A traveltime computation in 3-D anisotropic media by a finite-difference perturbation method

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ABSTRACT

A combination of finite-difference (FD) traveltime algorithm and first-order perturbation theory is used for fast 3-D computation of traveltimes of P-waves in arbitrary anisotropic medium. An isotropic medium as a reference medium works well for weak anisotropy. Using media of ellipsoidal anisotropy as a background medium in the perturbation approach allows to consider stronger anisotropy without loosing the computational speed because the traveltime computation in such a medium using FD eikonal solver is fast and accurate. Traveltimes in the unperturbed reference medium are computed with an FD eikonal solver, while perturbed traveltimes are obtained by adding a traveltime correction to the traveltimes of the reference medium. To compute the traveltime correction, the raypaths between source and receivers in the reference medium must be known. The FD eikonal solver computes traveltimes on a discrete grid assuming local plane wavefronts inside the grid cells. Corrected rays are not determined in this method. Therefore, we suggest to approximate rays by ray segments corresponding to the plane wavefronts in each cell. We compute the traveltime correction along these segments. Numerical examples show that the reference model with ellipsoidal anisotropy allows to consider perturbed model of strong anisotropy with a higher accuracy of the FD perturbation method.

INTRODUCTION

Robust and efficient methods for the traveltime computation are important in many seismic and inversion applications. There are two major approaches which can be used for computing traveltimes: ray tracing methods and methods which are based on a direct numerical solution of the eikonal equation using finite-differences (e.g. Vidale (1988), Vidale (1990), Qin et al. (1992)). The ray tracing methods are complicated and time consuming when applied in anisotropic media because for each propagation step an eigenvalue problem must be solved. Similar to the isotropic case the ray methods in anisotropic media fail in shadow zones or in the vicinity of a caustic. In the isotropic case this problem disappears when using FD methods. FD eikonal solvers were extended to anisotropic media by Dellinger (1991), Eaton (1993), Lecomte (1993). Dellinger (1991) uses the upwind scheme of Van Trier and Symes (1991) for transversely isotropic media. Eaton (1993) applies an expanding-wavefront scheme on a hexagonal grid in 2-D anisotropic models. He approximates one component of the slowness vector using FD and finds the root of the sixth-order polynomial for the other component numerically. Lecomte (1993) applied the finite difference calculation of traveltimes for P-wave using the method of Podvin and Lecomte (1991) for a 2-D model with elliptical and orthorhombic symmetry.

The aim of our work is to compute traveltimes of P-wave in arbitrary anisotropic media without solving higher order polynomials numerically. Perturbation techniques are suitable tools to describe wave propagation in complicated media. We suggest to combine the perturbation method and the FD eikonal solver for the traveltime computation. Traveltime computation by perturbation with FD eikonal solvers in isotropic and weakly anisotropic media in 2-D was considered by Ettrich (1998). Here we examine the FD perturbation method in the 3-D case for media of strong anisotropy.

An isotropic medium as a reference medium works well for weak anisotropy. Using media of ellipsoidal anisotropy as a background medium in the perturbation approach allows to consider stronger anisotropy without loosing the computational speed because the traveltime computation in such media is fast and accurate using the FD eikonal solver . A basic routine for the FD eikonal solver for an elliptically anisotropic medium in the 3-D case was offered by Ettrich (1998). He tested his routine in an ellipsoidal medium with identical velocities along two direction (ellipsoid with the rotation symmetry). Our algorithm works in the ellipsoidal reference medium with three different velocities along axes. Therefore it is possible to use a broader class of ellipsoidal media as a reference medium.

To minimize errors in the perturbation approach, the reference medium should be chosen closely to the given anisotropic medium. For construction of a reference medium we use formulas for a best-fitting ellipsoidal reference medium which were derived by Ettrich et al. (2001). They obtained linear relations for the coefficients of the ellipsoidal medium that depend on the elastic coefficients of the anisotropic medium. In this study it is assumed that the polarization vector coincides with the phase normal vector. Therefore, only P-wave anisotropy is approximated.

FINITE-DIFFERENCE PERTURBATION METHOD

The conception of the 2-D FD perturbation method was suggested in a paper of Ettrich and Gajewski (1998). Our goal is the 3-D extension of this method and its implementation in the case of arbitrary anisotropic media (for P-waves). We consider a reference model (isotropic or elliptically anisotropic) and a perturbed arbitrary anisotropic model. For both models velocities or elastic parameters are calculated on the same regular grid. The reference medium is given by the P-wave velocity v_p^{ref} for the isotropic case or by 3×3 symmetric matrix R_{ij} for the elliptically anisotropic case. These parameters of the reference medium are determined from parameters of the considered perturbed medium. In this paper the perturbed medium is given by the density normalized elastic parameters c_{ijkl}. Traveltimes for the reference model are computed directly using the FD eikonal solver along expanding wavefront (Qin et al., 1992). For the perturbed model we perform the traveltime computation using perturbation techniques. The traveltime correction is calculated for every step of the FD scheme in every grid cell along the ray segment corresponding to the plane wave in this grid cell.

Reference medium

To minimize errors in the perturbation approach the reference medium should be chosen as close as possible to the given anisotropic medium. For the ellipsoidal reference medium construction we used formulas for a best-fitting ellipsoidal reference medium which were derived by Ettrich at al. (2001). They obtained linear relations for the coefficients of the ellipsoidal medium which depend on the elastic coefficients of the anisotropic medium. In this study it is assumed that the polarization vector of the wave coincides with the phase normal vector. Therefore, only P-wave anisotropy can be considered. To construct the isotropic reference model we use formulas for the best fitting isotropic medium derived by Fedorov (1968) by minimizing the norm of differences between elastic coefficients of the anisotropic and the isotropic medium.

