Multiparametric traveltime inversion

R. Biloti, L. T. Santos, and M. Tygel

email: *biloti@ime.unicamp.br*

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ABSTRACT

In conventional seismic processing, the classical algorithm of Hubral and Krey is routinely applied to extract an initial macrovelocity model that consists of a stack of homogeneous layers bounded by curved interfaces. Input for the algorithm are identified primary reflections together with normal moveout (NMO) velocities, as derived from a previous velocity analysis conducted on common midpoint (CMP) data. This work presents a modified version of the Hubral and Krey algorithm that is designed to extend the original version in two ways, namely (a) it makes an advantageous use of previously obtained common reflection surface (CRS) attributes as its input and (b) it also allows for gradient layer velocities in depth. A new strategy to recover interfaces as optimized cubic splines is also proposed. Some synthetic examples are provided to illustrate and explain the implementation of the method.

INTRODUCTION

The CRS stacking method (see, e.g., Müller et al. (1998)) is a recent technique that is establishing itself as a better alternative to the conventional NMO/DMO stacking. As recently shown in Trappe et al. (2001) (see also more references therein), the CRS stack is able to provide, in a number of cases, significantly improved stacked sections that represent simulated zero-offset sections. The CRS stacking method provides, in addition to a better stacking, a set of parameters (called the CRS attributes) that convey more information of the propagating medium than the single parameter, the NMO-velocity, that results from the NMO/DMO stack. The present paper is concerned with the use of CRS attributes for velocity model inversion.

One of the important aims of seismic processing is the construction of a depth velocity model that is consistent with the traveltimes of previously identified primary reflections. In this sense, the classical algorithm of Hubral and Krey (1980), is routinely used to produce a layered homogeneous velocity model that makes use of NMO analysis of CMP data. The natural question is, can the additional information provided by the CRS attributes, that naturally result from the application of the CRS stacking, be advantageously used for model inversion purposes?

Following earlier initial publications (Majer (2000) and Vieth (2001)), we provide a positive answer to that question. As shown below (see also Bilot et al. (2001) and Biloti (2001)), we can readily adapt the Hubral and Krey algorithm to use the CRS attributes, that naturally result from the application of the CRS stacking method to the given multicoverage data, so as to also obtain a layered model that is consistent with identified reflections. Because of the cleaner sections that result from the CRS stack, the input reflections tend to be easier to identify and select, leading to a more stable and reliable procedure.

An important feature of the proposed algorithm is that it allows for velocity gradients within the layers, thus enlarging the application possibilities for subsequent imaging procedures. Moreover, special schemes designed for keeping track of the correct selection of attributes in the presence of complicated regions, such as caustic triplications, guarantees the stable recovery of the interfaces that define the layer model.

Hyperbolic Traveltime and CRS parameters. For a given fixed, reference ray, the hyperbolic traveltime moveout expression (see, e.g., Schleicher et al. (1993);, Tygel et al. (1997)), relates the (squared) traveltime of that ray with the (squared) traveltime of any other ray (of the same code) in its vicinity. The denominations *central ray* and *paraxial ray* are commonly employed to designate the reference and vicinity rays, respectively. The important property of the traveltime moveout expression under consideration is that it is completely given in terms of (a) the initial and end point relative positions of the paraxial with respect to the central ray and (b) the dynamic properties of the central ray only. The latter are specified by means of the so-called surface-to-surface propagator matrix of the central ray.

