Stress dependences of seismic velocities in porous and fractured rocks

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ABSTRACT

Using quite general results of the theory of poroelasticity we attempt to analyze the influence of the confining pressure and of the pore pressure on seismic velocities in rocks. In the first approximation the seismic velocities as well as the porosity depend on the differential stress, i.e., the difference between the confining pressure and the pore pressure only. The stress dependence of the porosity controls the elastic moduli and velocity changes with stress. Here, the most important role plays the compliant porosity which can be just a very small part of the total porosity. The stress dependence of the compliant porosity can be derived from the theory of poroelasticity under several, quite natural assumptions. This result provides the seismic velocities as functions of the differential stress. Corresponding equations coincide with experimentally observed exponentially saturating seismic velocities for increasing differential stresses.

INTRODUCTION

The pore pressure dependence of seismic velocities has an importance for interpretation of very different seismic data, ranging from AVO and velocity analysis to overpressure prediction and 4D seismic monitoring of reservoirs. Usually, this dependence along with the velocity dependence on confining stress is phenomenologically modeled by the following simple relationship (Zimmerman et al, 1986, Prasad and Manghnani, 1997, Khaksar et al, 1999, and Carcione and Tinivella, 2001):

$$V(P) = A + KP - B\exp(-PD), \tag{1}$$

where P is the differential pressure and coefficients A, K and D are fitting parameters for a given set of measurements. Equation (1) describes well the reality. However, some times more complex models related to attempts to define a type of the geometry of the porous space are used. E.g., spherical contacts Mindlin's theory based models (Duffy and Mindlin, 1957 and Merkel et al, 2001) or crack contacts Gangi's theory based models (Gangi and Carlson, 1996 and Carcione and Tinivella, 2001) have been used in different publications. In this paper we will show how equation of the form of (1) can be derived from a rather general consideration as well as clarify the physical meaning of its coefficients.

DIFFERENTIAL STRESS AS A CONTROLLING FACTOR

For simplicity we consider isotropic porous or fractured rocks with connected porosity and a homogeneous dry skeleton material (i.e., the grain material). It is very well known that in the reflection seismic frequency range the Gassmann's formula (Gassmann 1951, Mavko et al, 1998) describes well the seismic velocities dependences on fluids saturating porous rocks. The Gassmann's formula provides us with an explicit rule how to compute the bulk elastic modulus of rocks K_r using the porosity ϕ, the fluid bulk modulus K_f, the bulk modulus of the dry rock skeleton K_{dr} (also called the drained bulk modulus) and the bulk modulus of the grain material K_{gr}:

$$K_r = K_{dr} + \alpha^2 \left[\frac{\phi}{K_f} + \frac{\alpha - \phi}{K_{gr}} \right]^{-1}, \tag{2}$$

where the quantity α is

$$\alpha = 1 - \frac{K_{dr}}{K_{gr}}. \tag{3}$$

All the input parameters of the Gassman's formula to some extend depend on the confining or fluid pressure. However, the stress dependence of the K_{gr} is weak in the range of stresses less than several hundreds of MPa. A pressure dependence of the K_f can be relatively easy computed using state equations or empiric relationships (Batzle and Wang, 1992) and thus, taken into account. Further, in the case of a pure water or brine saturation this dependence is weak. Moreover, the influence of the stress dependence of K_f is strongly reduced in the case of small porosities. However, in all these situations the stress dependence of K remains significant. Therefore, the most important factors controlling the bulk modulus stress dependence are the modulus K_{dr} and the porosity of rocks.

A general result of the poroelasticity theory is that if the rock strain is a single valued (i.e., non-hysteresis) function of the confining pressure P_c and of the pore pressure P_p then the bulk compressibility $C_{dr} = 1/K_{dr}$ of the dry rock skeleton must be a function of the differential stress $P = P_c - P_p$ only (see Zimmerman, et al., 1986; Detournay and Chang, 1993; Gurevich 2001). Therefore, at least in the first approximation of the non-hysteresis character of rock deformations the differential pressure controls the stress dependence of the drained bulk modulus.

The stress dependence of the porosity can be characterized using another very general result of the poroelastic theory, which follows from definitions of porous rock compressibilities and the Betti reciprocal theorem. This is a differential equation directly relating porosity changes with the changes of the differential stress (Zimmerman, et al., 1986; Detournay and Chang, 1993):

$$\frac{d\phi}{dP} = \frac{1}{K_{gr}} - (1 - \phi) C_{dr}. \tag{4}$$

Again we note that the stress dependence is reduced to the dependence on the differential stress only. We have already intimated that the bulk modulus K_{gr} is practically independent of P.