FD scheme

This section will briefly outline finite-difference scheme. In WIT report 2000 (p. 239–248) we presented the algorithm of the FD eikonal solver for an 3-D elliptically anisotropic media in detail.

We developed a FD eikonal solver for the traveltime computations in the reference medium of ellipsoidal anisotropy Ellipsoidal anisotropy means three different velocities along the main axes. The eikonal equation for this type of medium reads:

$$(\mathbf{p}, \mathbf{R}\,\mathbf{p}) = 1, \tag{1}$$

where $\mathbf{p}$ is a slowness vector, $\mathbf{R}$ is a 3×3 symmetric matrix of parameters of the ellipsoidal reference medium, ( ) denotes scalar product. The eikonal equation allows for the computation of one slowness vector component if the other two and the medium parameters are known. In our algorithm the approximating formulas of the eikonal equation for an elliptically anisotropic medium are used. This formulas were suggested by Ettrich (1998). They are analogous to Vidale's approximating formulas of the eikonal equation for 3-D isotropic media (Vidale, 1990).

To retain causality and to guarantee stability we expand wavefronts (Qin et al. 1992). In anisotropic media two kinds of velocities need to be considered: the ray velocity and the phase velocity. The propagation of energy and, therefore, the causal continuation of computation is governed by the ray velocity vectors. In isotropic models where the group and the phase velocity vectors coincide, the causality is achieved by sorting the outer points of the irregular volume of timed points with respect to traveltime from minimum to maximum. In anisotropic media the group velocity vector and the phase velocity vector do not coincide. Therefore it is not sufficient to carry out the computation from the point with minimum traveltime to the point with maximum traveltime. For every step of the FD scheme we must compare the direction of the scheme of expansion with the direction of the group velocity. The traveltime in a given grid point has been successfully computed if the directions coincide, otherwise the point with minimum traveltime is not a point for a casual expansion and we have to consider the next timed point.

The phase velocity is determined from the eikonal equation, but the ray velocity in elliptically anisotropic media must be calculated from $\mathbf{v}_g = \mathbf{R}\,\mathbf{p}$.

Using a homogeneous model we tested the accuracy of the traveltime computation by the 3-D FD eikonal solver for an elliptically anisotropic medium. Figure 1 shows the numerically calculated wavefronts for the elliptically anisotropic models which are the best-fitting ellipsoidal reference medium for triclinic sandstone (*a*) and orthorhombic olivine (*b*). The parameters of the ellipsoidal media used are given in equations (4) and (6). Because it is 3-D model we give two slices: with zero (on the left-hand side) and non-zero offset (on the right-hand side). The underlying grayscale images show the relative errors.

We consider a reference and a perturbed model for which elastic parameters are defined on the same regular grid. Traveltimes must be calculated at all grid points for both models. We consider the arbitrary anisotropic medium as a perturbed one and the elliptically anisotropic medium as a reference one. The perturbed medium is given by the density normalized elasticity tensor λ_{ijkl}, the parameters of the reference medium which define an ellipsoid are constructed from λ_{ijkl} using the coefficients for the approximation of an arbitrary anisotropic medium from Ettrich et al. (2001). The traveltimes in reference medium are computed directly using the FD eikonal solver method for an elliptically anisotropic model. For the perturbed medium we perform traveltime computations using perturbation techniques.

Basic perturbation formulas

To compute traveltimes for an arbitrary anisotropic medium a perturbation scheme is embedded into the FD eikonal solver: traveltimes at every grid point of the reference model are computed and the perturbation method is used for the computation of the traveltime corrections to yield traveltimes in the perturbed (arbitrary anisotropic) model.

To compute the traveltime correction, the raypath between source and receivers in the reference medium must be known. Rays are not determined in the FD method. In the FD method traveltimes are computed on a discrete grid assuming local plane wavefronts inside the grid cell, therefore, we compute the traveltime correction along the ray segments corresponding to the plane wavefronts in each cell. The ray segment is a straight line between the grid point (see Figure 2), where the traveltime is to be found (point A7), and the point where the ray crosses the cell boundary (point N) which is defined using the ray velocity vector. The ray velocity for the reference ellipsoidal medium is given by the simple formula: $\mathbf{v}_{gr} = \mathbf{R}\,\mathbf{p}$. Here $\mathbf{R}$ is the matrix of the medium parameters (see equation 1), $\mathbf{p}$ is slowness vector computed for this grid cell. The traveltime at a point on the cell boundary (point N) is obtained by linear interpolation between the corner points of the cell. Therefore, the traveltime at point A7 for the perturbed model is:

$$t_{pert}(A7) = t_{ref}(A7) + \Delta t'(N) + \Delta t(N, A7),$$

where t_{pert} is the traveltime in the perturbed model and t_{ref} is the traveltime in the reference model, $\Delta t' = t_{pert}(N) - t_{ref}(N)$ is the difference between the traveltimes at point N in perturbed and reference models, and $\Delta t(N, A7)$ is the traveltime correction derived by Červený (1982):

$$\Delta t(N, A7) = -\frac{1}{2} \int\limits_{t_{ref}(N)}^{t_{ref}(A7)} \Delta c_{ijkl} p_i p_l g_j g_k dt. \tag{2}$$

