19). For very high stresses also the stiff porosity will obey an exponentially saturating decreasing behavior.

STRESS DEPENDENCES OF ELASTIC PROPERTIES

Let us now consider an arbitrary elastic characteristic E (e.g., a seismic velocity, a stiffness or a compliance) of a porous body. We will assume that the grain material is isotropic and homogeneous. The first consequence of this assumption is that in equation (23) the last term becomes zero due to the equivalence $C_{mt} = C_p$. Correspondingly, equations (26) and (27) are simplified.

Another consequence of the isotropy and homogeneity of the matrix material is that the elastic anisotropy of the porous body is only due to the geometry of the connected porosity. This geometry does not change under an isotropic homogeneous change of load. However, the elastic characteristic E will be a function of the porosity. We assume that the characteristic E can be approximated by a Taylor series expansion at the point $\phi = \phi_{s0}$ relative to the stiff and the compliant porosity (this should be valid for all such characteristics like seismic velocities and elastic moduli):

$$E(\phi_{s0} + \phi_s, \phi_c) = E_0 + \theta_{sE}\phi_s + \theta_{cE}\phi_c, \tag{28}$$

where we neglected higher order terms. Furthermore,

$$\theta_{sE} = \frac{\partial E}{\partial \phi_s}, \quad \theta_{cE} = \frac{\partial E}{\partial \phi_c}, \tag{29}$$

with the derivatives taken at $\phi_s = 0$ and $\phi_c = 0$, respectively. By substituting equation (26) and (27) into equation (28) we obtain:

$$E(P) = E(0) - \theta_{sE}(C_{drs} - C_{gr})P + \theta_{cE}\phi_{c0}\exp(-\theta_c P C_{drs}). \tag{30}$$

In the case of the drained compressibility, substituting equations (26) and (27) into equation (17) gives:

$$C_{dr}(P) = C_{drs}\left[1 - \theta_s(C_{drs} - C_{gr})P + \theta_c\phi_{c0}\exp(-\theta_c P C_{drs})\right]. \tag{31}$$

In terms of the compliances this yields:

$$S_{ijkl}(P) = S_{0ijkl} + \theta_{sijkl}(C_{drs} - C_{gr})P - \theta_{cijkl}\phi_{c0}\exp(-\theta_c P C_{drs}). \tag{32}$$

A comparison of these results with equation (1) shows that they all have the same form: $A + KP - B\exp(-PD)$. Moreover, we should expect that for dry as well as for saturated rocks in the first approximation all coefficients D are identic:

$$D_{ijkl} = D. \tag{33}$$

We call the non-dimensional quantity θ_c the piezosensitivity.

APPLICATION

We applied the piezosensitivity approach to a set of laboratory data measured at the Mineralogical-Geological Institut of the University of Kiel, Germany. In a cubic pressure cell three P- and six corresponding S-wave velocities were measured in three orthogonal directions at different hydrostatic stress levels ranging from 25 MPa up to 600 MPa. All measurement were conducted on dry rock samples. These rocks were sampled in the pilot hole of the German Continental Deep Drilling Project (KTB). Approximately 90 % of the rock mass penetrated by the pilot hole consist of gneisses. Therefore, the cubic samples were cut from the cores with respect to their macroscopically dominant texture in a way that the bedding plane of the phyllosillicates is oriented perpendicular to one of the measurement axes. Figure 1 illustrates the spatial orientation of the samples and the symmetry plane with respect to the measured velocities. Here, for a velocity V_{ij} index i denotes the direction of propagation and index j the direction of particle motion. Note, the used notation here with axis 1 as the symmetry axis does not agree with the notation usually used when dealing with VTI, where axes 3 denotes the symmetry axis.