In the case the central ray zero-offset (or normal reflection) ray at X_0, called the central point (see Figure 1) and sources and receivers are distributed on a single, horizontal seismic line (2-D situation), the hyperbolic moveout formula reads

$$T^2(x,h) = \left(t_0 + \frac{2x\sin\beta_0}{v_0}\right)^2 + \frac{2t_0\cos^2\beta_0}{v_0}\left(K_N\, x^2 + K_{NIP}\, h^2\right). \tag{1}$$

Here, x and h are the midpoint and half-offset coordinates of the source and receiver pair in the vicinity of the normal ray. More specifically,

$$x = (x_G + x_S)/2 - x_0 \quad \text{and} \quad h = (x_G - x_S)/2\,,$$

where x_S and x_G are the horizontal coordinate of the source and receiver pair (S,G) near X_0. Moreover, t_0 is the zero-offset traveltime at X_0 and β_0 is the angle of emergence at the zero-offset ray with respect to the surface normal at the central point X_0. Finally, the quantities K_{NIP} and K_N are the wavefront curvatures of the normal-incident-point wave (NIP-wave) and the normal wave (N-wave) (Hubral (1983)), respectively, measured at the central point X_0. The traveltime formula (1) is of fundamental use in the CRS method. Therefore, those three parameters, β_0, K_N and K_{NIP} are called CRS parameters.

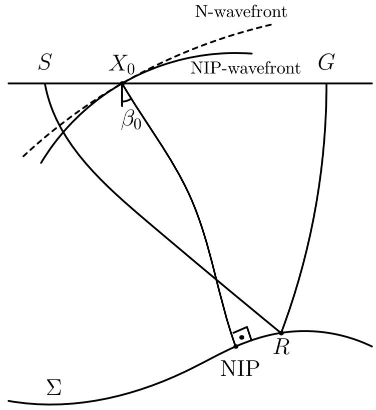

Figure 1: CRS Parameters for a normal central ray X_0 NIP X_0: the emergence angle β_0 and the NIP- and N-wavefront curvatures. Σ is the reflector, X_0 is the central point coordinate, and S and G are the source and receiver positions for a paraxial ray, reflecting at R.

THE HUBRAL AND KREY ALGORITHM

Our inversion method is based on the well-established algorithm described in Hubral and Krey (1980). There, the velocity model to be inverted from the data is assumed to consist of a stack of homogeneous layers bounded by smoothly curved interfaces. The unknowns are the velocity in each layer and the shape of each interface. These unknowns are iteratively obtained from top to bottom by means of a layer stripping process.

The main idea of the Hubral and Krey algorithm is to backpropagate the NIP-wave down to the NIP located at the bottom interface of the layer to be determined (see Figure 2). This means that the velocities and the reflectors above the layer under consideration have already been determined. Since the NIP-wave is due to a point source at the NIP, the backpropagation through this last layer gives us a focusing condition for the unknown layer velocity.

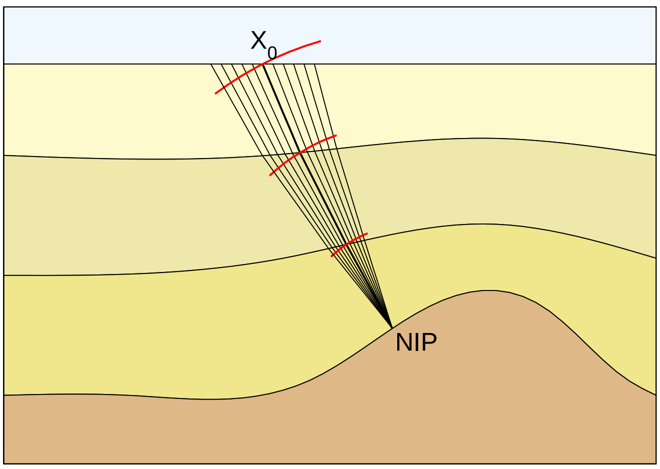

Figure 2: NIP-wavefront associated to the central zero-offset ray X_0 NIP X_0.

To describe the wavefront curvature along a ray path that propagates through the layered medium, Hubral and Krey (1980) consider two distinct situations: (a) the propagation occurs inside a homogeneous layer and (b) transmission occurs across an interface.

