Extending the $\mathbf{T^2 - X^2}$ method to 3-D heterogeneous media

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keywords: *traveltimes, $T^2 - X^2$, interpolation*

ABSTRACT

Traveltimes or time fields contain informations on wave field properties of seismic waves. The wavefront curvature corresponds to one of the most important wave field properties. It is related to the normal moveout and the geometrical spreading and therefore leads to many important applications in seismic data processing, e.g., computation of migration weights, NMO corrections, Fresnel zones as well as an accurate and economical interpolation of traveltimes. We present a technique, which is based on the assumption that any arbitrarily shaped wavefront can locally be approximated by a sphere. No particular type of model is assumed. This corresponds to a hyperbolic expansion of traveltimes and leads to traveltime hyperbolae. If an isotropic horizontally stratified medium is assumed, the obtained relations reduce to the well known NMO relation. This relation is used in the $T^2 - X^2$ method to determine the NMO velocity for this particular type of model. Therefore, the described technique can be understood as an extension of the $T^2 - X^2$ method to arbitrary 3-D heterogeneous (and even anisotropic) media. A numerical example demonstrates the efficiency and accuracy of the application of the technique to traveltime interpolation. Further applications of the method are, e.g., in amplitude preserving migration where all required quantities are determined alone from coarse gridded traveltime tables.

INTRODUCTION

The $T^2 - X^2$ method is a standard technique to determine NMO velocities of seismic reflections. For a horizontally stratified medium the traveltime corresponds (in the short offset approximation) to a hyperbola of the form

$$T = \sqrt{T_0^2 + \frac{X^2}{V_{\mathrm{RMS}}^2}} \quad . \tag{1}$$

Here V_{RMS} is the RMS velocity which controls the move out, T_0 is the two way traveltime at the zero offset receiver and X is the distance from the shot. The RMS velocity can be determined from the parameters of the horizontally stratified medium (Taner and Koehler, 1969). In the $T^2 - X^2$ domain the moveout is linear and can be directly determined from the slope of the T^2 curve, which is a straight line. The NMO velocity is related to the wave front curvature (Hubral and Krey, 1980). This wave field property is obtained by simple differencing in the $T^2 - X^2$ domain. Moreover, the wave front curvature is related to geometrical spreading, i.e., applications with respect to NMO, divergence corrections and amplitude preserving migration are possible.

The classical $T^2 - X^2$ method was derived for monotypic reflections under the assumption of a horizontally stratified subsurface. In the following sections we will consider traveltimes or time fields without assuming a particular model or wave type. The results represent an extension of the $T^2 - X^2$ method to 3-D heterogeneous media.

METHOD

In the following description we consider traveltimes and time fields synonymously. Time fields are characterized by certain properties, like the traveltime itself, its slope (i.e., slowness or first derivative of traveltime) and curvature (i.e., second derivative of traveltime). These quantities are general properties of time fields and can be described without assuming a particular model or wave type.

We assume locally smooth traveltime fields. This is a natural assumption, since a band limited signal averages the subsurface over the dimension of the first Fresnel zone, leading to smooth traveltimes in this area. Any locally smooth function can be expanded into a Taylor series. Here we drop all terms which are higher than second order. Since we are inspired by traveltime hyperbolae in the classical $T^2 - X^2$ method, we do not expand T, but T^2, which leads to the following expression:

$$\tau^2(\hat{\mathbf{s}}, \hat{\mathbf{g}}) = (\tau_0 - \hat{\mathbf{p}}_0 \triangle\hat{\mathbf{s}} + \hat{\mathbf{q}}_0 \triangle\hat{\mathbf{g}})^2 - \tau_0 \, \triangle\hat{\mathbf{s}}^\top \underline{\hat{\mathsf{S}}} \, \triangle\hat{\mathbf{s}} \, + \, \tau_0 \, \triangle\hat{\mathbf{g}}^\top \underline{\hat{\mathsf{G}}} \, \triangle\hat{\mathbf{g}} - 2 \, \tau_0 \, \triangle\hat{\mathbf{s}}^\top \underline{\hat{\mathsf{N}}} \, \triangle\hat{\mathbf{g}} \, + \, \mathcal{O}(3) \quad . \tag{2}$$