In a first, quite natural assumption, one can treat the gneisses as intrinsically transversal isotropic (TI) media due to the preferred orientation of the phyllosillicates. In such a TI medium, the velocities of waves propagating within the symmetry plane are independent from the direction of propagation. Furthermore, waves being polarized within the symmetry plane should also show the same velocities. If the symmetry

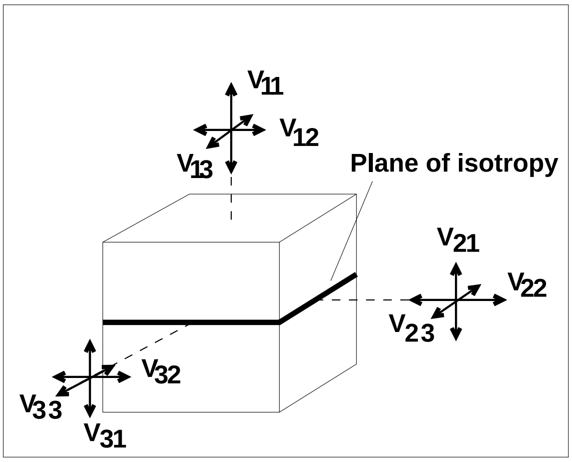


Figure 1: Illustration of the sample geometry and the orientation and notation of the measured velocities.

axes corresponds to the 1 direction, $V_{12} = V_{13} = V_{21} = V_{31}$, $V_{23} = V_{32}$ and $V_{22} = V_{33}$. To satisfy these theoretical characteristics of a TI medium, we averaged the measured velocities according to the mentioned scheme.

As a consequence of the orthogonally aligned velocity measurements, we are limited to four independent velocities. Therefore, we are not able to determine the complete compliance tensor of a TI medium since such a tensor consists of five independent parameters.

We perform the fitting of equation 1 separately to every of the four velocity vs. pressure relations in terms of a least squares fit. The nonlinearity of equation 1 requires the application of an iterative fit procedure, like, e.g., the Levenberg-Marquardt algorithm.

The theoretical requirement of a constant parameter D imposes the necessity on the fitting process to be conducted as a two step process. In a first step, the velocity-pressure data for all directions are fitted separately using all four parameters A, K, B, and D. In other words, for n velocity stress relations one obtains the fitting parameters A_i, K_i, B_i, D_i, with $i = 1...n$.

In the next step, we calculate the mean (D_{mean}) of the quantity D_n. Then, the velocity-pressure relations are fitted again, but only A, K, and B remain as fitting parameters and D_{mean} is kept constant for all velocities.

In general, a widely used measure for the quality of the fitting process is the deviation δ_i of the fitted value y_f from the observation y_m at every point x_i, i.e.,

$$\delta_i = y_f(x_i) - y_m(x_i) \,, \tag{34}$$

and the sum of the squared deviations χ, according to

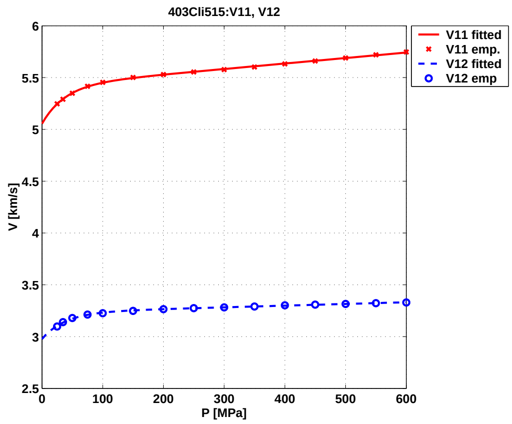
$$\chi = \sum_{i=1}^{n} \delta_i^2 \,. \tag{35}$$

The following example should illustrate the fitting process and gives results for this gneiss from the KTB pilot hole. Four velocities were fitted: V_{11}, V_{12}, V_{22}, and V_{23}. As indicated above V_{12} represents the mean of the measured velocities V_{12_m}, V_{13_m}, V_{21_m}, and V_{31_m}, and V_{22} the mean of V_{22_m} and V_{33_m}.

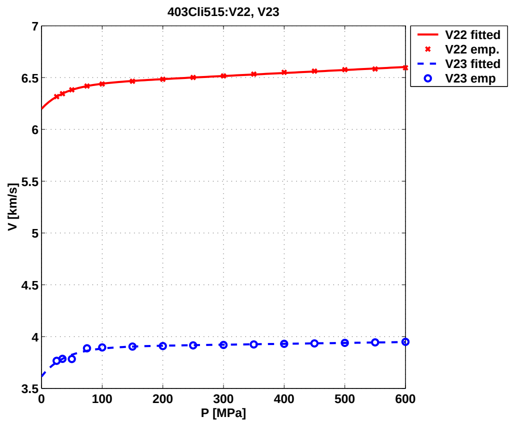
From the first fitting we obtained for A, K, B, and D the values listed in tabular 1. Obviously, the variation of the parameter D in the different directions is very small. Furthermore, the parameter K is much smaller than the other parameters, as shown by Shapiro (2002).

