Wave Inversion Technology, Report No. 4, pages 215-226

True-amplitude migration weights from traveltimes

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keywords: *traveltimes, geometrical spreading, migration weights, amplitude preserving migration*

ABSTRACT

3-D amplitude preserving pre-stack migration of the Kirchhoff type is a task of high computational effort. A substantial part of this effort is spent on the calculation of proper weight functions for the diffraction stack. We propose a method to compute the migration weights directly from coarse gridded traveltime data which are in any event needed for the summation along diffraction time surfaces. The method employs second order traveltime derivatives that contain all necessary information on the weight functions. Their determination alone from traveltimes significantly reduces the requirements in computational time and particularly storage. Application of the technique shows good accordance between numerical and analytical results.

INTRODUCTION

Kirchhoff migration is a standard technique in seismic imaging. During the last decade its objective has changed from the conventional diffraction stack migration that produces 'only' an image of reflectors in the subsurface to a modified diffraction stack, e.g., to peform AVO analysis, lithological interpretation and reservoir characterization. In the modified diffraction stack specific weight functions are applied which countermand the effect of geometrical spreading. Following from this, amplitudes in the resulting migrated image are proportional to the reflector strength if the weight functions are chosen correctly. Three different theoretical approaches (Bleistein, 1987; Keho and Beydoun, 1988; Schleicher et al., 1993) have lead to formulations of the weight functions. As (Docherty, 1991) and (Hanitzsch, 1997) have shown these results are closely related.

Since weight functions can be expressed in terms of second order spatial derivatives of traveltimes they were until now computed using dynamic ray tracing (Cervený and deCastro, 1993; Hanitzsch et al., 1994). Although (Schleicher et al., 1993) state that the modulus of the weight function can also be determined from traveltimes, (Hanitzsch,

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1997) points out that computing traveltime derivatives along dipping surfaces is expensive and numerically unstable. In this paper we propose an alternative algorithm for determining traveltime derivatives that does not suffer from the instability of numerical differentiation. It is based on a local spherical approximation of the wavefront leading to a hyperbolic expansion of the traveltimes. We determine the complete weight functions from traveltimes sampled on a coarse grid that are at the same time used for the computation of the diffraction time surface needed for the stack. This makes the algorithm computationally efficient in time and particularly in storage. Dynamic ray tracing is not required. For the special case of 2.5-D symmetry we use a simple expression for the out-of-plane spreading that can also be determined from traveltimes and not, as it is usually done, from an integral along the raypath.
Following an outline of true amplitude migration using a weighted diffraction stack we will give an expression for the actual form of migration weight functions as they were employed in this work. We will then demonstrate our method by applying it to two examples. Comparison of our results to analytical values will confirm its quality which we also summarize in our conclusions.

METHOD

(Schleicher et al., 1993) show that a diffraction stack of the form

$$V(M) = -\frac{1}{2\pi}\iint_A \mathrm{d}\xi_1\,\mathrm{d}\xi_2\, W_{3\mathrm{D}}(\xi_1,\xi_2,M)\frac{\partial U(\xi_1,\xi_2,t)}{\partial t}\bigg|_{\tau_D(\xi_1,\xi_2,M)} \tag{1}$$

yields a true amplitude migrated trace if proper weight functions $W_{3\mathrm{D}}(\xi_1,\xi_2,M)$ are applied. In equation (1) A is the aperture of the experiment (assumed to provide sufficient illumination), $\partial U(\xi_1,\xi_2,t)/\partial t|_{\tau_D(\xi_1,\xi_2,M)}$ is the time derivative of the input seismic trace in terms of the trace coordinates (ξ_1,ξ_2) at the diffraction traveltime τ_D for a diffractor at a subsurface point M. $U(\xi_1,\xi_2,t)$ is given by

$$U(\xi_1,\xi_2,t) = \frac{\mathcal{R}\mathcal{A}}{\mathcal{L}}\,F(t-\tau_R(\xi_1,\xi_2)) \quad . \tag{2}$$

In equation (2) $F(t)$ is the shape of the analytic source pulse, τ_R the reflection traveltime and $\mathcal{L}$ the geometrical spreading. $\mathcal{R}$ is the plane wave reflection coefficient and $\mathcal{A}$ expresses transmission losses. The integral (1) cannot generally be analytically solved. It can, however be transformed to the frequency domain and for high frequencies be approximately evaluated by the stationary phase method. This solution is then transformed back to the time domain and compared to the analytic true amplitude signal

$$U_{TA}(t) = \mathcal{L}\,U(\xi_1,\xi_2,t+\tau_R(\xi_1,\xi_2)) = \mathcal{R}\mathcal{A}\,F(t) \quad . \tag{3}$$

