True amplitude migration in the presence of a statistically heterogeneous overburden

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ABSTRACT

This paper describes an extension to true amplitude prestack Kirchhoff migration which takes into account scattering losses due to the propagation through a statistically heterogeneous overburden. An additional weighting factor is introduced which is a function of the length of the travel path between source and receiver and the statistical parameters of the overburden (standard deviation and correlation length of the velocity heterogeneities). The practical aspects of the implementation are described and the method is tested on a synthetic data set. This example shows that it is important, simple and efficient to account for scattering losses during migration.

INTRODUCTION

The analysis of amplitudes of seismic sections is becoming increasingly important in the exploration industry as well as in crustal seismology. Impedance contrasts at a reflector and amplitude variations along it are used to quantify the physical and geological causes for reflections. For exploration issues the inference on reflector properties allows for instance to make a decision on possible oil/gas/water-contacts. In crustal seismology the characterization of bright-spots and their fluid or melt based origin is one example of the application of amplitude studies.

A widely accepted observation in exploration or crustal seismology is that in general the complexity of the reflected signal increases with the depth of the corresponding reflector. One possible reason is that the reflectors themselves exhibit an increased complexity due to their ongoing breakup on their way down to greater depths. Another reason might be that for instance a statistically heterogeneous overburden affects the wave to a large extent on its way from the source to the reflector and back to the receiver. In the latter case the wave 'accumulates complexity' and loses energy due to scattering on its way even if the structure of the actual reflector itself is simple. If one attempts to use such a recorded signal for amplitude studies one will therefore underestimate the true reflection coefficient and will misinterpret the migrated section in terms of the internal structure of the reflector. In this paper a method is given that describes how to estimate the amount of scattering loss for given statistical parameters of the overburden. With this information one can correct for the amplitude loss due to scattering which in turn allows a for more reliable estimation of reflection coefficients and AVO trends.

True amplitude prestack Kirchhoff depth imaging is the state-of-the-art technique for generating sections on which amplitude interpretations are based. The usual weight function used within the Kirchhoff integral accounts only for geometrical spreading losses between source and receiver. Some attempts have been made to incorporate effects such as absorption and dispersion (Mittet et al., 1995; Toutou, 1997), or the effect of a thinly layered medium (Wapenaar and Herrmann, 1996; Widmaier et al., 1996). Furthermore, the effect of a statistically heterogeneous overburden on the amplitudes of reflections has been recognized (Henstock and Levander, 2000) but no attempts have been made to correct for the corresponding amplitude loss. Here, the effect of a statistically heterogeneous medium on the amplitudes is described in terms of the generalized O'Doherty-Anstey theory in 2-D and a procedure is proposed which accounts for the above mentioned scattering losses in the form of an additional weighting function in the Kirchhoff integral. In the following the derivation of this weighting function, its incorporation into the true-amplitude imaging concept and some details regarding the implementation are described.

METHODOLOGY

Scattering losses in random media

Fundamental properties of seismic waves in rocks are attenuation and dispersion. Theoretical methods developed in order to describe the pulse propagation and to quantify scattering attenuation include the meanfield theory using the Born approximation or the traveltime-corrected meanfield formalism (Sato and Fehler, 1998). The meanfield theory overestimates the scattering attenuation, whereas the traveltime-corrected meanfield excludes large wavenumbers so that scattering on large-scale heterogeneities is not taken into account. Moreover, it requires a heuristically chosen cut-off wavenumber which can only be determined by numerical tests. Shapiro and Hubral (1999) developed the generalized O'Doherty-Anstey theory to describe primary wavefields in single realizations of 1-D random media.