As mentioned above we calculate the mean of the different D values gives ($D_{mean} = 0.0262$), repeat the fitting and obtain the values listed in tab. 2.

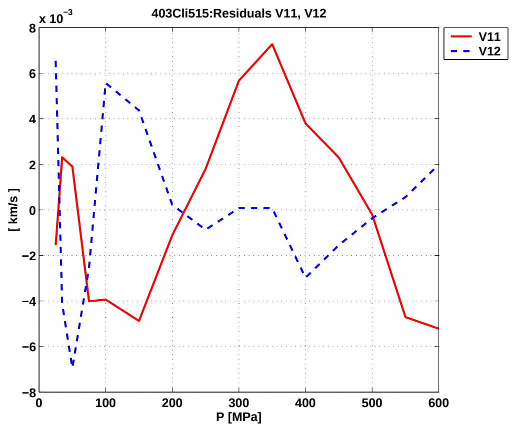
The results of the second fit do not vary significantly from the first fit. This could be expected since the deviations of the D values from the mean are very small. Figure 2 shows a comparison between the



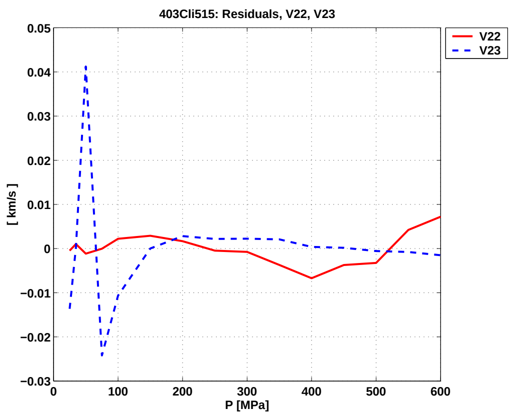
(a)



(b)



(c)



(d)

Figure 2: Comparison between measured (crosses and circles) and best-fit velocities (dashed and solid lines) (fig. 2(a) and fig. 2(a)) and the corresponding deviations (fig. 2(c) and 2(d)) as obtained from the second fit. The velocities are grouped as P- and S-wave pairs according to their direction of propagation. Fitted velocities match well the observed with a mean deviation less than 0.01 (2(c)).

Table 1: Resulting values of A, K, B, and D for the first fit. χ is the sum of the squared deviations.

Dir.	A $[km/s]$	K $[km/s/MPa]$	B $[km/s]$	D $[1/MPa]$	$\chi[(km/s)^2]$
V11	5.425333	0.000526	0.377304	0.026736	2.27e-04
V12	3.233694	0.000162	0.267389	0.027460	1.68e-04
V22	6.426458	0.000294	0.229628	0.026842	1.86e-04
V23	3.899291	0.000079	0.266897	0.023923	2.60e-03

Table 2: Second fit results with D kept fixed.

$D = D_{mean} = 0.0262[1/MPa]$				
Dir.	A $[km/s]$	K $[km/s/MPa]$	B $[km/s]$	$\chi[(km/s)^2]$
V11	5.423855	0.000530	0.369313	2.25e-04
V12	3.233357	0.000164	0.257072	1.81e-04
V22	6.428483	0.000290	0.230645	1.72e-04
V23	3.896205	0.000085	0.281312	2.60e-03

observed and best-fit velocities and the corresponding deviations at the observations points according to equation 34.

The same procedure was applied to nine additional data sets from the KTB pilot hole. For all data sets the best-fit velocities match the observations in a similar good manner as in the shown example. Furthermore, a similar constance of the fitting parameter D was found for all data sets. Our fitting results agree well with the theoretical prediction that the pressure dependence of elastic properties of porous and fractured rocks is, indeed, mainly controlled by one single parameter, D, for both, isotropic and anisotropic media.

In order to investigate weather the parameter D shows a characteristic magnitude for different rock types we plot Dp vs. Ds (the index denotes if a P- or a S-wave was fitted) as obtained from the first fitting step for the crystalline samples (circles) together with 80 results from sandstones (Eberhart-Phillips et al., 1989; Jones, 1995), indicated by crosses 3.