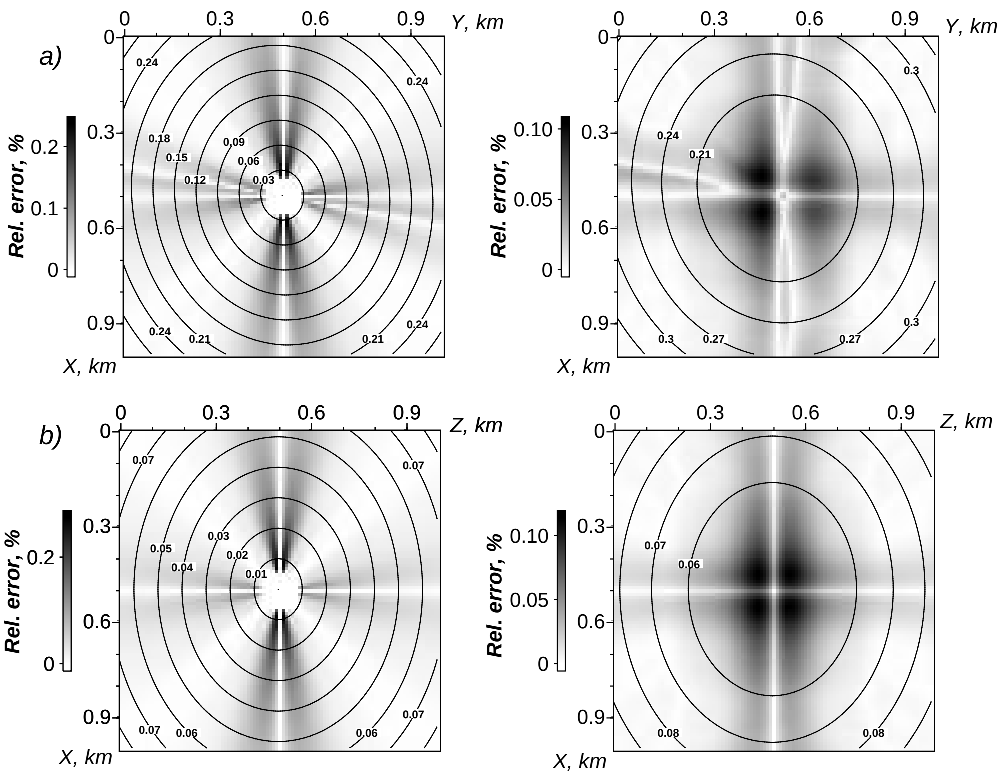

Figure 1: Wavefronts in a homogeneous elliptically anisotropic model: *(a)* — in the reference medium for triclinic from equation (6); *(b)* — in the reference medium for olivine from equation (4). For each model there are two slices with offset 0 km (left) and 0.4 km (right) from the source located at point (0.5, 0.5 and 0.5 km). An underlying grayscale images shows the relative errors of the traveltime computation. Maximum of relative errors in the whole 3-D model does not exceeds 0.35% for the case *(a)* and 0.45% for the case *(b)*.

where:

$$\Delta c_{ijkl} = c^{pert}_{ijkl} - c^{ref}_{ijkl}.$$

The parameters $c^{(pert)}_{ijkl}$ are the density normalized elastic coefficients of the anisotropic medium, p_i are the components of the slowness vector, and g_j are the components of the quasi P-wave polarization vector. The vectors $\mathbf{p}$ and $\mathbf{g}$ depend on the reference medium. For isotropic reference medium the P-wave polarization is a unit vector which is normal to the wavefront and the density normalized elastic coefficients are defined from $c^{ref}_{ijkl} = \frac{\lambda}{\rho}\delta_{ij}\delta_{kl} + \frac{\mu}{\rho}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$. In the case of the ellipsoidal reference medium we substitute the wavefront normal vector $\mathbf{n}$ instead of the unknown polarization vector $\mathbf{A}$ (see Ettrich et al., 2001), therefore, the traveltime correction reads:

$$\Delta t(N, A7) \approx -\frac{1}{2} \int\limits_{t_{ref}(N)}^{t_{ref}(A7)} (\lambda_{ijkl} n_j n_k - R_{il}) p_i p_l dt, \tag{3}$$

where R_{il} is a 3×3 matrix as given in equation (1) of an ellipsoidal reference medium. The assumption that the vector of polarization coincides with the wavefront normal introduces a additional error, which is quantified by numerical examples in the following section.

NUMERICAL RESULTS

To illustrate the traveltime computation by the FD perturbation method in 3-D anisotropic media, we choose two types of symmetry: triclinic and orthorhombic. To make an estimate of accuracy we consider homogeneous media. For comparison, traveltimes for all models are computed for isotropic and elliptically anisotropic reference models. In the figures, traveltimes are presented

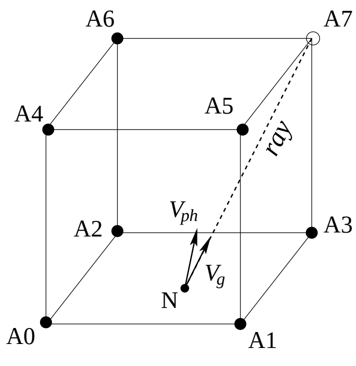

Figure 2: Grid cell with timed points A1 through A6. A local plane wavefront is assumed to propagate through the cell to calculate the traveltime at A7. The raypath in the cell defined from the direction of the ray velocity V_g is the length between the intersection point N and A7. V_{ph} is a phase velocity.

by first-arrival wavefronts. Traveltimes in a perturbed medium are called exact traveltimes (exact wavefronts) if they are computed directly by anisotropic ray tracing or analytically. They are considered to be exact with respect to the proved accuracy of the method used.