For the 3-D case, the expansion has to be carried out in 6 variables: the 3 components of the source position vector $\hat{\mathbf{s}} = (s_1, s_2, s_3)^\top$ and those of the receiver position $\hat{\mathbf{g}} = (g_1, g_2, g_3)^\top$. The values of $\hat{\mathbf{s}}$ and $\hat{\mathbf{g}}$ in the expansion point are $\hat{\mathbf{s}}_0$ and $\hat{\mathbf{g}}_0$ with the traveltime τ_0 from $\hat{\mathbf{s}}_0$ to $\hat{\mathbf{g}}_0$. The variations in source and receiver positions $\triangle\hat{\mathbf{s}}$ and $\triangle\hat{\mathbf{g}}$ are such that $\hat{\mathbf{s}} = \hat{\mathbf{s}}_0 + \triangle\hat{\mathbf{s}}$ and $\hat{\mathbf{g}} = \hat{\mathbf{g}}_0 + \triangle\hat{\mathbf{g}}$. $\hat{\mathbf{p}}_0$ and $\hat{\mathbf{q}}_0$ correspond to the slowness vectors (first derivatives of traveltime) at the source and receiver position, i.e. (in index notation)

$$p_{0i} = - \left. \frac{\partial \tau}{\partial s_i} \right|_{\hat{\mathbf{s}}_0, \hat{\mathbf{g}}_0} \qquad q_{0i} = \left. \frac{\partial \tau}{\partial g_i} \right|_{\hat{\mathbf{s}}_0, \hat{\mathbf{g}}_0} \qquad . \tag{3}$$

The second order derivatives in the expansion are given by the matrices $\underline{\hat{\mathsf{S}}}$, $\underline{\hat{\mathsf{G}}}$ and $\underline{\hat{\mathsf{N}}}$ with

$$\begin{aligned} S_{ij} &= - \left. \frac{\partial^2 \tau}{\partial s_i \partial s_j} \right|_{\hat{\mathbf{s}}_0, \hat{\mathbf{g}}_0} = S_{ji} \\ G_{ij} &= \left. \frac{\partial^2 \tau}{\partial g_i \partial g_j} \right|_{\hat{\mathbf{s}}_0, \hat{\mathbf{g}}_0} = G_{ji} \\ N_{ij} &= - \left. \frac{\partial^2 \tau}{\partial s_i \partial g_j} \right|_{\hat{\mathbf{s}}_0, \hat{\mathbf{g}}_0} \neq N_{ji} \quad . \end{aligned} \tag{4}$$

Provided that the distance to the expansion point is small, the Taylor series yields a good approximation for the original traveltime field. The size of the vicinity describing ‘small’ distances depends on the scale of variations in the input model (but please note, that no particular type of model is assumed yet since we consider time fields). A result similar to Eq. (2) for reflection traveltimes was presented by Ursin (1982). Physically, the second order expansion described above corresponds to a local approximation of an arbitrarily shaped wave front by a sphere.

Now we consider a CMP-situation. Here $|\hat{\mathbf{s}}| = -|\hat{\mathbf{g}}| = \frac{r}{2}$ is the half offset. For the zero offset receiver Eq. (2) reduces to

$$\tau^2 = \tau_0^2 + \frac{1}{2}\,\tau_0 N r^2 = \tau_0^2 + \frac{r^2}{v_{\mathrm{NMO}}^2} \qquad , \tag{5}$$

with

$$v_{\mathrm{NMO}}^2 = \frac{2}{\tau_0 N} \quad , \tag{6}$$

where N depends on the matrix elements N_{ij}.

Eq. (5) describes a traveltime hyperbola. Since we have not made any assumption on the model yet, we can draw a general conclusion: *For the CMP configuration the concept of traveltime hyperbolae is universal and independent of the model and wave type under consideration*. The moveout of this reflection hyperbola is controlled by the moveout velocity v_{NMO} which depends on the 2nd derivatives of traveltimes N (which is related to curvature) and the zero offset time (see also Hubral et al. 1993). With regard to time processing and NMO there is no particular distinction between time fields from isotropic and anisotropic media and the particular wave type considered. Both are described by a NMO velocity and zero offset time.

If we consider an isotropic laterally homogeneous layered model and monotypic waves, v_{NMO} turns into V_{RMS} – the RMS velocity. In this sense, Eq. (5) can be considered as an extension of the $T^2 - X^2$ method to arbitrary reflected waves of any type in 3-D heterogeneous isotropic and anisotropic media. For the layer cake model a closed form relation between the NMO velocity and the model parameters exists (Taner and Koehler, 1969), which can be even inverted through Dix’s formula (see, e.g., Yilmaz (1987) or any other text book on seismic data processing). For more complicated models (laterally heterogeneous, anisotropic, etc.) this relation can only be obtained through modeling. In terms of time processing, however, there exists no distinction.

Eq. (2) represents an even further generalization of the NMO equation (5) since it is not restricted to the CMP and zero offset situation. Therefore, it can be considered as a general NMO formula. This allows to choose the point of expansion at any offset and the move out remains hyperbolic. Only in the case of a large distance from the expansion point non-hyperbolic moveout has to be considered. However, since the point of expansion can be arbitrarily chosen in Eq. (2), we can always locate it close to the receiver under consideration. Therefore, non-hyperbolic moveout is a not an issue using the NMO formula Eq. (2). However, if the expansion point corresponds to the zero offset receiver like in Eq. (5) and we want to consider far away receivers, the well known non-hyperbolic moveout problems arise. In the next section, we will discuss some practical considerations concerning the generalized move out formula.

