Frequency and travel-distance dependencies of seismic scattering attenuation revealed by a weak fluctuation approximation and numerical experiments

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ABSTRACT

We present a scattering attenuation model based on the statistical wave propagation theory in random media. It is suitable for the weak wavefield fluctuation regime and has practically no restriction in the frequency domain. The presented formulas allow to quantify scattering attenuation in complex geological regions using simple statistical estimates from well-log data. This knowledge is important for further petrophysical interpretations of reservoir rocks. To test our theory we perform numerical simulations of seismic wave propagation in 2-D random media using a finite-difference solution of the elastodynamic wave equation. From the synthetic seismograms we determine the quality factor Q with help of spectral decay methods. We find good agreement of the frequency- and traveldistance-dependent Q values and the theoretical predictions.

INTRODUCTION

Deterministic approaches are not suitable to describe the complex structures of reservoirs. In contrast to this, stochastic models provide an interesting alternative and are ideally used complementary with deterministic models. It is of great importance for the interpretation of seismic data to quantify the magnitude as well as the frequency dependence of attenuation. Usually the attenuation of seismic wavefields are characterized with help of the quality factor Q. Analytical results are obtained within the framework of wave propagation theory in random media. Theoretical methods developed in order to quantify scattering attenuation include the meanfield theory using the Born approximation or the traveltime-corrected meanfield formalism which is commonly used in seismological studies (Sato and Fehler, 1998). The meanfield theory overestimates the scattering attenuation, whereas the traveltime-corrected meanfield excludes large wavenumbers so that scattering on large-scale heterogeneities is not taken into account. It requires a heuristically chosen cut-off wavenumber (or equivalently a scattering angle) which can be only determined by numerical tests. In addition, numerous numerical studies characterized the scattering attenuation in random media (e.g. Frankel and Clayton, 1986, Shapiro and Kneib, 1993, Frenje and Juhlin, 2000).

It is the purpose of this study to present a model of scattering attenuation. Within the framework of the extension of the O'Doherty-Anstey theory to 2-D and 3-D random media (Müller and Shapiro, 2001a, Müller et al., 2001) we obtain formulas for scattering Q. To test this theory, we perform numerical simulations of wave propagation in 2-D random media. With help of a spectral decay analysis of the recorded seismograms we determine frequency- and traveldistance-dependent Q^{-1} values and find good agreement with the theoretical predictions.

SEISMIC SCATTERING ATTENUATION

Based on the Rytov and Bourret approximations and the causality principle, we give a description of scattering attenuation for plane waves propagating in 2-D and 3-D weakly heterogeneous elastic solids (for a detailed derivation see Müller et al., 2001). For plane wave propagation in 3-D we obtain

$$Q^{-1}(k,L) = 4\pi^2 k \int_0^\infty d\kappa\, \kappa\, \Phi(\kappa) \left[H(2k-\kappa) - \frac{\sin(\kappa^2 L/k)}{\kappa^2 L/k} \right] , \tag{1}$$

where $k = \frac{\omega}{c_0}$ denotes the wavenumber, c_0 is the constant background velocity, L the travel-distance. $\Phi(\kappa)$ is the fluctuation spectrum which contains the second-order statistics of the medium's fluctuations, i.e. the variance σ_n^2 and the correlation length a of the P-wave (S-wave) velocity in rocks. H denotes the Heaviside step function. Note that the corresponding results in 2-D can be obtained by skipping κ in the integral over κ and dividing by π. The validity range of equation (1) in terms of the wave parameter $D = 2L/(ka^2)$ is $\max\{\frac{\lambda}{a}, \frac{\lambda^2}{a^2}\} \leq \pi D \leq \left(\frac{L}{a}\right)^2 \min\{1, \frac{\lambda}{a}\}$, where λ denotes the wavelength. Note that equation (1) is also restricted to the weak wavefield fluctuation regime. We emphasize that the scattering attenuation estimate (1) is dependent on the travel-distance. Another Q^{-1} estimate, which proves to be useful in order to verify the numerical results, is obtained by averaging equation (1) over the travel-distance interval $[L_0, L_1]$ under consideration:

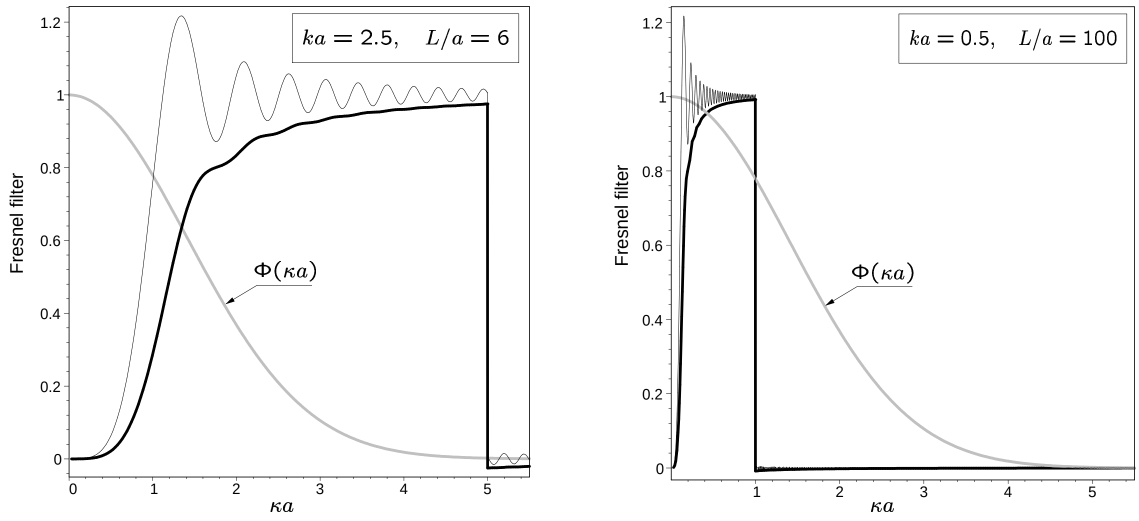
$$\langle Q^{-1} \rangle_L \equiv \frac{1}{|L_1 - L_0|} \int_{L_0}^{L_1} Q^{-1}(L')dL' \,. \tag{2}$$

For $L_0 = 0$ we simply obtain

$$\langle Q^{-1}(k,L) \rangle_L = 4\pi^2 k \int_0^\infty d\kappa\, \kappa\, \Phi(\kappa) \left[H(2k-\kappa) - \frac{\mathrm{Si}(\kappa^2 L/k)}{\kappa^2 L/k} \right] , \tag{3}$$

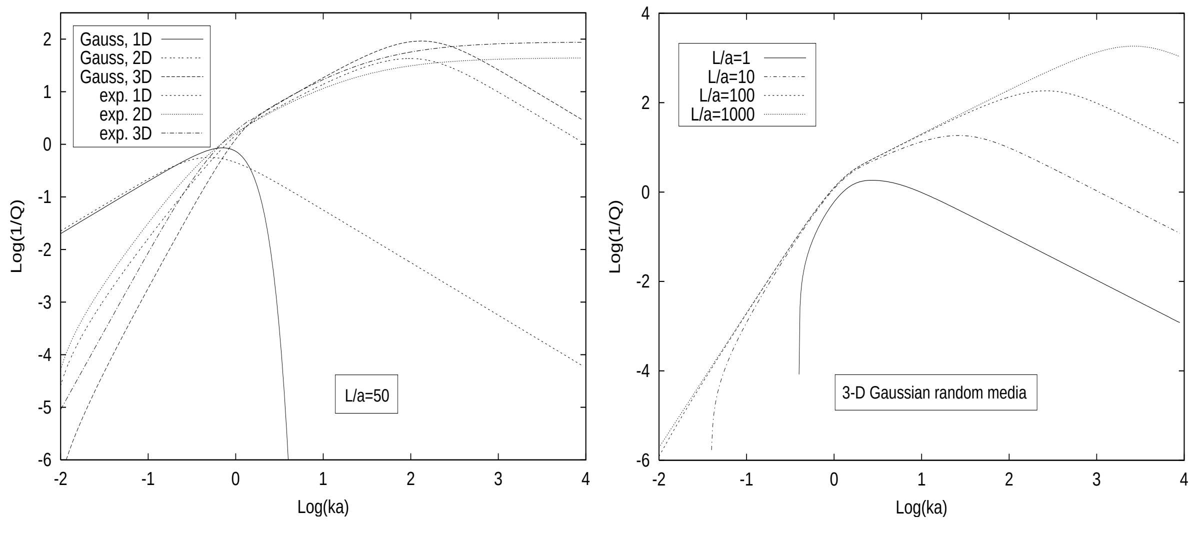
where $\mathrm{Si}(\cdot)$ denotes the sine integral and the subscript of L is omitted.