Based on the Rytov and Bourret approximations Müller and Shapiro (2001) described the seismic pulse propagation in 2-D and 3-D random media. This work was extended using the causality principle in order to derive Green's functions for transient plane waves propagating in randomly heterogeneous elastic solids (Müller et al., 2001). It can be understood as an extension of the O'Doherty-Anstey theory to 2-D and 3-D random media. In the case of a point source in 2-D we find the following approximation for the Green's function by analogy

$$G(t,L) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \, e^{-\alpha L + i\varphi L} e^{-i\omega t} \, , \tag{1}$$

where α is the scattering attenuation coefficient

$$\alpha \;\approx\; 2\pi k^2 \int_0^\infty d\kappa \, \Phi^{2D}(\kappa)\Big[H(2k-\kappa) - \frac{\cos(\pi/2A^2)\,C(A) + \sin(\pi/2A^2)\,S(A)}{A}\Big] \, , \tag{2}$$

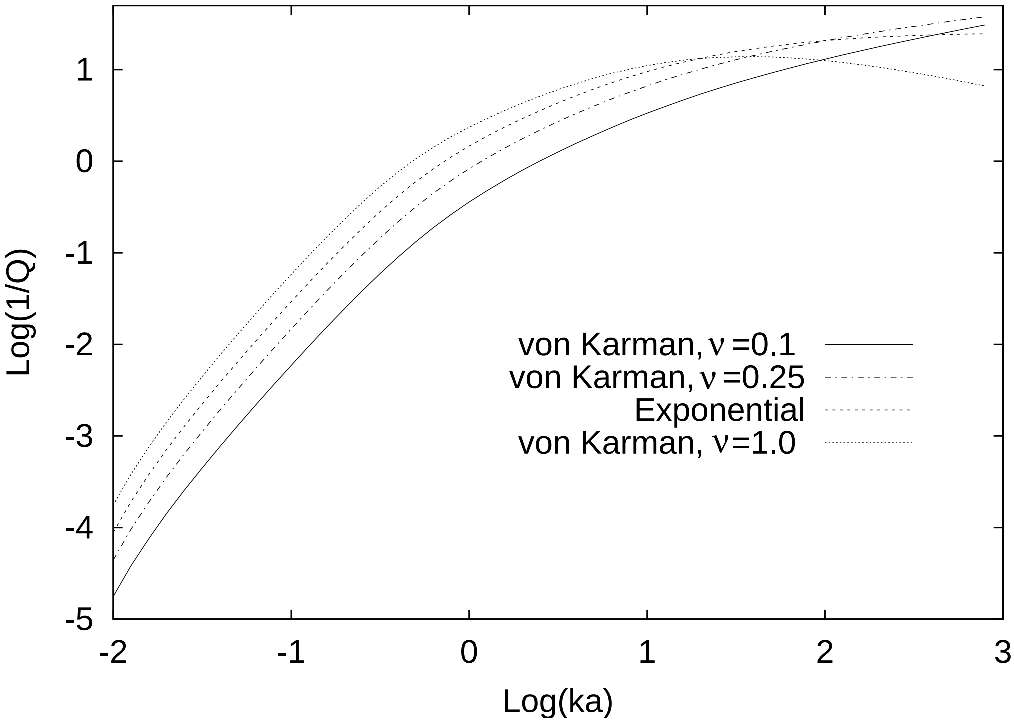

Figure 1: $1/Q$ as a function of ka for fixed $L/a = 50$ for waves radiated from a point-source in 2-D random media with different fluctuation spectra.

and φ denotes the phase increment

$$\varphi \approx k + 2\pi k^2 \int_0^\infty d\kappa \, \Phi^{2D}(\kappa) \left[\frac{2kH(2k-\kappa)}{\sqrt{\kappa^2 - 4k^2}} - \frac{\sin(\pi/2A^2)\, C(A) + \cos(\pi/2A^2)\, S(A)}{A} \right] \tag{3}$$

In equations (1)-(3) we use $A = \sqrt{\kappa^2 L/2\pi k}$, where $k = \frac{\omega}{c_0}$ denotes the wavenumber, c_0 is the constant background velocity and L is the travel distance. $\Phi^{2D}(\kappa)$ is the fluctuation spectrum which contains the second-order statistics of the medium's fluctuations i.e., the variance σ^2 of the P-wave (S-wave) velocity in rocks and the correlation length a. Moreover, H denotes the Heaviside step function, and functions C and S denote the Fresnel cosine and sine integrals, respectively. The validity range of the Green's function (1) in terms of the wave parameter $D = 2L/(ka^2)$ is $\frac{1}{\pi L/\lambda} \leq D \leq \left(\frac{L}{a}\right)^2$ if $L > \max\{\lambda, a\}$, where λ denotes the wavelength. Note that equation (1) is also restricted to the weak wavefield fluctuation regime.

