Reflection Impedance

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ABSTRACT

AVO is now an established technology and has been widely deployed as a lithology indicator and also as a direct hydrocarbon indicator. In recent years this technology has become a routine processing and its application to large 3D volumes has relied on the use of near- and far-offset stack volumes. These volumes greatly reduce the amount of pre-stack information that needs to be stored for standard AVO processing. Additionally, these volumes are easily converted into usual AVO attributes, like intercept and gradient, which can then be interpreted in terms of anomalies and calibrated with well logs. Reservoir characterization studies make use not only of these traditional AVO attributes but also impedance volumes. The near-offset, or the intercept, stack volume offers a natural way of obtaining acoustic impedance volume through the use of post-stack inversion algorithms. However, to invert far-stack volume one needs an approach capable of estimating impedances for a variable incidence angle. This approach has been described in the elastic impedance function presented by Connolly (1999). In this work we propose an approach called reflection impedance, which is based on constant ray parameter and a power relationship between density and S-wave velocity. This new method proved to be of better accuracy for angular impedance estimation and reflection coefficient recovery when compared with the elastic impedance approach.

INTRODUCTION

In recent years there has been an enormous increase in the amount of 3D seismic data processed with AVO purpose. The most economical form of processing large volumes of seismic data to obtain AVO attributes involves obtaining near- and far-offset stacks. These stacks have been intensively used not only to obtain traditional AVO attributes (e.g., intercept and gradient) but also as input of post-stack inversion algorithms to yield acoustic impedance (AI) volumes that help in reservoir characterization. The near-offset stack can be tied to synthetics obtained from acoustic impedance changes derived from well logs. After calibration, the near-offset stacks can then be inverted back to acoustic impedances using off-the- shelf post-stack inversion algorithms, which use the well log impedances as constraints. The missing part of this process was how to invert the far-offset stacks? The answer to the question came from the elastic impedance (EI) approach presented by Connolly (1999), which generalizes the acoustic impedance concept for variable incidence angle. In other words, the EI provides a way to calibrate and invert nonzero-offset seismic data just as AI does for zero-offset data. One advantage of the EI method is that it correlates directly to rock properties, like α/β ratio (P- to S-wave velocity ratio), instead of being an attribute that relates to contrasts of elastic properties of neighboring rocks (like most AVO attributes). In this work we demonstrate a new approach to obtain nonzero-offset impedance estimates to be used as calibration for nonzero-offset seismic data. We called this approach reflection impedance (RI). Basically, RI is based on constant ray parameter, opposed to constant incidence angles as proposed by Connolly (1999). Also, the new approach assumes a power relation between density and S-wave velocity while the EI approach assumes a constant $K = \beta^2/\alpha^2$. As a result, the new approach greatly improves the accuracy of the impedance estimates, which can be critical in case of subtle amplitude anomalies.

NORMAL INCIDENCE: ACOUSTIC AND ELASTIC

For a given normal reflected ray, parameterized by the *traveltime* τ, the normal P-P *reflection coefficient* is given by

$$R_0(\tau, \Delta\tau) = \frac{AI(\tau + \Delta\tau) - AI(\tau)}{AI(\tau + \Delta\tau) + AI(\tau)} \ , \tag{1}$$

where

$$AI(\tau) = \rho(\tau)\alpha(\tau) \tag{2}$$

is the *acoustic impedance* function, $\rho(\tau)$ is the density function, $\alpha(\tau)$ is the P-velocity function, and $\Delta\tau$ is the traveltime increment, chosen to be sufficiently small. Observe that we also consider that the elastic parameters are being parameterized by the traveltime.

We will also consider the P-P *Reflectivity* function,

$$\mathcal{R}_0(\tau) = \lim_{\Delta\tau \to 0} \frac{R(\tau, \Delta\tau)}{\Delta\tau} = \frac{1}{2} \frac{AI'(\tau)}{AI(\tau)} \ , \tag{3}$$

where the prime denotes derivative with respect to to τ.