The action of the so-called Fresnel filter, the term in brackets in equations (1) and (3) on the fluctuation spectrum, is illustrated in Figure 1(a) and 1(b) by the the thin and thick black curves,



(a) **(b)**

Figure 1: Behavior of the Fresnel filter: For large frequencies and small travel-distances (small wave parameter D), the Fresnel filter (thin and thick black curves corresponding to the term in brackets of equations (1) and (3), respectively) excludes only the low wavenumber components of the fluctuation spectrum $\Phi(\kappa a)$ indicated by the grey curve (a). The opposite behavior can be observed for low frequencies and large travel-distances (large D) (b).



(a) **(b)**

Figure 2: The reciprocal quality factor $1/Q$ as a function of ka for Gaussian and exponentially correlated random media in 1-D, 2-D and 3-D (according to equations (4.18) in Shapiro and Hubral (1999) and (1), respectively), (a). The L/a dependency of the reciprocal quality factor is shown for a 3-D Gaussian random medium, (b). Note that all curves are normalized by $\frac{\sqrt{\pi}}{2}\sigma_n^2$.

respectively. Figure 2(a) depicts the reciprocal quality factor as function of the dimensionless wavenumber ka for plane waves propagating in 2-D and 3-D (exponentially and Gaussian correlated) random media. Additionally we compute Q^{-1} for waves propagating in 1-D random media according to the generalized O'Doherty-Anstey formalism of Shapiro and Hubral (1999). Note that the scattering attenuation for frequencies larger than $ka \geq 1$ is more pronounced in 2-D and 3-D than in 1-D random media. The opposite is true for frequencies smaller than $ka < 1$. The travel-distance dependency of Q^{-1} according to equation (1) is shown in Figure 2(b).

NUMERICAL EXPERIMENTS

FD simulations in random media

In order to explore the accuracy of the scattering attenuation model (1) we simulate a plane wave propagating in a single random medium realization characterized by an isotropic exponential autocorrelation function (correlation distance=40m). Similar transmission simulations in 2-D acoustic random media were performed by Frankel and Clayton (1986), Shapiro and Kneib (1993). 3-D acoustic finite-difference modeling was done by Frenje and Juhlin, 2000. Here, we present results from 2-D random media modeling based on the elastodynamic wave equation using a finite-difference method (Saenger et al., 2000). The geometry as well as the medium parameters are of the order of reservoir scales and rocks in hydrocarbon exploration, respectively. The reservoir P-wave velocity fluctuations have an average velocity of 3km/s and a standard deviation of 4 percent. Density and S-velocities were derived from P-velocities using empirical relations for sandstones. The source wavelet is a 42Hz Ricker wavelet.

Scattering attenuation estimates

In order to obtain scattering attenuation estimates we apply the so-called spectral decay method. More specifically, we consider the decay of the logarithm of amplitude spectrum with travel-distance. The slopes are then directly linked with a global scattering Q^{-1} estimate. However, care should be taken how the analysis is done. Shapiro and Kneib (1993) investigated the influence of the different processing steps. Stacking all seismograms of the common-travel-distance gather in the time domain and analyzing the logarithm of the amplitude spectra vs. L, yields the the meanfield attenuation. Averaging the amplitude spectra and then analyzing its logarithm vs. L corresponds to the traveltime-corrected attenuation estimate ${}^{ttc}Q^{-1} = -\frac{2}{k}\frac{\ln(\langle A(\omega)\rangle)}{L}$, where the angular brackets denote the averaging operator and $A(\omega)$ the amplitude spectrum. Changing the order of operations, we obtain the scattering attenuation estimate corresponding to the mean of the log-amplitude spectra $Q^{-1} = -\frac{2}{k}\frac{\langle \ln(A(\omega))\rangle}{L}$, which corresponds to equation (1).

Moreover, the determination of Q^{-1} values from the first arrivals in seismograms is very sensitive to the applied window length around the primary arrivals (Frenje and Juhlin, 2000). We applied a window length corresponding to 1.5 times the source wavelength for $L = 0$ and that increases with L to account for the broadening and the traveltime fluctuations of the primary wavefield. The Q^{-1} estimates are shown in Figure 3. The overall frequency dependence as well as the magnitude of Q^{-1} can be well explained using our scattering attenuation model. The travel-distance dependency is somewhat more difficult to determine from the synthetic data. However, averaging the Q^{-1} estimates over the travel-distances, which corresponds to a Q^{-1} estimate according to equation (3), yields a reasonably good agreement with the theory (see thick black and red curves in Figure 3).