PRACTICAL CONSIDERATIONS

How can we take advantage of the NMO formula Eq. (2)? Let us consider a multi fold experiment where the fold is higher than the number of unknowns (first and second derivatives of traveltimes) in Eq. (2). Obviously we can solve for the unknowns here (referred to as coefficients in the following). This means we can solve for first (slownesses) and second derivatives (curvatures) of traveltime. These are wave field properties which lead to many important applications like the determination of geometrical spreading, migration weights, Fresnel zones, optimized migration apertures, and an efficient and accurate (up to the second order) interpolation of traveltimes, just to name a few. More details on this procedure and how the above mentioned applications are linked to first and second derivatives of traveltimes is described in Vanelle and Gajewski (2001b) and references therein.

For the above mentioned application traveltimes are needed. Thus, Eq. (2) is most easily applied if traveltimes are directly available, like for Kirchhoff migration, where traveltime tables are generated for the diffraction stack. In reflection data, traveltimes can be determined by picking. However, picking is a time consuming and unstable procedure. Fortunately the NMO formula Eq. (5) is closely related to the common reflection surface (CRS) stack (Jäger et al., 2001) and Eq. (2) is related to the common offset reflection surface stack (Zhang and Bergler, 2001). Thus, the coefficients in the NMO equations can also be determined by stacking.

APPLICATION

As an application of the extended $T^2 - X^2$ method we present an interpolation of traveltimes from a coarse grid to a fine grid (e.g., a migration grid) using Eq. (2) where the coefficients are determined from coarse gridded traveltime tables. We use a 3-D extension of the Marmousi model shown in Figure 1. First arrival traveltimes were computed with a 3-D-FD eikonal solver using the Vidale (1990) algorithm on a 12.5m fine grid. These were used as reference data as

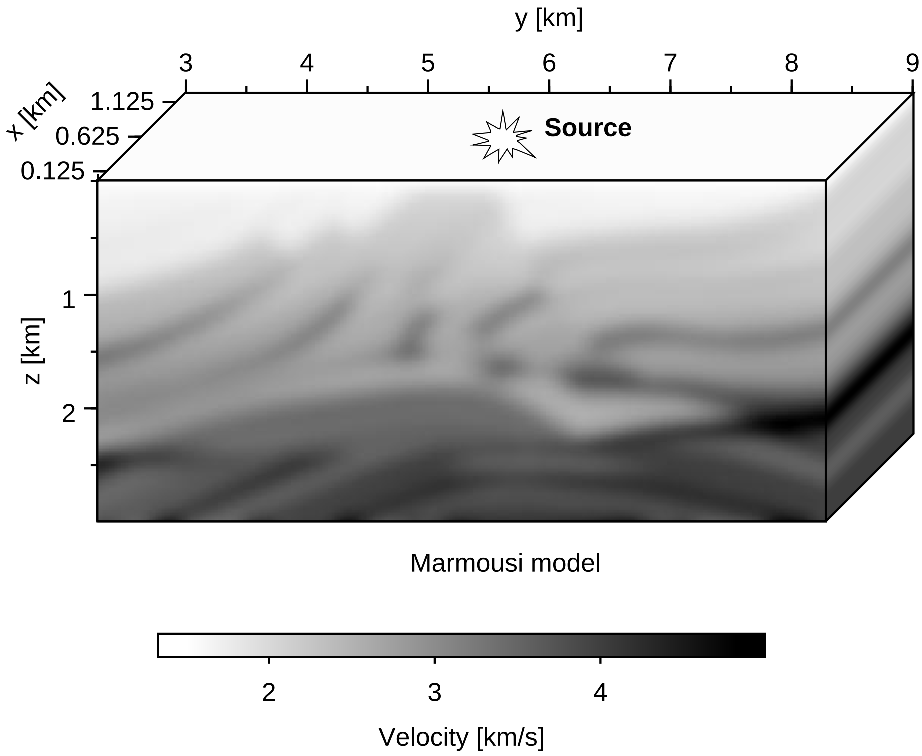

Figure 1: Marmousi model extended to three dimensions. The model was 100-fold smoothed. Only a part of the original model was used. The grid spacing is 12.5m in either direction. The source is placed on top of the model at 625m in x- and 6km in y-direction.

well as for the input traveltime tables, which were obtained from the fine grid data by resampling them onto a 125m coarse grid. The resulting interpolated traveltimes were compared to the directly computed reference data on the fine grid. The relative traveltime errors for the hyperbolic interpolation are shown in Figure 2. Except for a few areas a very good correspondence between interpolated and exact traveltimes is observed (the median error is 0.025 %). Please note, that the coefficients used here are the same as for the above mentioned other applications.