NON-NORMAL INCIDENCE: ACOUSTIC

For a general reflection, not necessarily normal, the *acoustic reflection coefficient* can be written as

$$R_A(\tau, \Delta\tau) = \frac{I(\tau + \Delta\tau) - I(\tau)}{I(\tau + \Delta\tau) + I(\tau)} \ , \tag{4}$$

with

$$I(\tau) = \rho(\tau)\ \alpha(\tau)\ \sec\ \theta(\tau)\ , \tag{5}$$

where $\alpha(\tau)$ now denotes the acoustic velocity, $\theta(\tau)$ is the incidence angle and, as before, $\rho(\tau)$ is the density. The quantity $I(\tau)$ will be called acoustic *reflection impedance*.

Analogously to the case of normal incidence, we can use the above-defined acoustic reflection impedance, $I(\tau)$, to express the *acoustic reflectivity* function, $\mathcal{R}_A(\tau)$. Using the previous equation (3), we find,

$$\mathcal{R}_A(\tau) = \lim_{\Delta\tau \to 0} \frac{R_A(\tau, \Delta\tau)}{\Delta\tau} = \frac{1}{2} \frac{I'(\tau)}{I(\tau)} \; . \tag{6}$$

Once we have computed the acoustic reflectivity function, a first-order approximation for the acoustic reflection coefficient can be readily expressed by

$$R_A(\tau, \Delta\tau) \approx \mathcal{R}_A(\tau) \; \Delta\tau \; . \tag{7}$$

NON-NORMAL INCIDENCE: P-P ELASTIC

For elastic P-P reflection under general incidence angles, the reflection coefficient, $R(\tau, \Delta\tau)$, has a much more complicated form than its acoustic counterpart, $R_A(\tau, \Delta\tau)$ in equation (4). Note that the classical expression for the P-P elastic reflection coefficient (see, e.g., Aki and Richards (1980)) replace the parameters τ and $\tau + \Delta\tau$ by indices 1 and 2, respectively. For example, ρ_1 replaces $\rho(\tau)$; α_2 replaces $\alpha(\tau + \Delta\tau)$, etc.

As recently proposed by Connolly (1999), it makes sense to look for a quantity $E(\tau)$ for which the complicated P-P elastic reflection coefficient assumes the simple form of equation (4), namely

$$R(\tau, \Delta\tau) = \frac{E(\tau + \Delta\tau) - E(\tau)}{E(\tau + \Delta\tau) + E(\tau)} \; . \tag{8}$$

As shown below, there exists no function $E(\tau)$ for which equation (8) exactly holds, for any choice of velocities and densities. Connolly (1999) proposed one approximation called *elastic impedance*. We find the term a little misleading because it has already a different meaning in the literature, namely the product between the density and the shear wave (recall that the definition of the acoustic impedance as the product between the density and the pressure velocity).

In this paper, we propose a new quantity, different from the one in Connolly (1999), that, at least in a number of relevant cases, provides a better representation of the P-P reflection coefficient under the approximation (8). We call the new function *P-P elastic reflection impedance*.

To construct $E(\tau)$, we start by introducing the P-P *elastic reflectivity* function $\mathcal{R}(\tau)$. Correspondingly to the acoustic case, the P-P elastic reflectivity is determined from the P-P elastic reflection coefficient, $R(\tau, \Delta\tau)$, by means of the limit

$$\mathcal{R}(\tau) = \lim_{\Delta\tau \to 0} \frac{R(\tau, \Delta\tau)}{\Delta\tau} \; . \tag{9}$$

After some algebraic manipulations in the exact expression of $R(\tau, \Delta\tau)$, the reflectivity $\mathcal{R}(\tau)$ is *exactly* given by

$$\mathcal{R}(\tau) = \frac{1}{2}\Big[1 - 4\beta^2 p^2\Big]\frac{\rho'}{\rho} \; + \frac{1}{2}\Big[\frac{1}{1 - \alpha^2 p^2}\Big]\frac{\alpha'}{\alpha} - \Big[4\beta^2 p^2\Big]\frac{\beta'}{\beta} \; , \tag{10}$$

where β is the S-velocity function, and p is the ray parameter, given by Snell's law

$$p = \frac{\sin\theta}{\alpha} = \frac{\sin\phi}{\beta} \; , \tag{11}$$

which is constant along the ray, and θ and ϕ are the P-P and P-S reflection angles, respectively.