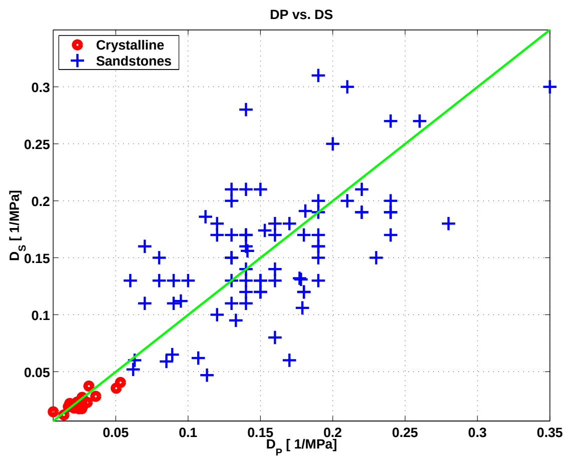


Figure 3: Dp versus Ds for crystalline rocks (circles) and sandstones (crosses). The solid line indicates $Dp = Ds$.

The distribution of the crystalline and sedimentary samples indicates a separation of both rock types at approx. 0.1 per MPa. Obviously, the theoretical requirement that Dp is equal to Ds seems to be more accomplished in crystalline rocks than in sedimentary. The strong difference between Dp and Ds for the sandstones can be caused by such factors like pore shape, grain size distribution, the degree of cementa-

Table 3: Resulting values of A, K, B, and D for the first fit. All nine measured velocities were fitted. Again, χ is the sum of the squared deviations.

Dir.	A $[km/s]$	K $[km/s/MPa]$	B $[km/s]$	D $[1/MPa]$	$\chi[(km/s)^2]$
V11	5.425333	0.000526	0.377304	0.026736	2.27e-4
V12	3.194678	0.000229	0.281762	0.043923	3.11e-4
V13	3.351793	0.000155	0.279077	0.025465	8.60e-5
V22	6.301271	0.000287	0.199504	0.020222	3.82e-4
V21	3.890744	0.000091	0.117543	0.025575	1.20e-5
V23	3.197202	0.000176	0.565104	0.044941	1.08e-4
V33	6.562397	0.000278	0.240769	0.027534	6.20e-5
V31	3.901571	0.000081	0.419581	0.024625	9.96e-3
V32	3.200580	0.000075	0.181381	0.011990	1.59e-4

Table 4: Second fit results with D kept fixed.

$D = D_{mean} = 0.027890[1/MPa]$				
Dir.	A $[km/s]$	K $[km/s/MPa]$	B $[km/s]$	χ
V11	5.420879	0.000536	0.381853	2.00e-4
V12	3.202880	0.000211	0.189312	5.10e-4
V13	3.348246	0.000163	0.293835	1.02e-4
V22	6.291994	0.000306	0.236985	5.33e-4
V21	3.889407	0.000094	0.123500	1.80e-5
V23	3.213761	0.000140	0.368053	9.21e-4
V33	6.562336	0.000278	0.243958	6.10e-5
V31	3.898130	0.000088	0.458373	1.02e-2
V32	3.160713	0.000158	0.216346	7.80e-4

tion and the loading/unloading history, which are less important for the low porosity metamorphic rocks. However, these interpretations have to be understood rather as a hint than a result since the amount of 10 crystalline samples does not represent a statistically representative base.

Since the piezosensitivity approach is valid for all isotropy classes we applied it also to the same data set shown before but without averaging the velocities ahead of the fit.

Tabular 3 shows the resulting parameters as obtained from the first fit step. In general, the numerical quality of both, the first and the second fit is as good as with averaged velocities. The first fit yields a mean χ of 1.30e-3 (its median is 1.5917e-04), the second fit a mean χ of 1.50e-3 and a median of 5.10e-04. The slight increase in χ is mainly caused by the relatively poor best-fit of V_{31} (tab. 3 and 4). In comparison to the fitting of the averaged velocities (tab. 1 and 2), the parameter D shows more distinct variations. However, in comparison to the sandstone fits (fig. 3) the variations of D are still very small and all observed velocities can be fitted successfully, as shown in figure 4. The mean value of D changes only slightly from 0.026 in the TI case to 0.028 when fitting all nine velocities. This clearly indicates that the piezosensitivity approach can deal with arbitrary symmetry classes.