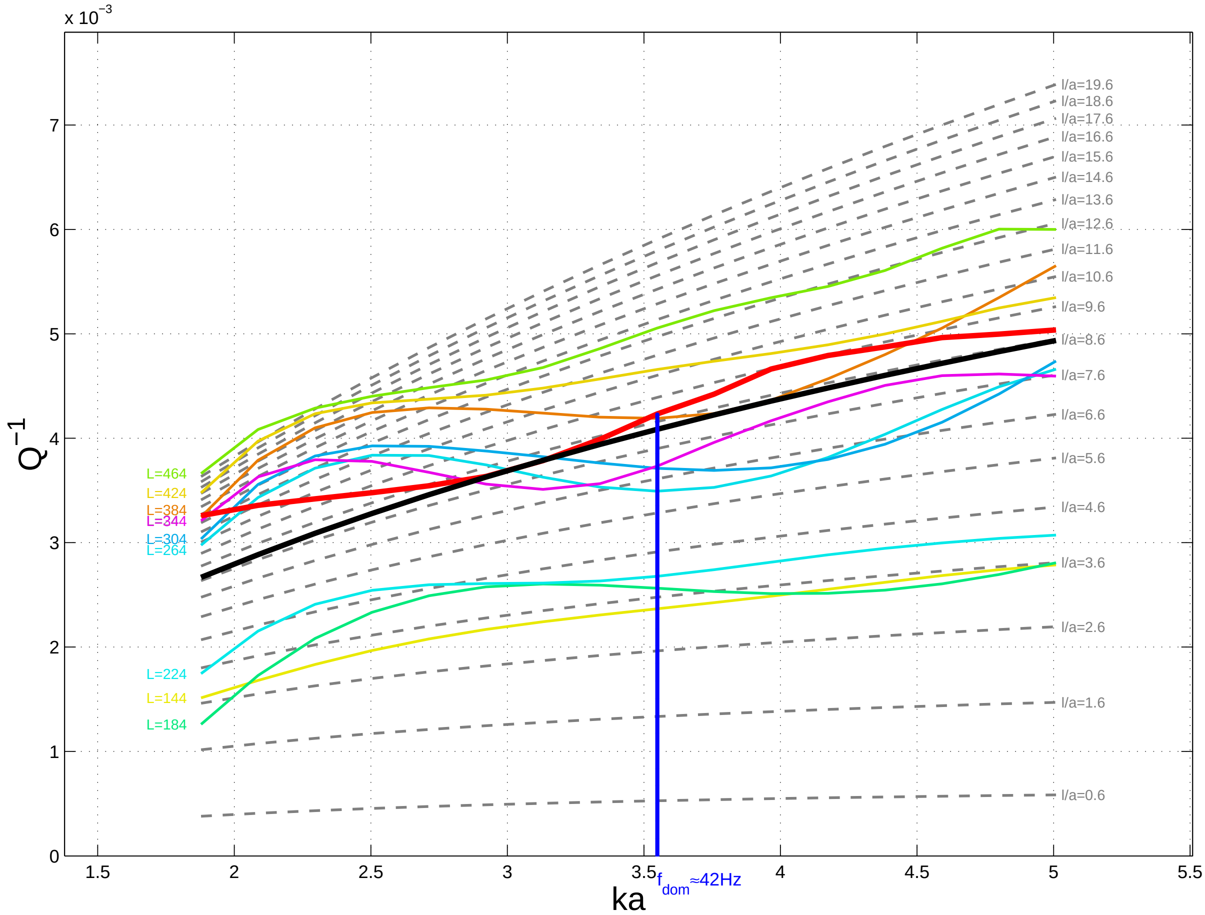


Figure 3: The numerically determined Q^{-1} estimates as a function of frequency ka and travel-distance L/a are illustrated by the colored lines (the corresponding travel-distance value is indicated on the lefthand side of the curve). The theoretical Q^{-1} values are given by the dashed curves (the corresponding travel-distance value is indicated on the righthand side of the curve). The thick, red curve results from averaging over the travel-distance of the numerically determined Q^{-1} estimates, whereas the thick, black curve results from averaging over the travel-distance of the displayed theoretical Q^{-1} curves (or equivalently, from equation (3)).

CONCLUSIONS

In this paper we present a new model of scattering attenuation valid in weakly heterogeneous elastic media. We perform finite-difference simulations of seismic wave propagation in 2-D random media to check the analytical results. The application of the spectral decay method proves to be somewhat difficult for the inversion of the travel-distance dependency of Q^{-1}. Nevertheless, an accurate data processing yields Q^{-1} values whose magnitude and frequency dependence can be explained with the proposed scattering attenuation model. To study the frequency dependence of Q^{-1} in a broader frequency range, more simulations are required.

PUBLICATIONS

Detailed results were published in Müller and Shapiro (2001a) and Müller et al. (2001).

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