In analogy with the acoustic case, the first-order approximation for $R(\tau)$ can be written as (see equation (7))

$$R(\tau, \Delta\tau) \approx \mathcal{R}(\tau)\ \Delta\tau\ . \tag{12}$$

Approximating the derivatives by their corresponding discrete differences, i.e., $f' \approx \Delta f/\Delta\tau$, and using the incidence angle θ instead of the ray parameter p, we have the following first-order approximation for R

$$R \approx \frac{1}{2}\left[1 - 4\frac{\beta^2}{\alpha^2}\sin^2\theta\right]\frac{\Delta\rho}{\rho} + \frac{1}{2}\left[\sec^2\theta\right]\frac{\Delta\alpha}{\alpha} - \left[4\frac{\beta^2}{\alpha^2}\sin^2\theta\right]\frac{\Delta\beta}{\beta}\ , \tag{13}$$

which is the well-known approximation of Aki and Richards (1980) and Shuey (1985).

REFLECTION IMPEDANCE

Let us now analyze the possibility of the existence of a function $E(\tau)$ satisfying equation (8) or, equivalently,

$$\mathcal{R}(\tau) = \frac{1}{2}\frac{E'(\tau)}{E(\tau)}\ . \tag{14}$$

Combining equations (10) and (14), we must search for a solution for the differential equation,

$$\frac{E'}{E} = \left[1 - 4\beta^2 p^2\right]\frac{\rho'}{\rho}\ + \left[\frac{1}{1-\alpha^2 p^2}\right]\frac{\alpha'}{\alpha} - \left[8\beta^2 p^2\right]\frac{\beta'}{\beta}\ , \tag{15}$$

for all possible choices of α, β and ρ. Clearly, the solution is not unique, since any multiple of a solution is also a solution.

Particular cases

We consider the solution of equation (15) for some particular choices of p and β.

Normal Incidence: Acoustic and Elastic

For a normal reflection ($p = 0$) equation (15) reduces to

$$\frac{E'}{E} = \frac{\rho'}{\rho} + \frac{\alpha'}{\alpha}\ , \tag{16}$$

whose solution is

$$E = C\ \rho\ \alpha = C\ AI\ , \tag{17}$$

where C is any real constant. Taking $C = 1$, we have that $E = AI$, as expected.

Non-normal Incidence: Acoustic

In the acoustic case ($\beta = 0$) equation (15) reduces to

$$\frac{E'}{E} = \frac{\rho'}{\rho} + \left[\frac{1}{1-\alpha^2 p^2}\right]\frac{\alpha'}{\alpha} \ , \tag{18}$$

whose solution is

$$E = \rho\ \alpha\ \frac{1}{\sqrt{1-\alpha^2 p^2}} = \rho\ \alpha\ \sec\theta = I \ , \tag{19}$$

where, again, we have taken the constant unitary.