The transmission losses are characterized by the scattering attenuation coefficient (2). Figure 1 depicts the reciprocal quality factor $1/Q = 2\alpha/k$ as a function of the dimensionless wavenumber ka according to equation (2) using different fluctuation spectra. The reciprocal quality factor in Figure 1 is normalized by σ^2.

True amplitude prestack Kirchhoff migration

Kirchhoff prestack depth migration is usually formulated as a weighted integral of the observed wavefield U over all receivers along the earth's surface along the corresponding diffraction curves t_{dif}:

$$M(\vec{x}) = \int\limits_{\vec{x}_{rcv}} W(\vec{x}, \vec{x}_{rcv})\; U(\vec{x}_{rcv}, t_{dif}) \tag{4}$$

The weighting function W depends on the subsurface coordinate $\vec{x}$ and the receiver coordinate $\vec{x}_{rcv}$ and accounts primarily for the geometrical spreading loss between source and receiver.

In order to correct for the scattering effect one has to simply deconvolve the observed wavefield with the inverse Green's function derived above. Then, the extended migration formula reads:

$$M_{ext}(\vec{x}) = \int\limits_{\vec{x}_{rcv}} W(\vec{x}, \vec{x}_{rcv})\; \left[G^{-1}(t, L) * U(\vec{x}_{rcv}, t_{dif})\right] \tag{5}$$

The Green's function depends on the length of the travel path L and therefore is a function of the coordinates of the source, the receiver and the subsurface point. This implies that in principle one has to compute the Green's function and to perform the deconvolution for each source, receiver and subsurface location. This would lead to an enormous increase in computing time. In the following subsection some approximations are introduced which finally lead to a practically applicable formula with only negligible increase in computing overhead.

Implementation

The first approximation is to avoid the integration over frequency in the computation of the Green's function. From the observed wavefield we instead select a dominant frequency ω_0, neglect the phase increment and compute a time-independent inverse Green's function factor:

$$G^{-1}(t, L) \rightarrow G^{-1}(L) \approx e^{\alpha(\omega_0, L)L} \tag{6}$$

This means that within the Kirchhoff integral the original deconvolution with the inverse Green's function becomes a multiplication with this factor:

$$M_{ext}(\vec{x}) \approx \int\limits_{\vec{x}_{rcv}} W(\vec{x}, \vec{x}_{rcv})\; \left[G^{-1}(L)\, U(\vec{x}_{rcv}, t_{dif})\right] \tag{7}$$

The most time-consuming part is now the computation of the scattering attenuation coefficient α, which depends directly on the length of the travel path L. Fortunately, the dependence is quasilinear so that it is not necessary to compute α for each subsurface location but rather use a precomputed coarse look-up table and to interpolate linearly during migration.

This finally leads to an efficient and relatively simple scheme and only small overhead during migration. The remaining question is which macro model to use during migration - a single realization of the statistically heterogeneous medium or a homogeneous background model. From the physical point of view it is clear that the latter is more reasonable as we will never be able to estimate a single realization of the medium. By determining a homogeneous background model one has to assure that the background velocity already incorporates the ’velocity-shift’ accounting for the fact that waves propagating in an heterogeneous medium are slightly faster than waves in a averaged-velocity medium (Gold et al., 2000).

SYNTHETIC EXAMPLE

The procedure described above is tested on a synthetic data set designed for use within the framework of the ANCORP96 profile, a deep seismic reflection survey across the central Andes, South America. The main purpose of this profile was to image the Nazca plate which is subducting under Chile. The model consists of a statistically heterogeneous overburden with an exponential autocorrelation function, a standard deviation of 1.5%, a correlation length of 1 km and a background velocity of 6 km/s (Figure 2). The Nazca plate is modeled as a dipping plane reflector between 60km and 72 km depth. The medium below this reflector is homogeneous.

For this medium a synthetic shot gather consisting of 251 receivers with a receiver spacing of 100 m was computed using a finite difference scheme. The reflection of the Nazca reflector appears between 21s and 23s two-way travel time (Figure 3). The complexity of the reflected signal itself increases with offset.