The traveltime errors in our examples are caused by three sources: the error of the numerical computation of traveltimes in a reference ellipsoidal (or isotropic) model using an FD method, the inherent error of the perturbation technique and the error by approximating the polarization vector by the wavefront normal vector (in the case of an ellipsoidal reference model) in the traveltime correction formula (equation 3).

A model cube of 100×100×100 is considered. The grid has 100 cells along each direction and the grid spacing is 10 m. The source is placed at the center of the models, at point (0.5; 0.5; 0.5 km). A cubic region of 11 grid points around the source is initialized using the exact solution. Since 3-D models are presented we always show two slices: with zero and non-zero offset from the source.

First, we will consider a test model with orthorhombic anisotropic elastic parameters of olivine. The density normalized parameters are:

$$\begin{pmatrix} 97.77 & 21.62 & 20.05 & 0. & 0. & 0. \\ & 71.0 & 22.83 & 0. & 0. & 0. \\ & & 59.68 & 0. & 0. & 0. \\ & & & 19.51 & 0. & 0. \\ & & & & 23.86 & 0. \\ & & & & & 23.77 \end{pmatrix} \rightarrow \begin{pmatrix} 91.99 & 0 & 0 \\ & 67.35 & 0 \\ & & 57.25 \end{pmatrix}; \quad V_p = 8.5\,km/s \qquad (4)$$

Coefficients for the best-fitting ellipsoid reference medium (Ettrich et al., 2001) and P-wave velocity for the best-fitting isotropic reference medium (Fedorov, 1968) are listed on the right. To compute traveltimes in this medium of strong P-wave anisotropy we use three types of reference media: isotropic, elliptically anisotropic and transversely isotropic with elliptical P-wavefront. The isotropic reference medium is constructed using formulas for the best fitting isotropic reference medium which were derived by Fedorov (1968). In this reference medium the polarization of the P-wave is a unit vector in the direction of the slowness vectors. Figures 3*a* and 4*a* display the exact wavefronts in the olivine model (solid lines) and the circular wavefronts in the best fitting isotropic reference medium (dashed lines). Referring to Figures 3*b* and 4*b* we observe the largest relative errors between exact (dashed lines) and FD perturbation traveltimes (solid lines) where the isotropic reference medium does not give the best fit. The maximum of the relative errors for the whole 3D model is 3.9 %. Such large errors are expected, because the anisotropy of the perturbed model is too strong to use any isotropic reference medium.

An elliptically anisotropic reference medium for the perturbation method allows to consider stronger anisotropy. However, for this type of reference medium we do not have the polarization vector to compute the traveltime correction (see equation (2)). The elliptically anisotropic medium is constructed using formulas for the best fitting elliptically anisotropic (ellipsoidal) medium (Ettrich et al. (2001)). (This formulas are linear relations for the coefficients of the ellipsoidal reference medium which depend on the elastic coefficients of the anisotropic medium, therefore, are convenient to use). To obtain this formulas the authors assumed that the polarization vectors coincides with the phase normal vector. We make the same assumption and use