Non-normal Incidence: P-P Elastic

Unfortunately, there is no general solution for $E(\tau)$ for any choice of the parameters. Indeed, it is possible to show that equation (15) admits a solution only if ρ has a functional dependence on β, i.e., $\rho \equiv \rho(\beta)$. Under this assumption, a normalized solution for $E(\tau)$ is given by

$$E \equiv RI = \frac{1}{\sqrt{1-\alpha^2 p^2}}\ \exp\left\{-4p^2\left[\beta^2 + \int \frac{\beta^2}{\rho} d\rho\right]\right\} \ , \tag{20}$$

We see that the P-P elastic reflection impedance is a natural extension of the acoustic reflection impedance after the introduction of a correcting factor. One reasonable dependence between ρ and β is the following

$$\rho = b\ \beta^\gamma \text{ , or equivalently, } \frac{\rho'}{\rho} = \gamma\ \frac{\beta'}{\beta} \ , \tag{21}$$

where b is some constant of proportionality and γ is a constant. With this assumption, equation (20) reads

$$RI = \frac{\rho\ \alpha}{\sqrt{1-\alpha^2 p^2}} \times \begin{cases} \mathrm{e}^{-2[2+\gamma]\beta^2 p^2} & , \ \beta' \neq 0 \\ \rho^{-4\beta^2 p^2} & , \ \beta' = 0 \end{cases} \tag{22}$$

We call the above function elastic *reflection impedance*. Observe that function RI automatically reduces to the acoustic reflection impedance, I, in the case $\beta = 0$, or to the acoustic impedance, AI, when $\theta = 0$.

CONNOLLY'S APPROACH

The *elastic impedance* EI proposed by Connolly (1999) is derived by taking equation (13) equal to $\Delta EI/2EI$ and applying difference calculus. The main assumption is that θ and the ratio $K = \beta^2/\alpha^2$ are constant. Such process is equivalent to solve equation (15) using the mentioned assumption. The differential equation (15) takes the form

$$\frac{EI'}{EI} = \left[1 - 4K\sin^2\theta\right]\frac{\rho'}{\rho} + \left[\sec^2\theta\right]\frac{\alpha'}{\alpha} - \left[8K\sin^2\theta\right]\frac{\beta'}{\beta} \ , \tag{23}$$

The normalized solution for the above equation is given by

$$EI = \rho^{1 - 4K\sin^2\theta}\, \alpha^{\sec^2\theta}\, \beta^{-8K\sin^2\theta} \;. \tag{24}$$

It is important to observe that although function EI reduces to the acoustic impedance AI in the case $\theta = 0$, the same does not occur for more general non-normal acoustic reflection. Indeed,

$$\lim_{\beta\to 0} EI = \rho\, \alpha^{\sec^2\theta} \neq I = \rho\, \alpha\, \sec\theta \;. \tag{25}$$

Moreover, the reflection impedance has the same dimension as the acoustic impedance whereas the elastic impedance has a dimension depending on the value of the angle θ.

APPROXIMATIONS FOR THE REFLECTION COEFFICIENT

By the use of formulas (8) and (22) or (24) we can construct approximations for the P-P reflection coefficient. Given a P-P elastic reflection at a point between two media with local parameters $\rho_1, \alpha_1, \beta_1$ and $\rho_2, \alpha_2, \beta_2$, R_{PP} can be approximated by

$$R_{PP} = \frac{E_2 - E_1}{E_2 + E_1} \;, \tag{26}$$

where, for the reflection impedance approximation

$$E_j \equiv RI_j = \frac{\rho_j\ \alpha_j}{\sqrt{1 - \alpha_j^2 p^2}} \times \begin{cases} \mathrm{e}^{-2[2+\gamma]\beta_j^2 p^2} & , \ \beta_1 \neq \beta_2 \\ \rho_j^{-4\beta_j^2 p^2} & , \ \beta_1 = \beta_2 \end{cases} \qquad j = 1, 2 \;, \tag{27}$$

with

$$\gamma = \frac{\ln(\rho_2/\rho_1)}{\ln(\beta_2/\beta_1)} \approx \frac{\Delta\rho/\rho}{\Delta\beta/\beta} \;, \quad \text{if } \beta_1 \neq \beta_2 \;, \tag{28}$$

and, for the elastic impedance approximation

$$E_j \equiv EI_j = \rho_j^{1 - 4K\sin^2\theta}\, \alpha_j^{\sec^2\theta}\, \beta_j^{-8K\sin^2\theta} \;. \tag{29}$$

with

$$K = \left(\frac{\alpha_1\beta_1 + \alpha_2\beta_2}{\alpha_1^2 + \alpha_2^2}\right)^2 . \tag{30}$$