In this example there are, however, areas with higher errors. These are no measure for the accuracy of the interpolation scheme because they only occur in the vicinity of ‘kinks’ in the isochrones. These indicate triplications of the wavefronts. The resulting errors are not surprising because the assumption of smooth traveltimes does not hold here. A triplication consists of wavefronts belonging to two different branches. These must be interpolated separately which was not implemented in the program version used.

The NMO equation (2) not only allows to interpolate between grid points but also between sources. This leads to considerable savings in storage of traveltime tables. If you consider only every 10th grid point and source position (provided your model allows such a sampling) savings

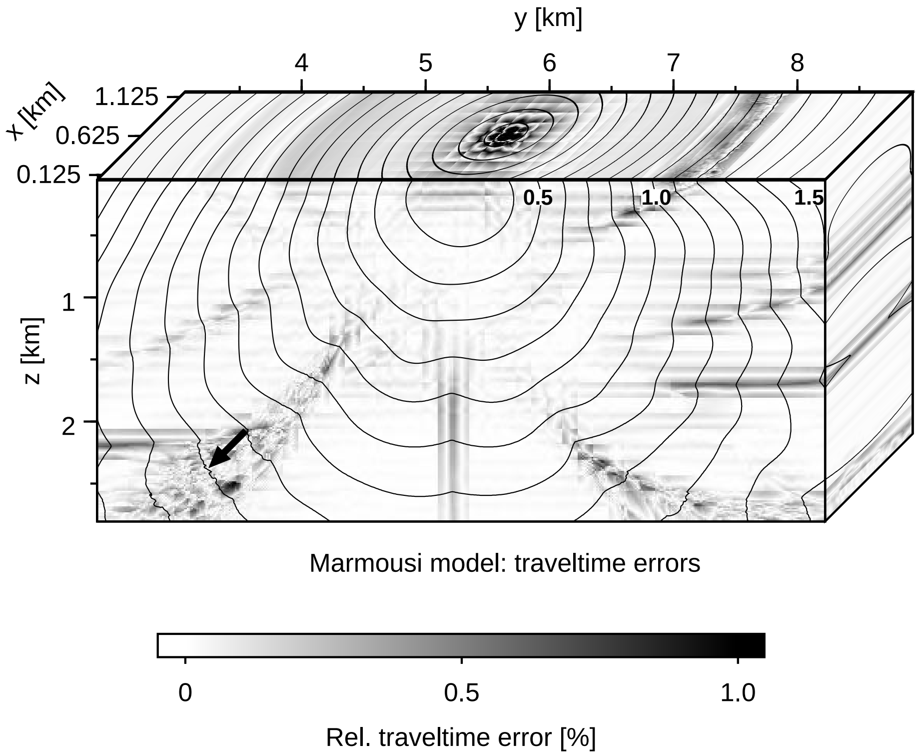

Figure 2: Relative traveltime errors for the Marmousi model using hyperbolic interpolation. Isochrones are given in seconds. The correlation of errors and 'kinks' in the isochrones is clearly visible. The arrow at the 1.2s isochrone indicates a higher error area that is caused by bad quality of the input traveltimes (due to a deficiency of the FD implementation used). This can be compensated by smoothing the input traveltimes.

in storage are 10^5 for a 3-D model and 10^3 for a 2-D model. These savings even allow to directly compute traveltimes from every subsurface position of the coarse grid to the surface of the model (in contrast to the usual top down approach, i.e., sources at the surface are considered and travel times at depth are computed by combining appropriate sources). This technique is much more suitable for sorting data into common angle gathers.

CONCLUSIONS

In a second order approximation traveltime hyperbolae are a general feature of seismic wave propagation. This is valid for 3-D heterogeneous (an)isotropic media and any type of wave. General NMO formulae were used to derive wave field properties like slownesses and curvatures from time fields. These properties lead to a manifold of applications in seismic processing and imaging, like traveltime interpolation, geometrical spreading, divergence corrections, dynamic corrections, Fresnel zones and migration weights and apertures. The technique is particularly suited for amplitude preserving prestack Kirchhoff migration leading to considerable savings in computational time and storage (for details see Vanelle and Gajewski 2001c).

PUBLICATIONS

More details on the traveltime interpolation and the determination of the coefficients can be found in Vanelle and Gajewski (2001c). The application of the method to amplitude-preserving migration was published in Vanelle and Gajewski (2001b). In Vanelle and Gajewski (2001a) the authors describe how the optimum migration aperture can be obtained from traveltimes.

ACKNOWLEDGMENTS

This work was supported by the German Research Society (DFG, Ga 350-10) and the sponsors of the Wave Inversion Technology (WIT) consortium. Continuous discussions with the members of the Applied Geophysics Group Hamburg are appreciated.

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