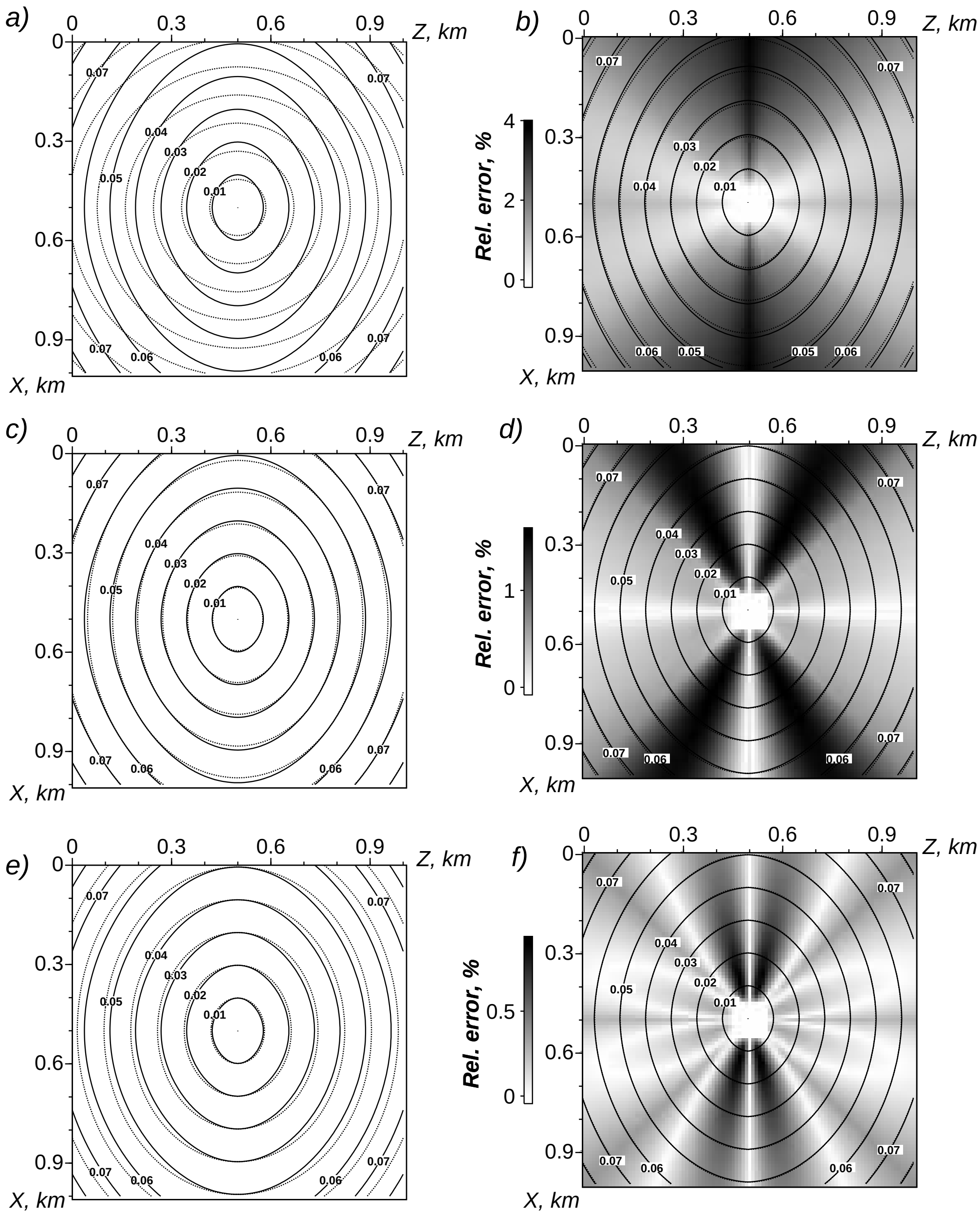

Figure 3: Wavefronts in a homogeneous model of olivine from equation (4); slice with zero offset from the source located at point (0.5, 0.5, 0.5 km). On the left-hand pictures solid lines show exact wavefronts and dashed lines show wavefronts in the reference models: *(a)* — isotropic model; *(c)* — elliptically anisotropic model; *(e)* — transversely isotropic model. On the right-side pictures the FD perturbation method wavefronts (solid) for the corresponding reference model and exact wavefronts (dashed) with the underlying grayscale images of relative errors.