CONCLUSIONS

In the first approximation elastic moduli, seismic velocities as well as the porosity depend on the differential stress, i.e., the difference between the confining pressure and the pore pressure only. The stress dependence of the porosity controls the elastic moduli and velocity changes with stress. Here, the most important property is the compliant porosity which is usually a very small part of the total porosity. The closure of the compliant porosity with increasing differential stress explains the experimentally observed exponentially saturating increase of seismic velocities. Coefficients of this relationship are defined by the compliant porosity dependence of the drained bulk modulus.

The dimensionless quantity θ_c defines the sensitivity of the elastic characteristics to the differential stress. We propose to call it the elastic piezosensitivity. The piezosensitivity is an important property of

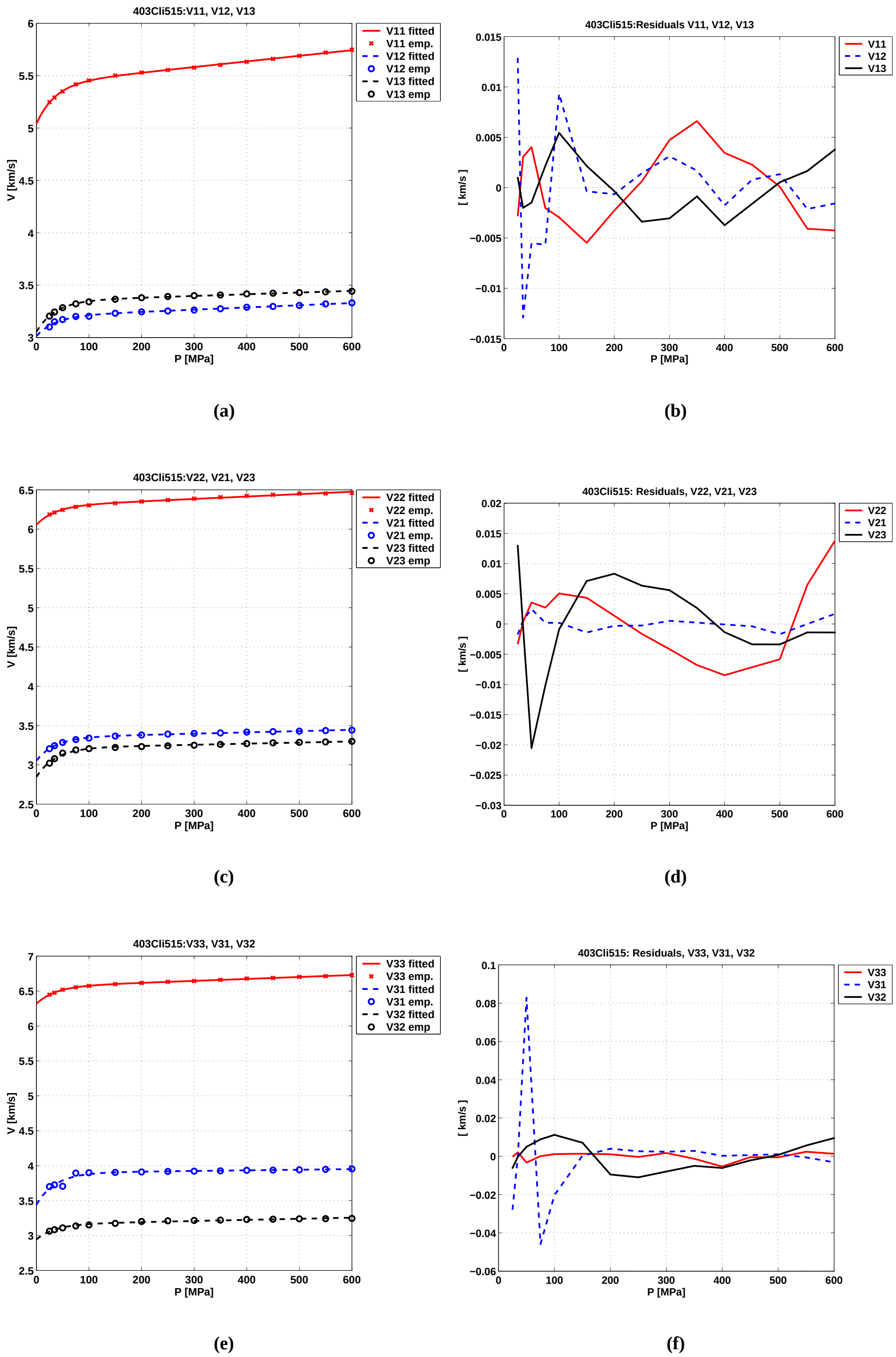


Figure 4: Comparison between measured (crosses and circles) and best-fit velocities (dashed and solid lines) (fig. 4(a), fig. 4(c), and fig. 4(e)) and the corresponding deviations (fig. 4(b), fig. 4(d) and 4(f)) as obtained from the second fit. All curves were fitted with $D = D_{mean} = 0.027890/MPa$.

rocks. From the derivation above it is clear that it is defined by the compliant porosity of rocks. Moreover, it is approximately proportional to an effective reciprocal aspect ratio of the compliant porosity.

The velocities of all 10 rock samples have been fitted successfully with a four parametric exponential equation. The four parameters A, K, B, and D represent combinations of different rock physical parameters. As derived from our theoretical considerations the parameter D is approx. constant for all observed velocities of a sample in any direction. Its magnitude ranges from approx. 0.01 to 0.05 per MPa. A comparison with data from the literature shows that D ranges in sandstones in general from 0.1 to 0.3 per MPa.

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REFERENCES

Brown, R. and Korringa, J. (1975). On the dependence of the elastic properties of a porous rock on the compressibility of the pore fluid. *Geophysics*, 40:608–616.