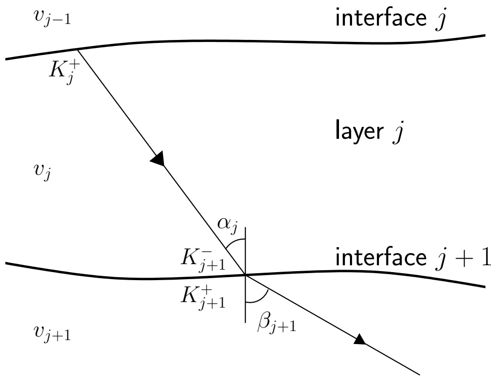

Figure 3: Ray propagation through homogeneous layer j.

Figure 3 depicts a ray that traverses the j-th homogeneous layer (of velocity v_j) being transmitted (refracted) at the interface $j+1$. Let us denote by K_j^+ the wavefront radius of curvature at the initial point of the ray (that is, just below the j-th interface). The wavefront of curvature, K_{j+1}^-, just before transmission, satisfies the relationship

$$\frac{1}{K_{j+1}^-} = \frac{1}{K_j^+} + v_j \,\Delta t_j \ , \tag{2}$$

where Δt_j is the traveltime of the ray inside the layer. The change in wavefront curvature due to transmission at the interface, as also shown in Hubral and Krey (1980), is given by

$$K_{j+1}^+ = \frac{v_{j+1}}{v_j}\frac{\cos^2\alpha_j}{\cos^2\beta_{j+1}}K_{j+1}^- + \left(\frac{v_{j+1}}{v_j}\cos\alpha_j - \cos\beta_{j+1}\right)\frac{K_{j+1}^I}{\cos^2\beta_{j+1}} \ , \tag{3}$$

where, α_j and β_{j+1} are the incident and transmission angles of the ray, respectively, and K_{j+1}^I is the interface of curvature, all these quantities being measured at the transmission point. Observe that Snell's law,

$$\frac{\sin\alpha_j}{v_j} = \frac{\sin\beta_{j+1}}{v_{j+1}} \, , \tag{4}$$

is valid. Assume now that the NIP is located at the $(N+1)$-th interface. This leads to the *focusing conditions*

$$\frac{1}{K_{N+1}^-} = 0 = \frac{1}{K_N^+} + v_N \,\Delta t_N \quad \text{and} \quad \Delta t_N = t_0 - \left(\sum_{j=1}^{N-1}\Delta t_j\right) \ , \tag{5}$$

that determine the velocity v_N. Here, K_{N+1}^- is the curvature of the wavefront at the NIP (it starts as a point source) and K_N^+ is the curvature of the wavefront after transmission across the interface

N. Note that K_N^+, as given by setting $j = N - 1$ in equation (3), has an implicit dependence on v_N and β_N. This is because, by Snell's law,

$$\sin \beta_N = \frac{\sin \alpha_{N-1}}{v_{N-1}} \, v_N \ . \tag{6}$$

Once v_N and β_N were determined, the segment of the zero-offset ray inside the N-th layer can be constructed. The sought-for NIP location is then such that its distance to that transmission point is $v_N \Delta t_N$.

Summary of the Hubral and Krey algorithm

It is instructive to discuss the key ideas involved in the preceding strategy. This is done below. We first present the main steps of the algorithm. Next we make some comments about the implementation of the various steps.

The method aims to extract a model composed by homogeneous layers separated by smoothly curved reflectors, corresponding to the well identified interfaces within the data only. This choice is made *a priori* by the user.

Determination of the first layer: The input data is, for each zero-offset ray, the traveltime t_0, the emergence angle β_0 and the wavefront curvature K_{NIP}. The velocity of the first layer is assumed to be known. Thus, only the reflector (the bottom of the first layer) should be determined. As explained below, this can be achieved in many different ways.