NUMERICAL EXPERIMENTS

In order to compare the accuracy of EI and RI functions presented above we use the approximation of the P-P elastic reflection at a point between two media as indicated by equations (26)–(30). Table 1 shows the parameters for the three models used in the computations. They were chosen from a suite of 25 sets of α, β and ρ measurements in adjacent shales, gas sands and brine sands given in Castagna and Smith (1994), and represent Class I, II and III. The results are shown in Figures 1–3.

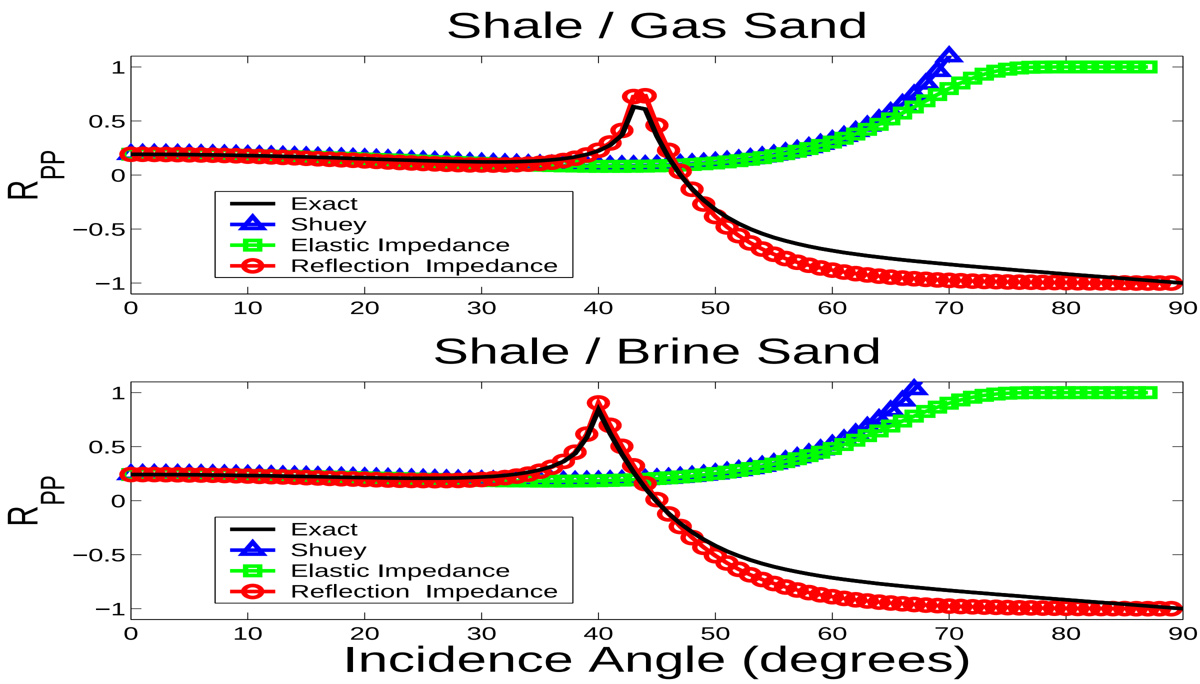

Figure 1: P-P reflection coefficient for Class I model given in Table 1: shale over gas sand (top) and shale over brine sand (bottom).

Class	Rock Type	α [km/s]	β [km/s]	ρ [g/cm^3]
I	Brine Sand	4.35	2.34	2.40
	Shale	2.77	1.52	2.30
	Gas Sand	4.05	2.38	2.32
II	Brine Sand	3.05	1.56	2.40
	Shale	2.77	1.27	2.45
	Gas Sand	2.69	1.59	2.25
III	Brine Sand	2.13	0.67	1.90
	Shale	1.83	0.40	2.02
	Gas Sand	1.44	0.58	1.53

Table 1: P- and S-wave velocities and densities for shale over brine sand and shale over gas sand, representing classes I, II and III.