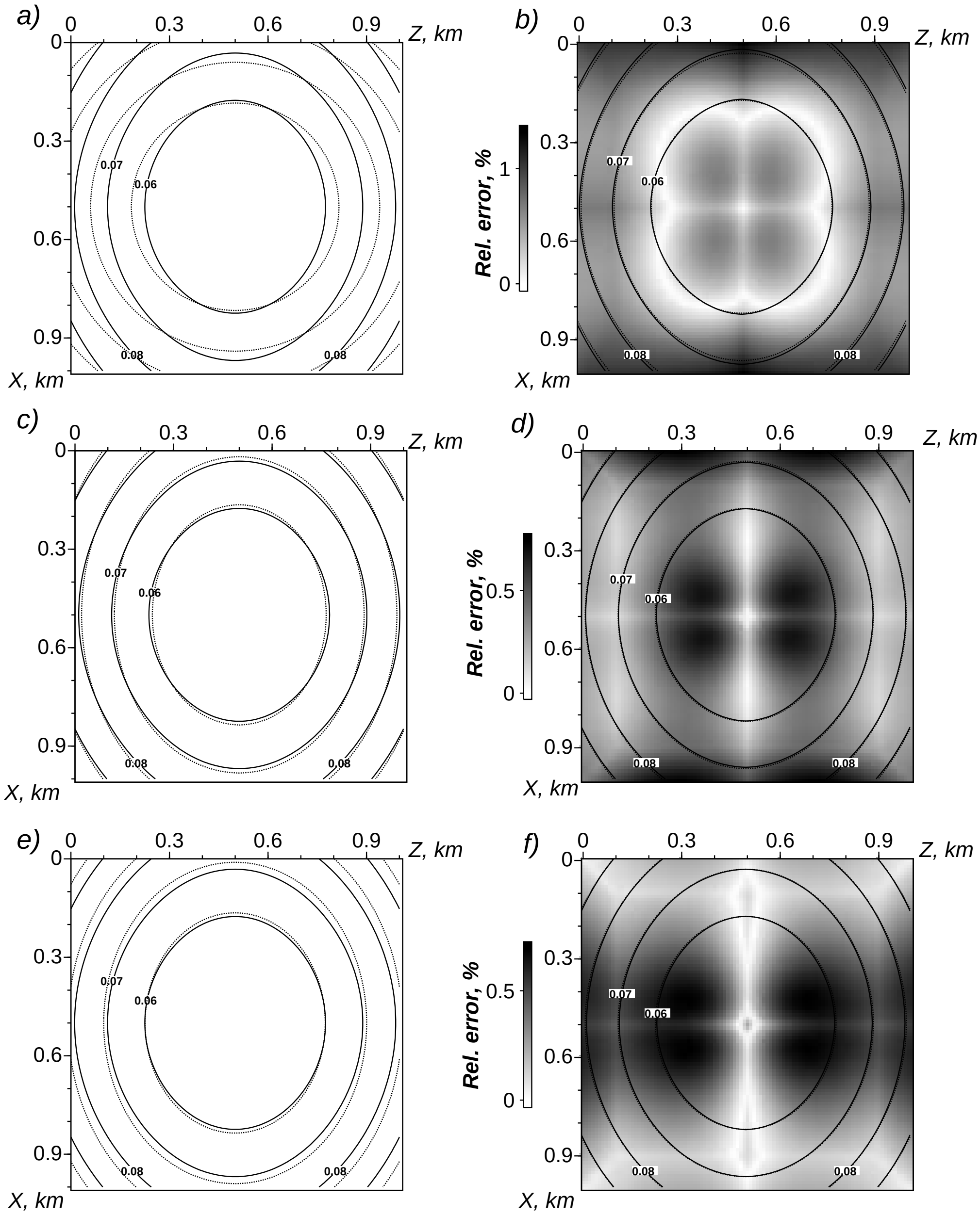

Figure 4: Wavefronts in a homogeneous model of olivine from equation (4); slice with offset 0.4 km from the source located at point (0.5, 0.5, 0.5 km). On the left-hand pictures solid lines show exact wavefronts and dashed lines show wavefronts in reference models: *(a)* — isotropic model; *(c)* — elliptically anisotropic model; *(e)* — transversely isotropic model. On the right-side pictures FD perturbation method wavefronts (solid) for the corresponding reference model and exact wavefronts (dashed) with the underlying grayscale images of relative errors.

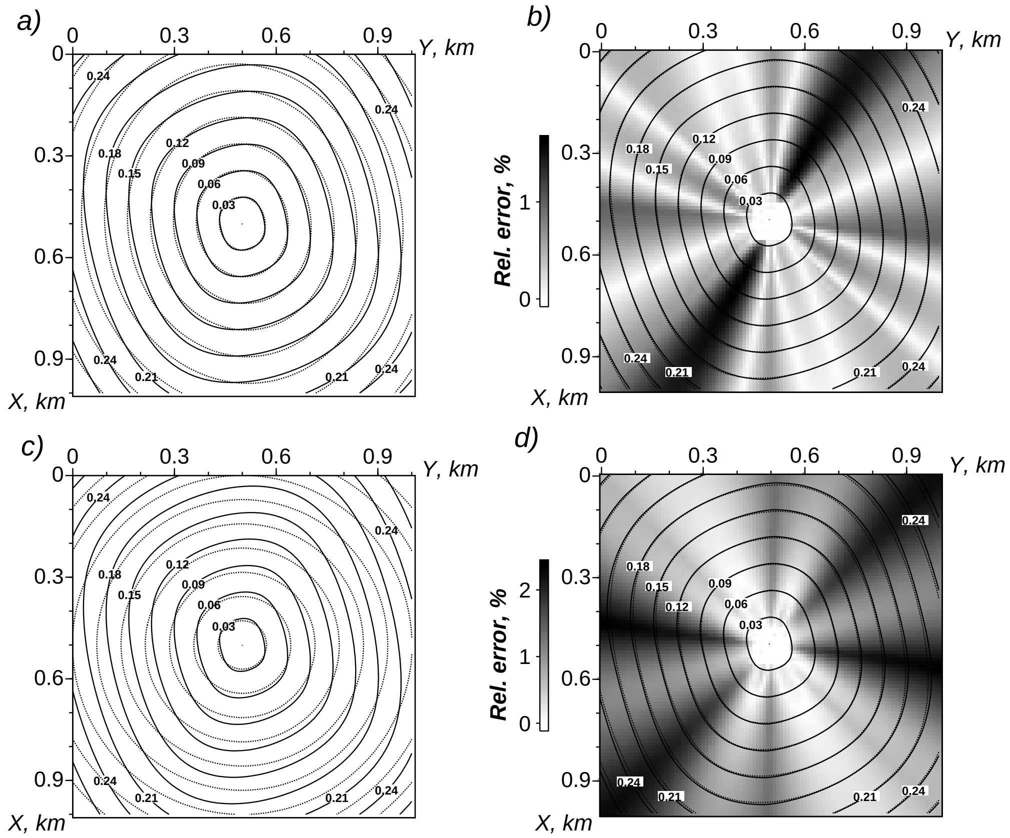

Figure 5: Wavefronts in a homogeneous model of triclinic sandstone from equation (6); slice with zero offset from the source located at point (0.5, 0.5, 0.5 km). On the left-hand side exact wavefronts (solid) and reference wavefronts (dashed): *(a)* — elliptically anisotropic model; *(c)* — isotropic model. On the right-hand side FD perturbation method wavefronts (solid) for the corresponding reference model and exact wavefronts (dashed) with underlying grayscale images of relative errors: *(b)* — elliptically anisotropic model; *(d)* — isotropic reference medium.

equation (3) instead of equation (2) to compute the traveltime corrections in the perturbed model. Figures 3*c* and 4*c* show the exact wavefronts in olivine (solid lines) and the reference wavefronts in the best fitting elliptically anisotropic (ellipsoidal) medium (dashed lines). This reference model gives the best fit to the perturbed medium. From Figures 3*d* and 4*d* we notice that the accuracy of the computation is higher than for the isotropic background media. The maximum of relative errors for the whole 3-D model is 1.89%. This error is mainly due to the approximation of the unknown polarization vector by the phase normal, while the error produced by the traveltime computation using the FD eikonal solver in the reference model does not exceed 0.45% (see Figure 1).