Determination of the jth-layer: Suppose that the model has been already determined up to the $(j-1)$th-layer. The method will proceed to the determination of the next layer, that is, the velocity of the jth-layer and the $(j+1)$th-interface. The input data is again, for each zero-offset ray reflecting at the interface $(j+1)$, the traveltime t_0, the emergence angle β_0 and the wavefront curvature K_{NIP}. Trace the zero-offset ray down to the j-th interface. Recall that this ray makes the angle β_0 with the surface normal at its initial point. Now, using equations (2) and (3), back-propagate the NIP-wave from the surface to the j-th interface along that ray. Now use the focusing conditions (5) to determine the layer velocity v_j, the angle β_j and the NIP.

The above procedure can, in principle, be done to each zero-offset ray. However, under the constraint that the layer velocity v_j is constant, we obtain an over-determined system for that unknown. How to deal with this problem will be discussed below.

BRIEF DISCUSSION OF THE ALGORITHM

We now comment on the above algorithm with regards to its accuracy and implementation. Our aim is to identify those aspects that can be improved upon the introduction of the CRS methodology.

- The quantities needed by the method (emergence angles, normal traveltimes and NIP-wavefront curvatures) are not directly available, but have to be extracted from the data. In the description in Hubral and Krey (1980), these quantities are obtained by conventional processing on CMP data.

- Note that the main idea of the method, the back-propagation of the NIP-wavefront, is carried out independently for each ray. Thus, in principle, each ray carries enough information to recover the layer velocity, that can be translated into many equations depending on the same unknown. Since we are assuming homogeneous layers, this implies an over-determination of the velocity. Of course, this question was faced on the original algorithm, but the methodology applied is not stated in the text. Hubral and Krey (1980) have pointed out that this "excess" of information could be used to improve the velocity distribution considered, for example, assuming a linear velocity variation.

- Note that the law of transmission for the wavefront curvatures depends on the curvature of the interface (K^I) at the transmission point, as we can see on formula (3). Hubral and Krey (1980) state that this can be obtained by a normal ray migration.

- As stated in Hubral and Krey (1980), after the determination of the velocity, the location of each NIP can be obtained by down propagating the last ray segment. Since each ray hits the interface normally, the local dip can also be determined.

In the next section we show, with the help of the CRS attributes, how most of the difficulties addressed above can be solved. In this way, a more accurate and efficient version of the algorithm can be obtained and, moreover, preserving its elegant structure.

THE REVISED HUBRAL AND KREY ALGORITHM

In this section we discuss how the CRS parameters can be used to fully supply the requirements of the Hubral and Krey algorithm and also how to address the involved numerical aspects. Then, we present a revised version of the original algorithm.

The obvious advantage of having the CRS parameters is that emergence angles and NIP-wavefront curvatures have been already determined. Thus, nor a velocity analysis neither a traveltime gradient estimation are required to obtain the input data for the inversion process. Moreover, with the help of the well-defined coherence section provided by the CRS method, it becomes easier to select the horizons of interest. We now proceed to point out the main features of the proposed algorithm.

Smoothing. Special care is to be taken when using estimated quantities as input data. We have to try to avoid or, at least, reduce the effect of the estimation errors on the inversion process. The strategy applied in our implementation is to smooth the parameter curves. This makes physical meaning, since no abrupt variations on the parameters can in general occur. The method used is stated in Leite (1998): For each five neighboring points on the curve, we fit a least-square parabola and replace the middle point by the corresponding one that belongs to the parabola (see Figure 4). This smoothing technique can be applied several times (we have used five times) to each parameter curve.

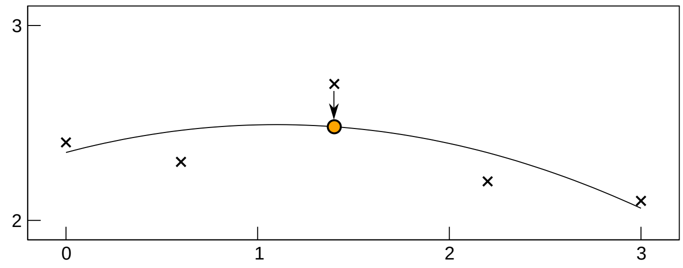

Figure 4: For each five points, the smoothing scheme fits a parabola and replaces the central point to its corresponding value at the parabola.