The response computed based on the elastic impedance method deviates from the exact Zoeppritz formula for reflection coefficients as the linearized approximation (Shuey (1985)) for Rpp. The response computed from the reflection impedance method agrees with the exact Zoeppritz formula for Rpp. Therefore there is a significant gain in accuracy provided by the reflection impedance method compared to the elastic impedance method.

Figure 4 depicts the well log data of an oil sand reservoir (dashed box) encased in marine shales. In Figure 5 we show the comparison of the AI curve with $EI(30^o)$ and $RI(30^o)$ curves.

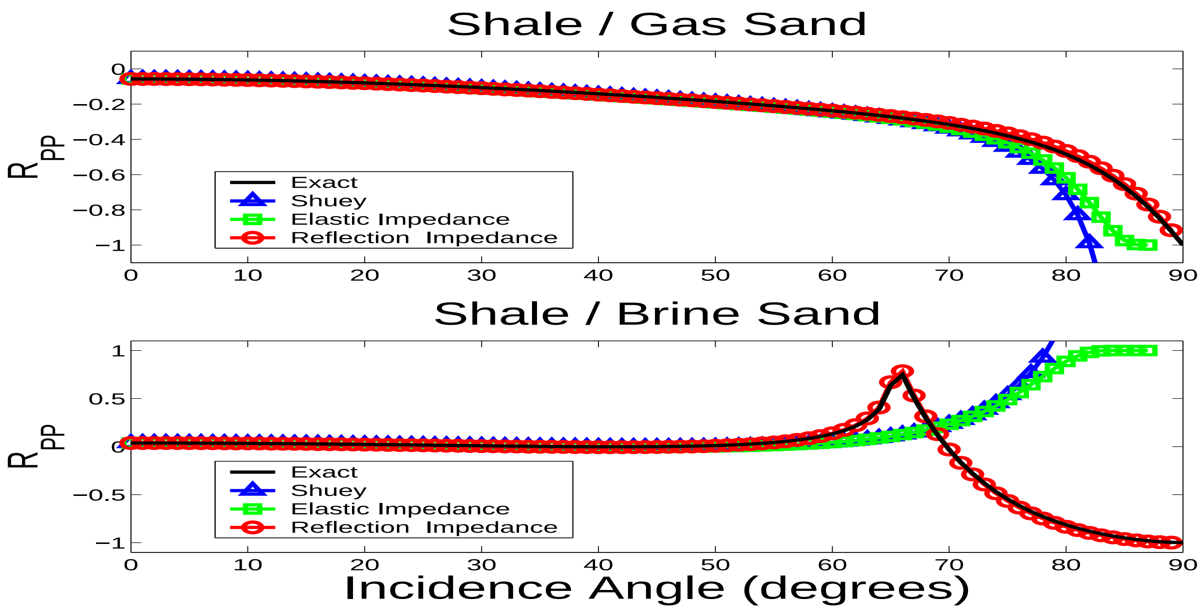

Figure 2: P-P reflection coefficient for Class II model given in Table 1: shale over gas sand (top) and shale over brine sand (bottom).