Following Burridge et al. (1983) there are three possibilities to simplify orthorhombic symmetry to ellipsoidal symmetry. One of these three possibilities is useful to construct a transversely isotropic medium with elliptical P-wavefront. For this case we can compute the correct polariza-

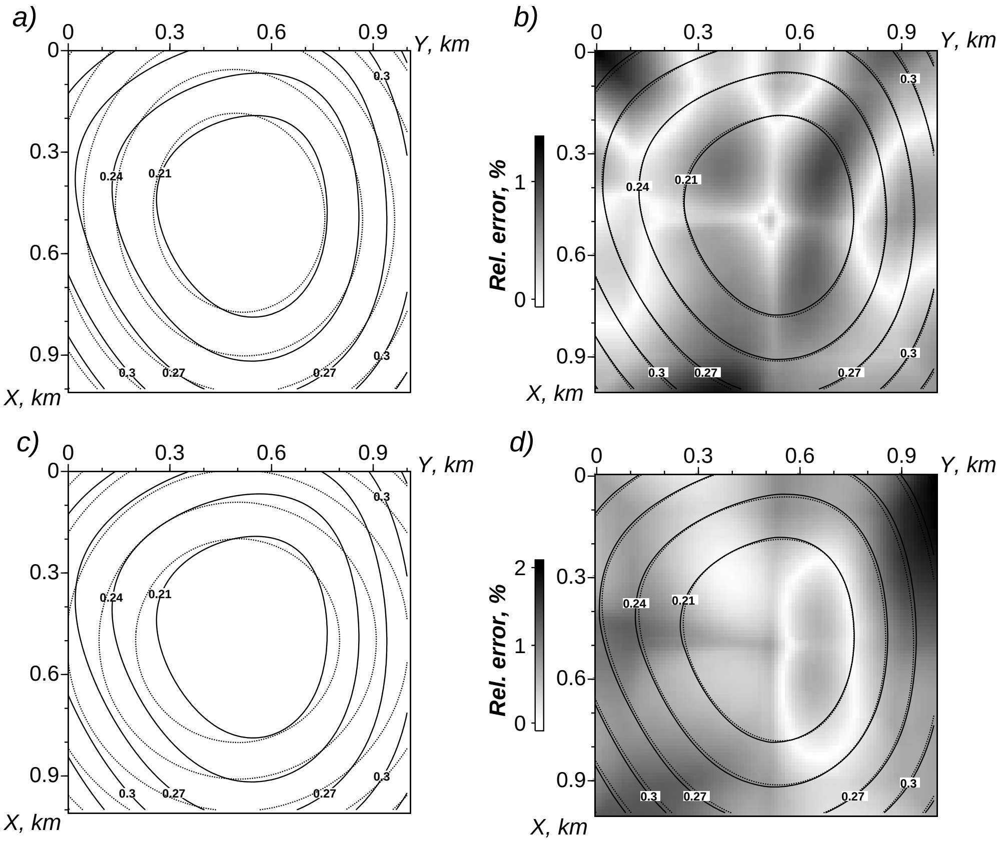

Figure 6: Wavefronts in a homogeneous model of triclinic sandstone from equation (6); slice with offset 0.4 km from the source located at point (0.5, 0.5, 0.5 km). On the left-hand side exact wavefronts (solid) and reference wavefronts (dashed): *(a)* — elliptically anisotropic model; *(c)* — isotropic model. On the right-hand side FD perturbation method wavefronts (solid) for the corresponding reference model and exact wavefronts (dashed) with underlying grayscale images of relative errors: (*(b)* — elliptically anisotropic model; *(d)* — isotropic reference medium.

tion vector for the elliptically anisotropic reference medium. The transversely isotropic medium with the axis of symmetry along X-axis direction has ellipsoidal symmetry if the non-zero elastic parameters satisfy the relations:

$$C_{22} = C_{33}; \quad C_{12} = C_{13}; \quad C_{55} = C_{66}; \quad C_{44} = \frac{1}{2}(C_{22} - C_{23}); \quad C_{55} = \frac{C_{11}C_{22} - C_{12}^2}{C_{11} + C_{22} + 2C_{12}}. \quad (5)$$

Because of the non-linear relations between the elastic parameters in equation (5) we construct the reference medium in the following simple manner. The transversely isotropic reference medium for olivine using (5) was constructed in the following way:

$$C_{11} = C_{11}^{oliv}; \quad C_{12} = \frac{1}{2}(C_{12}^{oliv} + C_{13}^{oliv}); \quad C_{22} = \frac{1}{2}(C_{22}^{oliv} + C_{33}^{oliv}); \quad C_{23} = C_{23}^{oliv}.$$

Figures 3*e* and 4*e* display the exact wavefronts in olivine and the wavefronts in the reference medium with transversely isotropic symmetry using equation (5). Underlying grayscale images in Figure 3*f* and 4*f* show the relative errors. The maximum of the relative errors in the whole 3-D model does not exceeds 1.2 %. We observe that the largest relative errors are located where bigger differences of the angle between the wavefront normals in perturbed and reference media occur (in other words, the raypaths are too different). The perturbation method gives high accuracy when the raypaths in both media are close to each other.