Unfolding criterion. The above-described smoothing method can be applied to any curve on the plane. All we need to know is how to follow the curve. In our case, the curves are parameterized by the central point coordinate. In caustic regions, more than one set of parameter values are associated to the same central point. This could generate a problem to find out the correct sequence. The reader could argue that perhaps the correct order is already known. However, since the CRS parameters are extracted from the parameter sections by some picking process, the method that we describe can be automatically built in the picking process. At the left side of Figure 5 we can see a situation where the parameter values are sorted by their central-point coordinates.

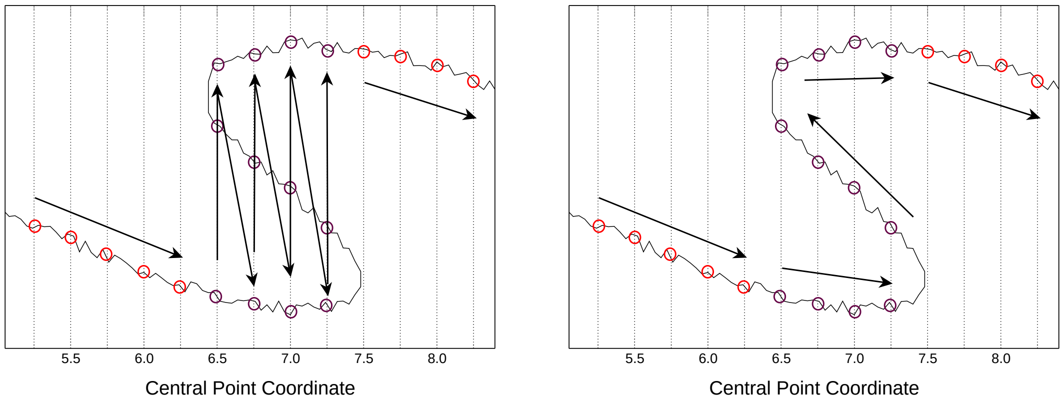

Figure 5: Both figures show a parameter curve within a caustic region. The small circles denote the sampled values of the parameter. At left, the points on the curve are sorted by their central-point coordinates. The arrows indicate the obtained sequence. At right, the same points were resorted to the correct sequence by the application of the unfolding criterion.

We have formulated a criterion to unfold the parameter curve. When the curve has more than one value for the same central point coordinate, the proposed criterion tries to keep the variations of the CRS parameters between two neighboring points on the curve as small as possible. This is a reasonable assumption since a smooth behavior of the CRS parameters is expected. The merit function which is to be minimized is

$$F(\mathbf{p}_j, \mathbf{p}_i) = \frac{|t_0^j - t_0^i|}{t_0^i} + \frac{|\beta_0^j - \beta_0^i|}{|\beta_0^i|}\,. \tag{7}$$

Here, $\mathbf{p}_i = (t_0^i, \beta_0^i)$ is a vector with components t_0^i (traveltime) and β_0^i (emergence angle), that refer to the central-point coordinate, x_0^i. Also, i is the index of the current point and j varies on the set of indices that specify their associated neighboring points. We calculate the function above for each point in the vicinity of point i. Thereafter, we assign the point that has achieved the minimum value of F to be the next one in the reordered sequence. We observe that we made additional tests including, in the merit function F, two more terms, one for K_{NIP} and one for K_N. However, we got worse results concerning the stability of the scheme. Recall that the above unfolding criterion is to be applied before the smoothing process. So it should work even if there is noise in the obtained parameter values. Several tests have confirmed the ability of the proposed criterion to effectively unfold the parameter curve.