Thus, in the equation above two quantities are stress dependent only: ϕ and C_{dr}. Therefore, in order to obtain a stress dependence of these two quantities one more equation relating them to the stress or just one mutually relating them is required.

COMPRESSIBILITY VERSUS STIFF AND COMPLIANT POROSITIES

Let us separate the porosity ϕ in two parts

$$\phi = \phi_c + [\phi_{s0} + \phi_s] \,, \tag{5}$$

where the first part, ϕ_c, is a compliant porosity supported by thin cracks, channels and grain contacts vicinities. The second part, $\phi_{s0} + \phi_s$ is a stiff porosity supported by isomorphic pores. This porosity is in turn separated into a part ϕ_{s0}, which is equal to the stiff porosity in the case of a zero differential stress, and to a part ϕ_s expressing changes of the stiff porosity due to changes in the differential stress. We assume that the relative changes of the stiff porosity, ϕ_s/ϕ_{s0}, are small. In contrast, the relative changes of the compliant porosity $(\phi_c - \phi_{c0})/\phi_{c0}$ can be very large, i.e., of the order of 1 (here we denoted by ϕ_{c0} the compliant porosity in the case of $P = 0$). Note, however, that ϕ_c and ϕ_{c0} are usually very small quantities. As a rule, (e.g., in porous sandstones) they are much smaller than the ϕ_{s0} and even than the absolute value of ϕ_s. Under such assumptions it is logic to assume the first, linear approximations of the compressibility C_{dr} as a function of the porosities:

$$C_{dr}(\phi_{s0} + \phi_s, \phi_c) = C_{drs}\left[1 + \theta_s\phi_s + \theta_c\phi_c\right], \tag{6}$$

where C_{drs} is the drained compressibility of a rock in the case of a closed compliant porosity (i.e., $\phi_c = 0$) and the stiff porosity equal to ϕ_{s0}. The coefficients

$$\theta_s = \frac{1}{C_{dr}}\frac{\partial C_{dr}}{\partial \phi_s}, \qquad \theta_c = \frac{1}{C_{dr}}\frac{\partial C_{dr}}{\partial \phi_c} \tag{7}$$

are assumed to be independent physical properties of rocks.

Approximation (6) implies that the both quantities $\theta_s\phi_s$ and $\theta_c\phi_c$ are smaller than 1. Further, numerous laboratory experiments and the practical experience show that the drained compressibility depends strongly on changes in the compliant porosity, and it depends much weaker on changes in the stiff porosity. We will express this empirical observation by the restriction

$$\theta_s\phi_s \ll \theta_c\phi_c. \tag{8}$$

If so, the approximation (6) is further simplified as follows:

$$C_{dr}(\phi_{s0} + \phi_s, \phi_c) = C_{drs}\left[1 + \theta_c\phi_c\right]. \tag{9}$$

Using this approximation, neglecting ϕ in comparison with 1 and neglecting $1/K_{gr}$ in comparison with $1/K_{drs}$ we obtain the following relationship instead of equation (4):

$$\frac{d\phi_s}{dP} + \frac{d\phi_c}{dP} = -C_{drs} - \theta_c\phi_c C_{drs}. \tag{10}$$

STRESS DEPENDENCES OF THE STIFF AND COMPLIANT POROSITIES

We assume that stiff porosity changes with stress are independent on the changes of the compliant porosity. This means also, that changes of the stiff porosity are independent on the fact if the compliant porosity is closed or not. If the compliant porosity is closed then $\phi_c = 0$ and we obtain from (10)

$$\frac{d\phi_s}{dP} = -C_{drs}. \tag{11}$$

However, if the assumption above is valid than this relationship will be valid also for an arbitrary (however, because of other assumptions, small) ϕ_c. Therefore,

$$\frac{d\phi_c}{dP} = -\theta_c \phi_c C_{drs}. \tag{12}$$

These two equations immediately provide us with the following approximations of the stress dependences of the stiff and compliant porosities:

$$\phi_s = -PC_{drs}. \tag{13}$$

$$\phi_c = \phi_{c0} \exp\left(-\theta_c P C_{drs}\right). \tag{14}$$

Note that equation (13) is not valid for very large P. This is explained by the fact that in previous steps we neglected the stiff-porosity dependence of the drained compressibility. The validity of this assumption as well as the validity of equation (13) are restricted by the condition (8). For very high P stresses also the stiff porosity will obey an exponentially saturating behavior.