The result of a prestack Kirchhoff migration is shown in Figure 4. In the upper part a homogeneous medium is used for computing the shot gather as well as for the migration. In the lower part the shot gather of Figure 3 together with a homogeneous macro model has been used for the migration.

Finally, Figure 5 shows the amplitude along the reflector for the cases considered here. The black line shows the homogeneous medium, the dashed red line the uncorrected heterogeneous case and the solid red line the corrected heterogeneous case, respectively. Amplitude losses for this example are on the order of 15 % although standard deviation is only 1.5 %. The length of the travel path is about 120 km and is responsible for this relatively large effect. The correction is obviously able to recover the true amplitude level and to allow further reliable estimates of reflection coefficients.

Beside this the covered part of the reflector seems to shrink and appears shifted to the left. The former is due to the increased complexity of the signal and the resulting incoherent summation within the Kirchhoff integral, the latter is probably due to some difference of the travel paths in the homogeneous and heterogeneous medium, respectively. However, the detailed investigation of both effects is subject of further studies.

CONCLUSIONS

A method has been presented which combines the theory of wave propagation in random media with the true amplitude imaging concept. The described technique accounts for amplitude losses due to scattering in a statistically heterogeneous overburden by introducing an additional weight function into the Kirchhoff integral. The weight function depends on the statistical parameters of the overburden, the dominant frequency of the signal and the length of its travel path. The implementation is straightforward and simple resulting in only a small computing overhead. The

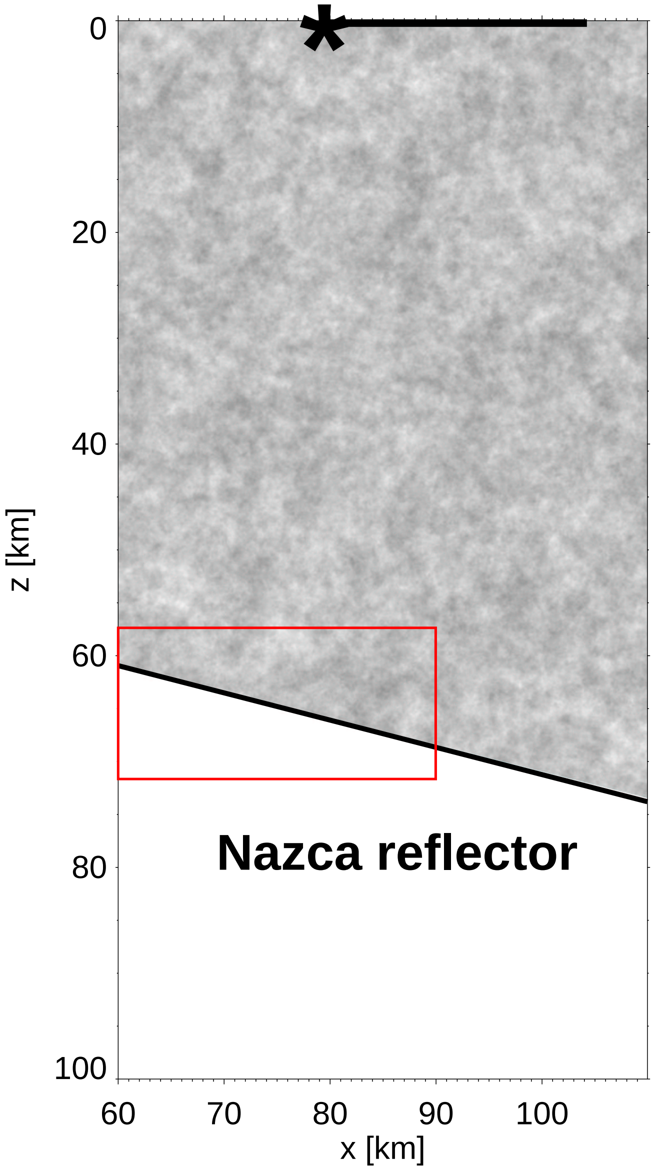

Figure 2: Model setup for the synthetic example. The red box marks the part of the model which is shown later in the migration result.