Model with vertical velocity gradients. As earlier described, the velocity determination is carried out at each normal ray, leading to an over determination problem. To approach the velocity over-determination, we consider a solution in the sense of least squares as described below.

Our proposed algorithm assumes that the layered velocity model to be constructed is such that the velocity, in each layer, has a constant gradient in depth z-direction (an affine function on the depth). In other words, the velocity of the jth-layer is

$$v_j(z) = a_j z + b_j\,, \tag{8}$$

with constant values of a_j and b_j. As previously indicated, this feature is a significant improvement of the original Hubral and Krey algorithm in the sense that the latter admits only constant-velocity layers. Use of more general velocity profiles as in equation (8), implies that the the formulas (2) and (3), designed for homogeneous layers, are no longer valid. To account for variations of velocity inside the layers, we have used standard ray-theoretical results (see, e.g., Červený (2001)), to derive the corresponding expressions for propagation and transmission of wavefront curvatures in layered media with velocity as in (8). For the propagation of wavefront curvatures from a point A to a point B we found

$$K(B) = \frac{v(B)}{v(A)}\left(K(A)^{-1} + \frac{\sigma(B,A)}{v(A)}\right)^{-1}\,, \tag{9}$$

where

$$\sigma(B,A) = \frac{\mathrm{sign}(\cos\gamma)}{ap_0^2}\left(\sqrt{1 - p_0^2 v(A)^2} - \sqrt{1 - p_0^2 v(B)^2}\right)\,, \tag{10}$$

γ is the angle between the tangent of the ray at A and the vertical axis, a is the angular coefficient of v, and p_0 is the ray parameter. The transmission law of curvatures across an interface is found to be

$$K^+ = \frac{v^+}{v^-}\left(\frac{\cos\alpha}{\cos\beta}\right)^2 K^- + \left(\frac{v^+}{v^-}\cos\alpha - \cos\beta\right)\frac{K^I}{\cos^2\beta} + \frac{2\cos\alpha(\sin\alpha)^2\sin\gamma^- a^-}{v^-(z)}\,, \tag{11}$$

where a^- is the angular coefficient of v^-. The superscripts - and + account for the quantities before and after the transmission through the interface, respectively.

Interface construction. Concerning the construction of the layer interfaces, our approach is to fit a cubic spline, in the sense of least squares, to a set of obtained NIPs. Remember that, after the determination of the layer velocity, each zero-offset ray can be propagate down to its correspondent NIP (see Figure 6). The optimization solver employed is the GENCAN, proposed

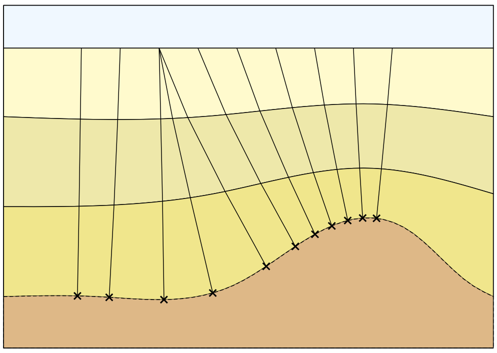

Figure 6: Several NIPs (crosses) obtained by propagation of the last ray segments.

by Birgin and Martínez (2001). GENCAN is an active-set method for smooth box-constrained minimization. The algorithm combines an unconstrained method, including a line-search which aims to add many constraints to the working set at a single iteration, with a recently introduced technique (spectral projected gradient) for dropping constraints from the working set.

IMPLEMENTATION OF THE REVISED HUBRAL AND KREY ALGORITHM

In the following, we present and comment our implementation scheme of the algorithm of revised Hubral and Krey as proposed in this work.

Input data. Recall that after the application of the CRS method, we obtain a simulated zero-offset section in which the events of interest (selected primary reflections) are well identified. This means that the traveltimes t_0 at each reflection are known. The identified primary reflections will provide the interfaces of the layered model to be inverted. Also recall that, attached to each point of the simulated zero-offset section, we have the three parameters β_0, K_{NIP} and K_N, that have been extracted from the multicoverage data. We finally note that the velocity of the medium in the vicinity of each central point is assumed to be *a priori* known.