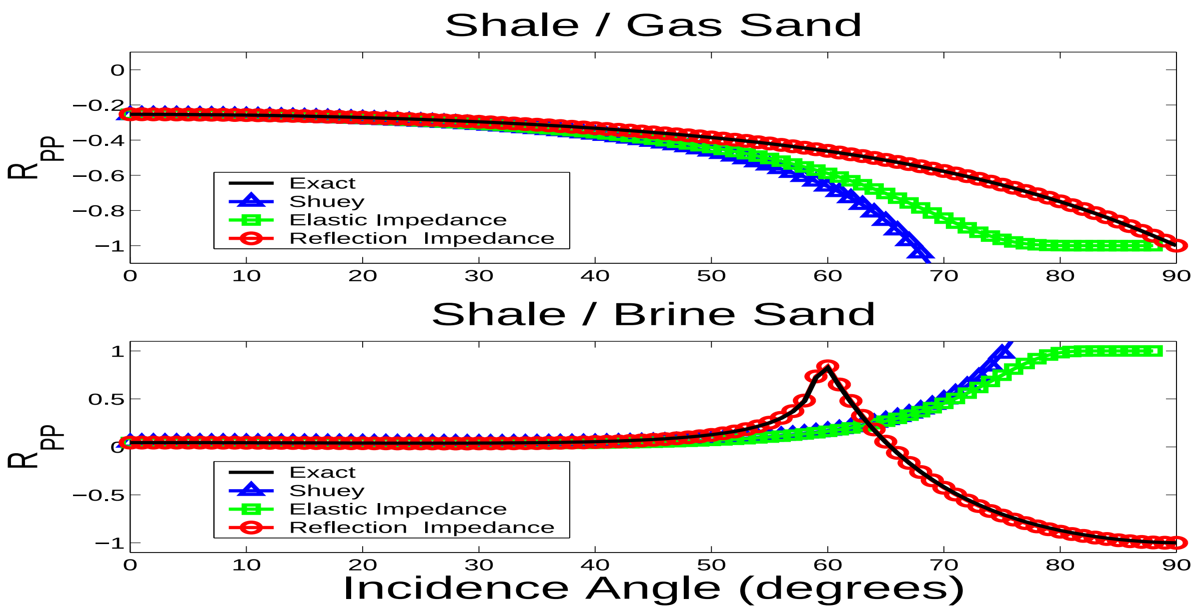

Figure 3: P-P reflection coefficient for Class III model given in Table 1: shale over gas sand (top) and shale over brine sand (bottom).