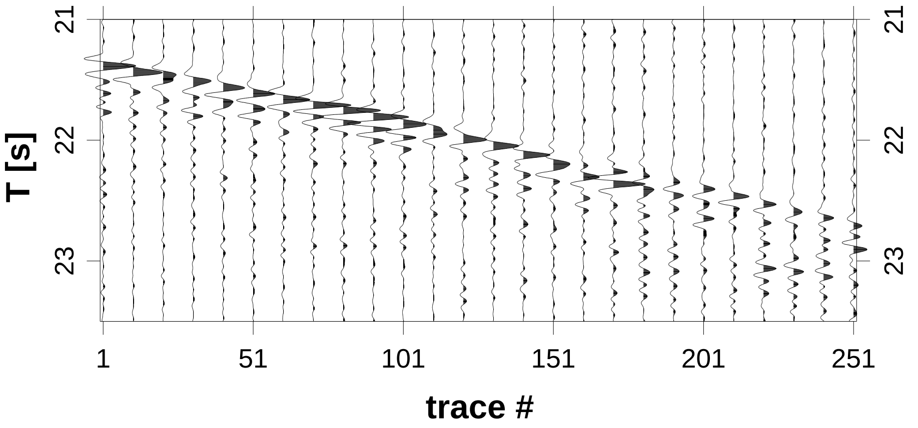

Figure 3: Synthetic shot gather for the statistically heterogeneous medium. Only every tenth trace is plotted. The reflection from the Nazca reflector appears between 21s and 23 s two-way travel time.

method has been successfully tested on a synthetic example designed for crustal seismology where scattering losses are on the order of 15 %. These scattering effects have to be taken into account in order to allow for a reliable estimation of reflection coefficients.

The method described here is not restricted to a homogeneous background medium. A laterally varying macro model containing varying statistical parameters can be used as long as the travel path through this medium and its length are known (which is possible by employing ray tracing techniques). Further investigations will include stronger fluctuations and the extension to 3-D. Within the framework of the ANCORP project studies of travel time fluctuations are planned in order to determine the statistical parameters of the medium above the Nazca plate. With the help of these parameters an estimation of the reflection coefficient of the Nazca plate becomes possible. Furthermore we intend to apply this method to exploration-type data sets (for instance sub-basalt). For this type of data the travel paths are shorter but the standard deviations are expected to be larger than in the example shown above resulting in comparable or even larger effects on the amplitudes of reflections.

PUBLICATIONS

Detailed results were published by Buske et al. (2002).

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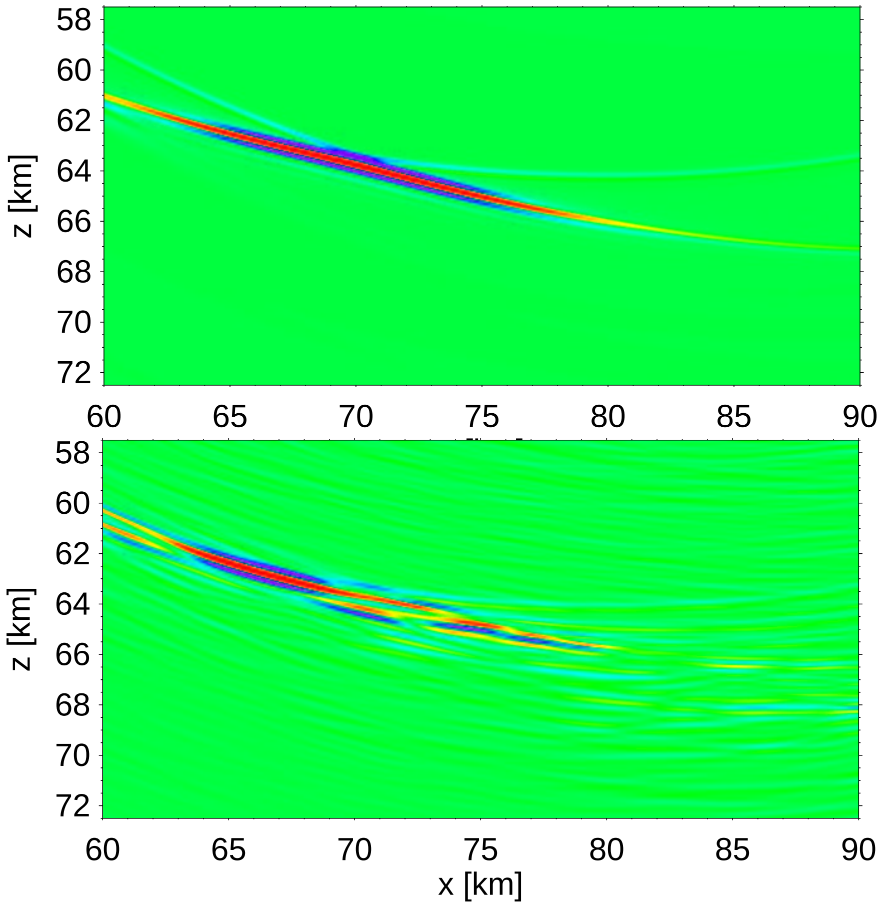