Determination of the first layer. For a given central point, X_0, let t_0 be the zero-offset traveltime of the primary reflection at the first interface. Also, let β_0 be the corresponding emergence angle. In the present form of the algorithm, we assume that the velocity of the first layer is known and has the form given by equation (8). We trace from X_0 a ray segment that makes an angle β_0 with the surface normal at X_0 and has length equals to $v_0 t_0/2$.

The extreme of that ray segment is the NIP. Do this for all central points that illuminate the first interface. Finally, we fit a cubic spline to the obtained NIPs, in a least-squares sense, to represent the interface. As a result of this process, the first layer is completely determined.

Subsequent layers. Let us assume that the model has been already determined up to the layer $(j-1)$. Note that this means that interface j has been already constructed. The input data are now the CRS parameters that refer to the zero-offset rays that reflect at the interface $(j+1)$. Trace the rays down to the interface j and back-propagate the NIP-wavefront along those rays. Applying the focusing conditions, estimate v_j by least squares. Using Snell's law calculate β_j and, knowing the remaining traveltimes, trace the last segment of each ray. To fit the many obtained NIPs an optimized cubic spline is estimated, representing, in this way, the interface $(j+1)$.

SYNTHETIC EXAMPLES

The algorithm was applied to the model depicted in Figure 7. That model consists of three interfaces separating four homogeneous layers. The second reflector has a synclinal region between 6.5km and 7.5km that generates caustics. The data was modeled by ray tracing, using the package Seis88, designed by Červený and Pšenčík (Červený (1985)). Noise was added to the data. The CRS parameters were estimated by the strategy described in Birgin et al. (1999). Finally, those parameters were inverted by the proposed method. The obtained model, depicted by solid lines in Figure 7, shows a very good agreement with the input model.

As a second example, we consider the model depicted in Figure 8(a). It is composed by four layers. The first and the fourth layers are homogeneous. The second and the third layers are inhomogeneous, in the sense that the velocity varies both in vertical and in horizontal directions. As before, the multicoverage data was modeled by ray tracing and noise was added to it. The estimated CRS parameters were inverted by the method. The result is the model depicted in Figure 8(b). Note that the resulting model has, of course vertical gradients. To better assess the accuracy of the recovered model, Figure 8(c) shows the percentual error between the synthetic and the inverted model. Note that the errors are, almost everywhere, below 5%. In fact, the euclidian norm of the relative error is approximately 3.9.

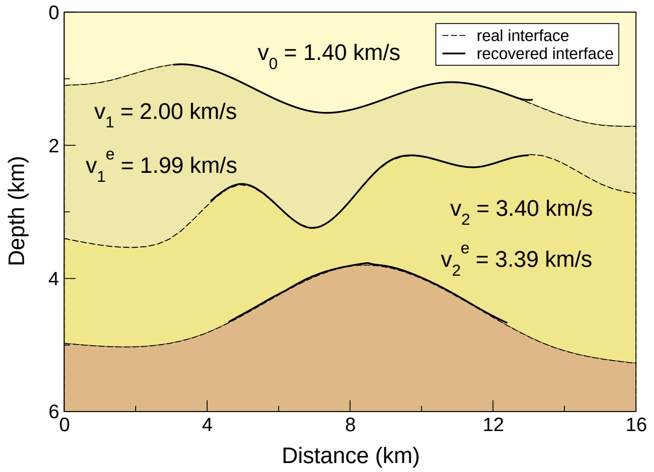

Figure 7: Synthetic model with four homogeneous layers separated by smooth interfaces (dashed lines). Inverted model composed by layers separated by the recovered interfaces (solid lines). v_j and v_j^e are the real and estimated velocities, respectively.