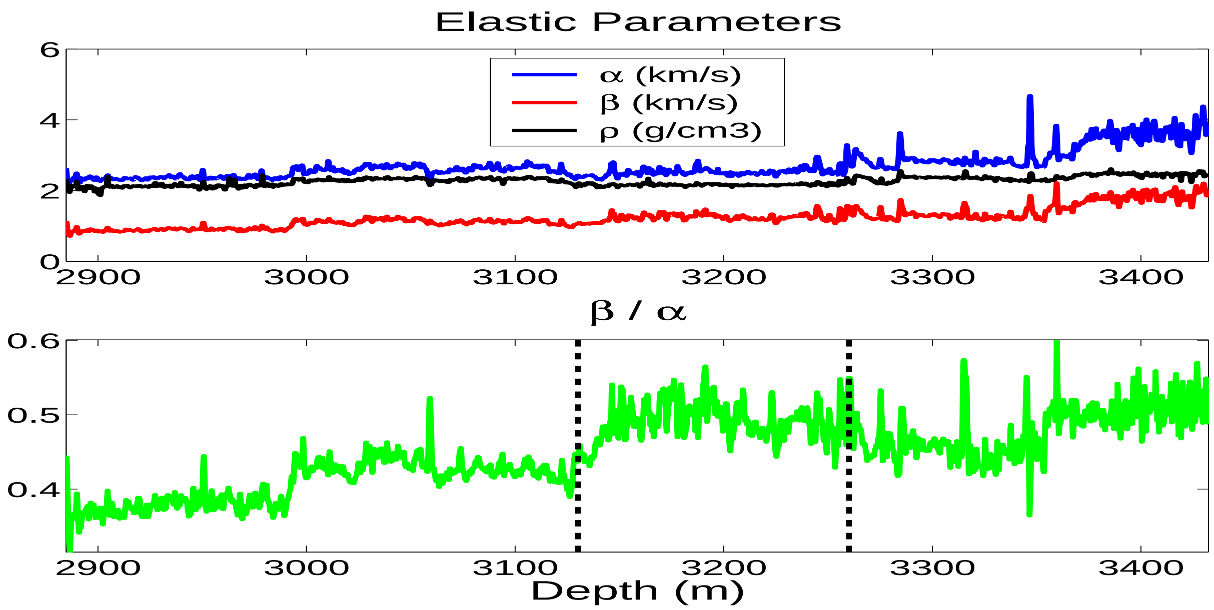

Figure 4: Well log data of an oil sand reservoir (dashed box) encased in marine shales.

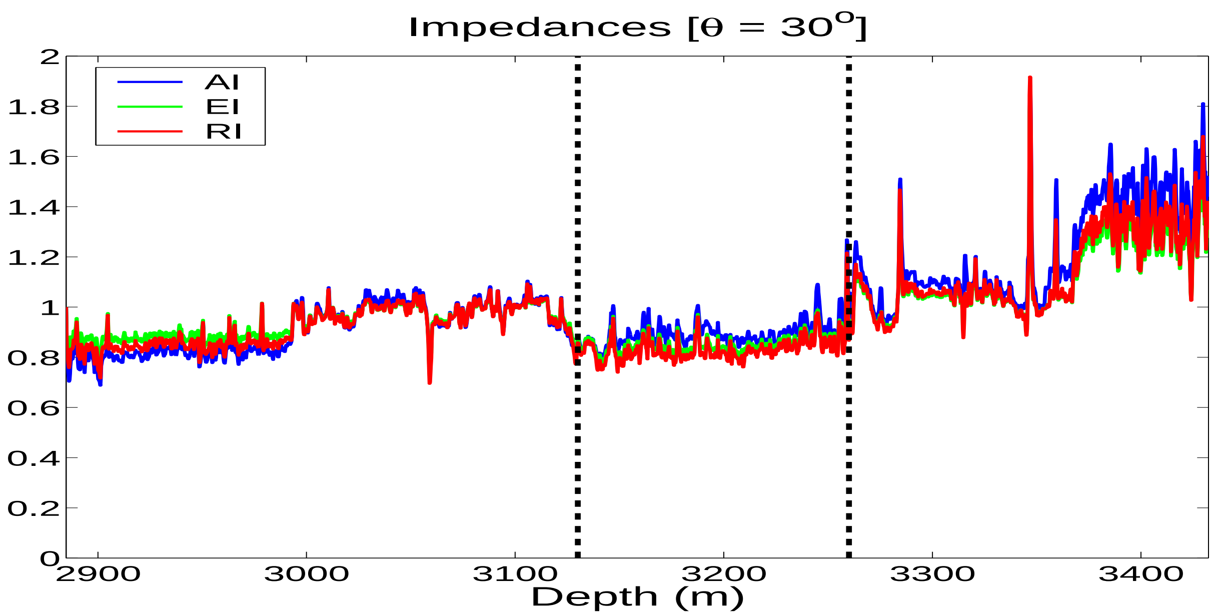

Figure 5: Comparison of the AI curve with the 30^o EI and RI curves for the well log data given in Figure 4. The values were normalized so that at the first simple $AI = EI = RI = 1$.

The values were normalized so that at the first simple $AI = EI = RI = 1$. The EI and RI curves are very similar outside the reservoir zone, but disagree in the reservoir zone, possibly because the RI method senses more the changes in the β/α ratio. Therefore, the observed differences are in part related to the higher degree of accuracy obtained by the RI method compared to the EI method. The apparent improved discrimination of the reservoir zone in the RI curve can be a key for the use of this method instead of the EI.

CONCLUSIONS

The RI method proved that it recovers back the exact reflection coefficient curve from a simple form of approximation. Additionally, when used to produce angle dependent impedances, the proposed RI method showed greater accuracy and improved degree of discrimination compared to the EI method.

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