Figure 4: The result of the prestack migration. Upper part: modeling and migration performed for homogeneous background velocity macro model. Lower part: shot gather computed for heterogeneous model, migration using homogeneous background velocity macro model.

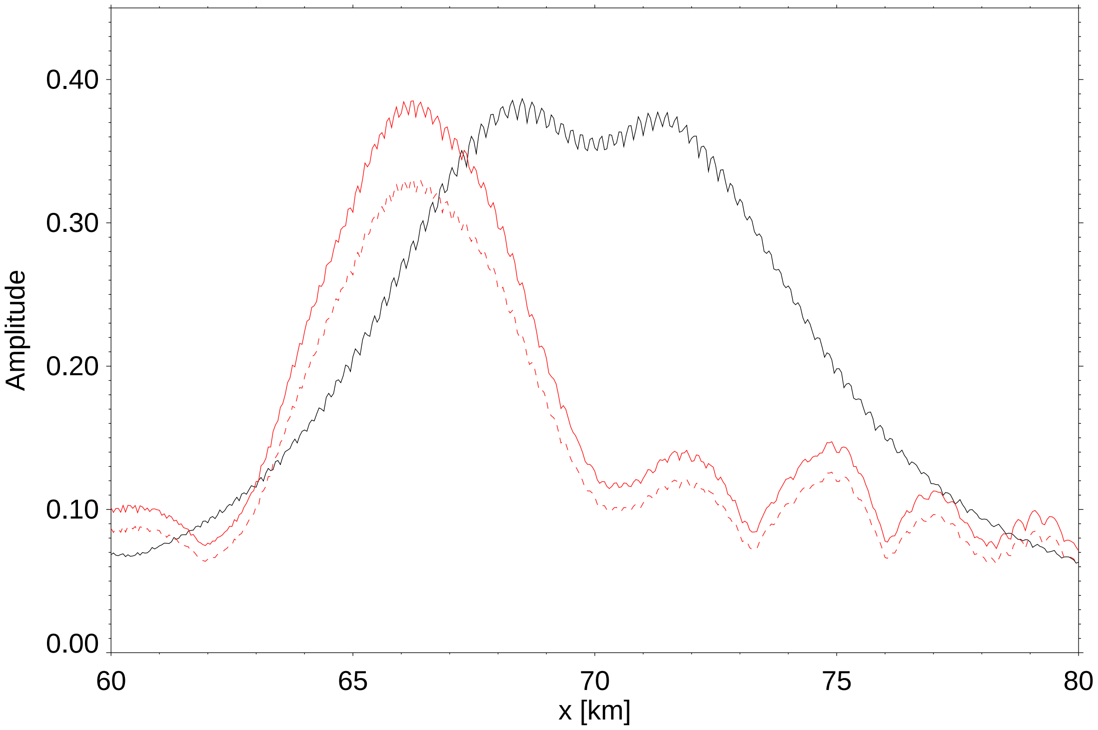

Figure 5: Amplitude along the reflectors of Figure 4. black: homogeneous, dashed red: heterogeneous uncorrected, solid red: heterogeneous corrected.

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