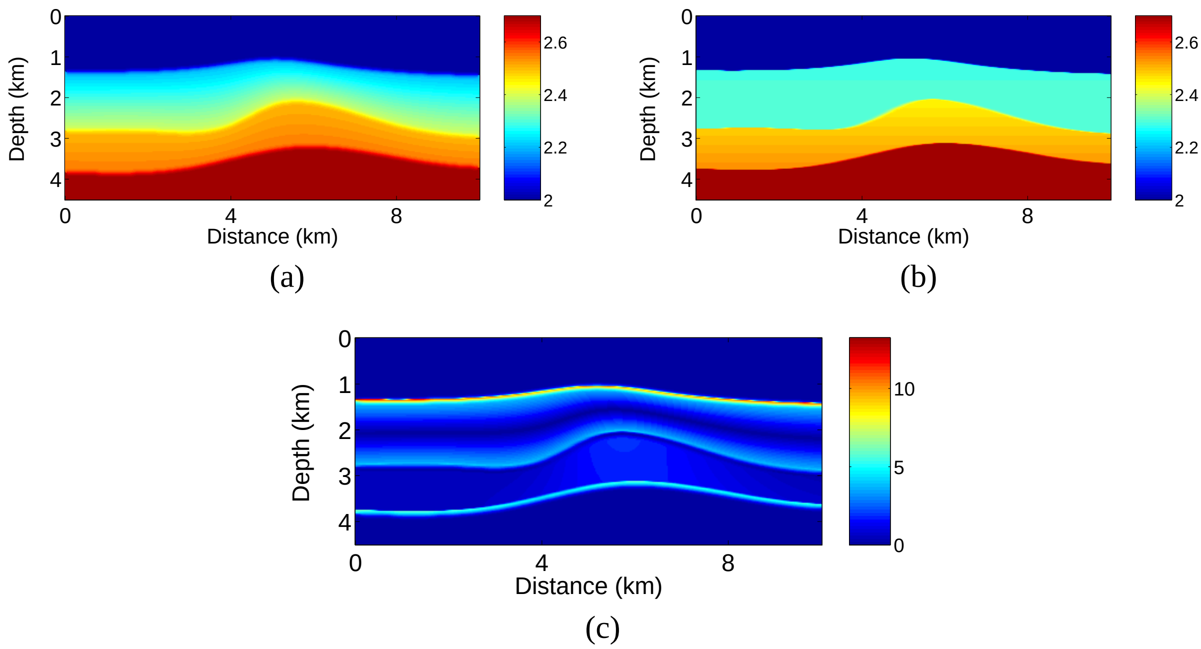

Figure 8: (a) Synthetic model with four layers, two homogeneous and two inhomogeneous. (b) Obtained model with gradients. (c) Percentual error, between the synthetic and the obtained model.

SUMMARY AND CONCLUSIONS

The contribution of this work is an extension of the classical Hubral and Krey velocity-model inversion algorithm. That extension uses CRS parameters, obtained from previous application of the CRS stacking method. A significant improvement of the proposed algorithm is that it allows for (vertical) gradient velocity profiles inside each layer. The algorithm is also able to efficiently and accurately consider caustic regions. Finally, the obtained interfaces are represented by optimized cubic splines, leading to very stable results. A discussion on the important aspects of the numerical implementation of the algorithm, as needed for an efficient application of the method, is provided.

Two synthetic examples are shown, where the main features of the method can be illustrated. The results are encouraging. Concerning more complex velocities profiles, we may consider the application of algorithm on separated parts of the domain. Thus, each inverted model would be composed by layers with gradient-type velocity profiles that could, in a next step, be put together to form a complete inverted model. Research on these and other aspects of the algorithm are under current investigation.

PUBLICATIONS

Detailed discussions on the various topics addressed in this paper are provided in Biloti (2001). Some of the material of the present paper has been presented at 7th International Congress of the Brazilian Geophysical Society (Bilot et al. (2001)).

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