Detournay, E. and Cheng, A.-D. (1993). Fundamentals of poroelasticity. In Hudson, J., editor, *Comprehensive Rock Engeneering: Principles, Practice, and Projects.*, chapter 5. Pergamon Press, Oxford.

Duffy, J. and Mindlin, R. (1957). Stress-strain relations and vibrations of a granular medium. *Journal of Applied Mechanics*, 24:585–593.

Eberhart-Phillips, D., Han, D.-H., and Zoback, M. (1989). Empirical relationship among seismic velocity, effective pressure, and clay content in sandstones. *Geophysics*, 54(1):82–89.

Freund, D. (1992). Ultrasonic compressional and shear velocity in dry elastic rocks as a function of porosity, clay contant and confining pressure. *Geophys. J. Int.*, 108:125–135.

Gangi, A. F. and Carlson, R. (1996). An asperity-deformation model for effective pressure. *Tectonophysics*, 256:241–251.

Goulty, N. (1998). Relationship between porosity and effective stress in shales. *First Break*, 16:413–419.

Gurevich, B. (2001). Effective stress and elastic properties of rocks. *Geophysics, in preparation*.

Hudson, J., Pointer, T., and Liu, E. (2001). Effective-medium theories for fluid-saturated materials with aligned cracks. *Geophysical Prospecting*, 49:509–522.

Jones, S. (1995). Velocities and quality factors of sedimentary rocks at low and high effective pressure. *Geophys. J. Int.*, 123:774–780.

Kirstetter, O. and MacBeth, C. (2001). Compliance-based interpretation of dry frame pressure sensitivity in shallow marine sandstones. In *Expanded abstracts*, pages 2132–2135.

Mavko, G. and Jizba, D. (1991). Estimating grain-scale fluid effects on velocity dispersion in rocks. *Geophysics*, pages 1940–1949.

Merkel, R., Barree, R., and Towle, G. (2001). Seismic response of Golf of Mexico reservoir rocks with variations in pressure and water saturation. *The Leading Edge*, 20:290–299.

Prasad, M. and Manghnani, M. (1997). Effects of pore and differential presure on compressional wave velocity and quality factor in berea and michigan sandstones. *Geophysics*, pages 1163–1176.

Shapiro, S. (2002). Piezosensitivity of porous and fractured rocks. *accepted to Geophysics*.

Thomsen, L. (1995). Elastic anisotropy due to aligned cracks in porous media. *Geophys. Prosp.*, 45:805–829.

Zimmermann, R., Somerton, W., and King, M. (1986). Compressibility of porous rocks. *J. Geophys. Res.*, 91:12765–12777.