STRESS DEPENDENCES OF ELASTIC PROPERTIES

Substituting equations (13) and (14) into equation (6) we obtain:

$$C_{dr}(P) = C_{drs}\left[1 + \theta_s \phi_{s0} - \theta_s C_{drs} P + \theta_c \phi_{c0} \exp\left(-\theta_c P C_{drs}\right)\right]. \tag{15}$$

For the bulk modulus this gives:

$$K_{dr}(P) = K_{drs}\left[1 - \theta_s \phi_{s0} + \theta_s C_{drs} P - \theta_c \phi_{c0} \exp\left(-\theta_c P C_{drs}\right)\right]. \tag{16}$$

Using for the skeleton shear modulus μ_{dr} an expansion similar to the expansion (6) we obtain

$$\mu_{dr}(P) = \mu_{drs}\left[1 - \theta_{s\mu} \phi_{s0} + \theta_{s\mu} C_{drs} P - \theta_{c\mu} \phi_{c0} \exp\left(-\theta_c P C_{drs}\right)\right], \tag{17}$$

where

$$\theta_{s\mu} = \frac{1}{\mu_{dr}} \frac{\partial \mu_{dr}}{\partial \phi_s}, \qquad \theta_{c\mu} = \frac{1}{\mu_{dr}} \frac{\partial \mu_{dr}}{\partial \phi_c} \tag{18}$$

General form of stress functions (16) and (17) coincides with equation (1). It is clear also, that if the differential stress is smaller or of the order of 10^2 MPa we can neglect the terms $\theta_{s\mu} C_{drs} P$ and $\theta_s C_{drs} P$ because C_{drs} is of the order of 10^{-4} MPa^{-1}. Finally, neglecting the ϕ_s- and ϕ_c-dependences of the density we obtain for the drained velocities

$$V_{Sdr} \approx V_{Sdrs0} - \frac{1}{2} V_{Sdrs0} \theta_{c\mu} \phi_{c0} \exp\left(-\theta_c P C_{drs}\right), \tag{19}$$

$$V_{Pdr} \approx V_{Pdrs0} - \frac{1}{2} V_{Pdrs0} H \theta_{c\mu} \phi_{c0} \exp\left(-\theta_c P C_{drs}\right), \tag{20}$$

where

$$H = \frac{K_{drs}\theta_c/\theta_{c\mu} + 4\mu_{drs}/3}{K_{drs} + 4\mu_{drs}/3}. \tag{21}$$

CONCLUSIONS

Let us draw preliminary conclusions of this consideration. In the first approximation the seismic velocities as well as the porosity depend on the differential stress, i.e., the difference between the confining pressure and the pore pressure only. The stress dependence of the porosity controls the elastic moduli and velocity changes with stress. Here, the most important role plays the compliant porosity which can be just a very small part of the total porosity. Closing compliant porosity with increasing differential stress explains the experimentally observed exponentially saturating increase of seismic velocities. Coefficients of this relationship are defined by the compliant porosity dependence of the drained bulk modulus.

In the first approximation, the coefficient K in the empiric equation (1) can be neglected. The modified such equations have similar forms for the both, P- and S- wave velocities:

$$V_{Pdr}(P) = A_P - B_P \exp\left(-PD_P\right), \tag{22}$$

$$V_{Sdr}(P) = A_S - B_S \exp\left(-PD_S\right). \tag{23}$$

They exactly coincide with the empiric equation (3) of Khaksar et al, (1999). Moreover, we should expect that

$$D_P \approx D_S. \tag{24}$$

This is also in a good agreement with the data of table 2 of Khaksar et al, (1999).

The dimensionless quantity θ_c can be then estimated as

$$\theta_c = D/C_{drs} = \rho(A_P^2 - 4A_S^2/3)D, \tag{25}$$

where ρ is the density of the dry rock. This coefficient defines the sensitivity of the elastic moduli to the differential stress. We propose to call it the piezosensitivity. The piezosensitivity is an important property of rocks. From the derivation above it is clear that it is defined by the compliant porosity of rocks. It can be related to the poroelastic coefficient α. Moreover, it can be also related to non-linear elastic moduli of rocks.

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