Now we will consider a test model of triclinic sandstone. The density normalized parameters are:

$$\begin{pmatrix} 6.77 & 0.62 & 1.0 & -0.48 & 0. & -0.24 \\ & 4.95 & 0.43 & 0.38 & 0.67 & 0.52 \\ & & 5.09 & -0.28 & 0.09 & -0.09 \\ & & & 2.35 & 0.09 & 0. \\ & & & & 2.45 & 0.0 \\ & & & & & 2.88 \end{pmatrix} \rightarrow \begin{pmatrix} 6.88 & 0.27 & 0.27 \\ & 5.10 & -0.05 \\ & & 5.08 \end{pmatrix}; \quad V_p = 2.38\, km/s \qquad (6)$$

Coefficients for the best-fitting ellipsoid reference medium (Ettrich et al., 2001) and P-wave velocity for the best-fitting isotropic reference medium (Fedorov, 1968) are listed on the right. This medium has strong P-wave anisotropy and irregular velocity surfaces caused by the relatively small non-orthorhombic coefficients. The left-hand sides of Figure 5 and Figure 6 display exact wavefronts in the triclinic sandstone (solid lines) and reference wavefronts (dashes lines) in the ellipsoidal model (Figures 5*a* and 6*a*) and in the isotropic model (Figures 5*c* and 6*c*). On the right-hand side, these figures display results of the numerical computation by the FD perturbation method with the ellipsoidal reference medium (Figures 5*b* and 6*b*) and with the isotropic reference medium (Figure 5*d* and Figure 6*d*). The underlying grayscale images show relative errors. The maximum of the relative errors in whole the 3-D model is 2.26% when the ellipsoidal reference medium is used and 2.40% for calculations with the isotropic reference medium. (The appropriate accuracy of the traveltime computation in the elliptically anisotropic reference medium is displayed in Figure 1*a*). Although the maximum errors in traveltimes with the elliptically anisotropic reference medium is close to the maximum errors in the traveltimes for the isotropic medium for this particular model, we observe for the other models in Figures 5 and 6 that the accuracy for the ellipsoidal reference medium is considerably better for most grid points.

CONCLUSIONS

We have presented a finite-difference perturbation method for the efficient computation of traveltimes for P-waves in arbitrary anisotropic 3-D media. An arbitrary anisotropic medium is considered as a perturbed model with respect to a simple reference medium. Traveltimes in the reference medium are computed using the FD eikonal solver which allows fast and accurate computation of traveltimes, while traveltimes in the perturbed medium are obtained by adding traveltime corrections to the traveltimes of the reference medium. Instead of the raypath between source and grid point we use the ray segments corresponding to plane wavefronts in each grid cell.

We suggest to use models of ellipsoidal symmetry as a reference model to compute traveltimes for strong anisotropic media. An elliptically anisotropic medium approximates a medium of strong anisotropic better than an isotropic one. The corresponding eikonal equation is only slightly more complex then for the isotropic case. The primary source of errors related to the elliptically anisotropic medium is the unknown polarization vector which is needed to carry out the traveltime correction with the perturbation method. We have substituted the unknown polarization vector by the phase normal vector to compute the traveltime correction and the compared accuracy of the traveltime computations with the accuracy in the case of an isotropic reference medium. The maximum of the relative errors in the whole 3-D model for the olivine model with respect to the isotropic reference model is 3.9%, while with respect to the elliptically anisotropic model it is 1.89%. In the case of the isotropic reference model, the deviation between the perturbed and the reference model is too large. On this case the inherent error of the perturbation scheme is larger than the error caused by the approximation of the unknown polarization vectors in the case of an elliptically anisotropic reference medium.

PUBLICATIONS

Previous results concerning traveltimes for anisotropic media using the FD perturbation method were published Soukina et al. (2001). A paper containing these results will be submitted to Geophysical Prospecting (Soukina et al. (2002).)

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REFERENCES

Dellinger, J. (1991). Anisotropic finite-difference traveltimes. In *Expanded Abstracts, 61st Annual International Meeting SEG,November 10–14, Houston, Texas*, pages 1530–1533. Soc. Expl. Geophys.

Eaton, D. W. S. (1993). Finite difference traveltime calculation for anisotropic media. *Geophysical Journal International*, 114:273–280.

Ettrich, N. (1998). FD eikonal solver for 3-d anisotropic media. In *Expanded Abstracts, 68th Ann. Int. Mtg. SEG*, pages 1957–1960.

Ettrich, N., Gajewski, D., and Kashtan, B. (2001). Reference ellipsoids for anisotropic media. *Geophys. Pros.*, 49:321–334.

Lecomte, I. (1993). Finite difference calculation of first traveltimes in anisotropic media. *Geophys. J. Int*, 113:318–342.

Van Trier, J. and Symes, W. W. (1991). Upwind finite-difference calculation of traveltimes. *Geophysics*, 56:812–821.

Podvin, P. and Lecomte, I. (1991). Finite difference computation of traveltimes in very contrasted velocity model: a massively parallel approach and its associated tools. *Geophys. J. Int*, 105:271–284.

Qin, F., Luo, Y., Olsen, K. B., Cai, W., and Schuster, G. T. (1992). Finite-difference solution of the eikonal equation along expanding wavefronts. *Geophysics*, 57:478–487.

Soukina, S., Gajewski, D., and Kashtan, B. M. (2001). Finite-difference perturbation method for 3-d anisotropic media. In *Expanded Abstracts, EAGE 63rd Conference & Technical Exhibition — Amsterdam, The Netherlands, 11 - 15 June 2001*, pages P–002. EAGE.

Soukina, S. M., Gajewski, D., and Kashtan, B. M. (2002). Traveltime computation for 3-d anisotropic media by a finite-difference perturbation method. *submitted to Geophys. Pros.*

Vidale, J. E. (1988). Finite-difference calculation of traveltimes. *Bull. Seis. Soc. Am.*, 78:6, 2062–2076.

Vidale, J. E. (1990). Finite-difference calculation of traveltimes in three dimensions. *Geophysics*, 55:521–526.