# **CRS stacking formula for 3-D acquisition**

G. Höcht and S. Bergler

email: Steffen.Bergler@gpi.uni-karlsruhe.de
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# ABSTRACT

To simulate a zero-offset (ZO) volume with a very improved signal-to-noise ratio from noisy multi-coverage reflection data from a 3-D acquisition, the CRS stack uses an eight-parameter traveltime formula. The parameters can be explained by means of the wavefronts of the normal-incidence-point (NIP) and normal wave. In this paper we shortly present the traveltime formula and its parameters without giving a derivation.

# INTRODUCTION

In the last years, three- and five-parameter traveltime formulas have been formulated in terms of kinematic wavefield attributes (see,e.g., Jäger et al., 2001; Zhang et al., 2001) that serve as stacking operator for seismic reflection data from 2-D acquisition<sup>1</sup>. We want to refer to these traveltime formulas as 2-D traveltime formulas although they are valid for both, 2-D as well as 3-D subsurface media. The Common-Reflection-Surface (CRS) stack uses the traveltime formulas to construct stacked zero-offset (ZO) and common-offset (CO) sections with a very improved signal-to-noise ratio from noisy multi-coverage pre-stack reflection data. As additional output to the stacked sections, one obtains kinematic wavefield attributes-the propagation direction and curvatures of wavefronts an detected at the measurement surface—which are useful for many seismic applications. With these wavefield attributes it is possible to describe an approximation of the wavefronts at the measurement surface. The extension of the traveltime formulas to use them to stack reflection data from 3-D acquisition is mainly the extension of the wavefront description. In this paper, we present a traveltime formula which can be applied to simulate a ZO volume from multi-coverage pre-stack data from 3-D acquisition. The traveltime formula is given in terms of (now eight) kinematic wavefield attributes. In the following, we call this formula the 3-D ZO traveltime formula.

<sup>&</sup>lt;sup>1</sup>For a 2-D acquisition shots and receivers are aligned along a line.

#### LOCAL DESCRIPTION OF A WAVEFRONT

A wavefront at an observation point on the ground surface is locally described by its orientation and curvatures. The orientation of the wavefront is given by the propagation direction of the wavefront, i.e. the ray direction along which the wave travels. The ray direction can, for instance, be described by the azimuth and polar angle  $\varphi_0$  and  $\varphi_1$ . By means of the ray direction we can construct an orthogonal ray-centered coordinate system on the ground surface. Thereby, a necessary condition for the ray-centered system is that its z-axis is defined by the ray direction vector at the observation point. Its x- and y-axis lie in the plane perpendicular to the z-axis but can be arbitrarily chosen. A suitable local second-order description of the wavefront is then given in ray-centered coordinates (where the first order terms vanish) by means of the symmetric curvature matrix  $\hat{A}$ :

$$\hat{z} = -\frac{1}{2}\hat{\mathbf{x}}\hat{\mathbf{A}}\hat{\mathbf{x}} \quad \text{with} \quad \hat{\mathbf{x}} = \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix}.$$
 (1)

 $\hat{x}$ ,  $\hat{y}$ , and  $\hat{z}$  denote the coordinates of the ray-centered coordinate (rcc) system. Although Equation (1) defines a paraboloid, this description only serves to determine the curvatures of the wavefront.

For a 3-D acquisition it is in principle possible to extract the angles  $\varphi_0$  and  $\varphi_1$  and the curvature matrix  $\hat{A}$  from traveltime measurements on the ground surface. For a 2-D acquisition, however, one can only determine the wavefront description in an observation plane. This means, it is only possible to extract the orientation (given by the angle  $\alpha$ ) and the curvature of the wavefront within the observation plane (see Figures 1 and 2). The observation plane on the ground surface is defined by the ray direction vector at the observation point and the direction vector of the seismic line (indicated by s<sub>i</sub> in Figure 2).

### **TRAVELTIME FORMULA**

The parameters of the 3-D ZO traveltime formula are given by the local description of the NIP and normal wave (Hubral, 1983) at the observation point (see Figure 3). A local description of both wavefronts in a rcc system is given by Equation (1) substituting the curvature matrix  $\hat{M}$  (NIP wave) and  $\hat{N}$  (normal wave), respectively, for  $\hat{A}$ . With the curvature matrices of the two wavefronts (2 × 3 parameters) and their coincident propagation direction (2 parameters) at the ground surface, the eight parameters of the traveltime formula are described.

In a local observation coordinate (loc) system, in which the coordinates of the shot and receivers are specified, the equation for a local description of the wavefront gets more complicated. The description in the loc system involves the introduction of transformation matrices between the loc system and the rcc system. The loc system has its origin at the observation point where the z-axis is perpendicular to the measurement plane. Its x- and y-axis lie in the measurement plane and can be arbitrarily chosen.

In midpoint and half-offset coordinates (measured in the loc system) the so-called hyperbolic



**Figure 1:** A wavefront (light grey) emerging at the ground surface. The seismic line (black line on the ground surface) and the direction of the central ray at the ground surface define the observation plane. The intersection of the wavefront with the observation plane yields the bold curve.



**Figure 2:** Observation plane: the parameters observed in the 2D case describe the 3D wavefront in the observation plane.



**Figure 3:** NIP (grey) and normal wavefront (light grey) propagating through the medium along a ray.

3-D ZO traveltime reads

$$t_{hyp}^{2} = \left(t_{0} + \frac{2}{v} \mathbf{w}_{\mathbf{z}} \cdot \mathbf{m}\right)^{2} + \frac{2t_{0}}{v} \mathbf{m}^{\mathrm{T}} \mathbf{T} \,\hat{\mathbf{N}} \,\mathbf{T} \,\mathbf{m} + \frac{2t_{0}}{v} \,\mathbf{h}^{\mathrm{T}} \,\mathbf{T} \,\hat{\mathbf{M}} \,\mathbf{T} \,\mathbf{h}$$
(2)

where

- $t_0$  is the two-way traveltime along a ZO ray,
- v is the near-surface propagation velocity in the vicinity of the observation point,
- m denotes the midpoint vector with respect to the observation point; h denotes the half-offset vector between shot and receiver,
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$$\mathbf{w}_{\mathbf{z}} = \begin{pmatrix} \cos\varphi_0 \sin\varphi_1\\ \sin\varphi_0 \sin\varphi_1\\ \cos\varphi_1 \end{pmatrix}, \qquad (3)$$

• T is the upper left  $2 \times 2$  submatrix of the  $3 \times 3$  transformation matrix which describes the transformation from the loc system to the rcc system.

For a detailed derivation of the traveltime formula as well as the description of wavefronts in 3-D media, we want to refer to Höcht (2001).

The 3-D ZO traveltime formula can be used as CRS stacking operator for a 3-D data acquisition to simulate a ZO volume from multi-coverage reflection pre-stack data. First tests to determine the eight parameters from traveltime measurements and to construct a ZO volume by means of the CRS stack have been carried out on synthetic data by Cristini et al. (2001).

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