The comparison of $U_{TA}(t)$ and the result of (1) shows that equation (1) yields indeed a true amplitude trace if the weight functions are chosen to be (Schleicher et al., 1993)

$$W_{3\mathrm{D}}(\xi_1,\xi_2,M) = \mathcal{L}\sqrt{|\det\mathcal{H}_F|}\,\mathrm{e}^{i\frac{\pi}{2}\left(1-\frac{\mathrm{sgn}\mathcal{H}_F}{2}\right)} \quad . \tag{4}$$

The matrix $\mathcal{H}_F$ is the Hessian matrix of the difference $\tau_F = \tau_D - \tau_R$ between diffraction and reflection traveltime at the stationary point ξ_1^*, ξ_2^*, where $\vec{\nabla}\tau_F = 0$, meaning that in this point the diffraction and reflection traveltime curves are tangent to each other. $\mathcal{H}_F$ can be expressed in terms of second order spatial derivative matrices of traveltimes which until now were computed by dynamic ray tracing. This is, however, not necessary since these derivatives can be extracted from traveltime data that is required for the construction of the diffraction traveltime surface for the stack anyway. Using these derivatives also gives an effective and highly accurate algorithm for interpolating traveltimes from the coarse input grid to the fine migration grid (Vanelle and Gajewski, 2000a). Also, the geometrical spreading can be written using second order derivatives of traveltimes (see Appendix C). The expression for the weight function we use is

$$W_{3\mathrm{D}}(\xi_1, \xi_2, M) = \frac{\sqrt{\cos\alpha_s \cos\alpha_g}}{v_s} \frac{\left|\det[\hat{\mathcal{N}}_1^\top \Sigma + \hat{\mathcal{N}}_2^\top \Gamma]\right|}{\sqrt{\left|\det\hat{\mathcal{N}}_1 \det\hat{\mathcal{N}}_2\right|}} \,\mathrm{e}^{-i\frac{\pi}{2}(\kappa_1+\kappa_2)} \quad . \tag{5}$$

The angles α_s and α_g are the emergence and incidence angles at the source and receiver, v_s is the velocity at the source and κ_1 and κ_2 are the KMAH indices of the two branches of the traveltime curve. The matrices Σ and Γ describe the measurement configuration (e.g., common shot). $\hat{\mathcal{N}}_1$ and $\hat{\mathcal{N}}_2$ are second order derivative matrices of the traveltimes. All quantities and their determination from traveltimes are explained in detail in the appendices.

APPLICATIONS

In this section we will apply our method. A simple example was chosen in order to allow for comparison of numerically and analytically computed amplitudes. The method is, however, not limited to homogeneous velocity layer models. For convenience reasons we have restricted our example to what is commonly referred to as a 2.5-D geometry (Bleistein, 1986). The need to introduce this concept arises when seismic data is only available for sources and receivers constrained to a single straight acquisiton line. Processing of this data with techniques based on 2-D wave propagation does not yield satisfactory results because the (spherical) geometrical spreading in the data caused by the 3-D earth does not agree with the cylindrical (i.e., line source) spreading implied by the 2-D wave equation. The problem can be dealt with by assuming the subsurface to be invariant in the off-line direction. This symmetry is called to be 2.5 dimensional. Apart from the geometrical spreading the properties involved do not depend on the out-of-plane variable and can be computed with 2-D techniques. The geometrical spreading is split into an in-plane part that is equal to the 2-D spreading and an out-of-plane contribution. For the described symmetry the product of both equals the spreading in a true 3-D medium. We determine the out-of-plane spreading along with the migration

weight functions from traveltimes only. The actual expression and its derivation are given in Appendices B and D.
For the example the only input data used were the velocity model and traveltimes computed with a finite-difference eikonal solver (Vidale, 1990) using the implementation of (Leidenfrost, 1998) and stored on a coarse grid of 50m in either direction. These were used to compute the migration weights as well as to interpolate the diffraction traveltimes on a fine migration grid of 5m in z-direction. Traveltimes were interpolated using the hyperbolic approximation as described in (Vanelle and Gajewski, 2000a). The migration weights were also computed using the coefficients determined from the hyperbolic approximation.
The velocity model we used has a plane reflector with an inclination angle of 14°. The velocity is 5km/s in the upper part of the model and 6km/s below the reflector. The reflector depth under the source is 2500m. Ray synthetic seismograms were computed for 80 receivers with 50m distance starting at 50m from the point source. Figure 1 shows the migrated depth section. The reflector was migrated to the correct position and the source pulse, a Gabor wavelet, was reconstructed. Since there are no transmission losses caused by the overburden, the amplitudes of the migrated section should coincide with the reflection coefficients. Figure 2 shows the accordance between amplitudes picked from the migrated section in Figure 1 with theoretical values. Apart from the peaks at 900m and 2500m distance the two curves coincide. These peaks are aperture effects caused by the limited extent of the receiver line.

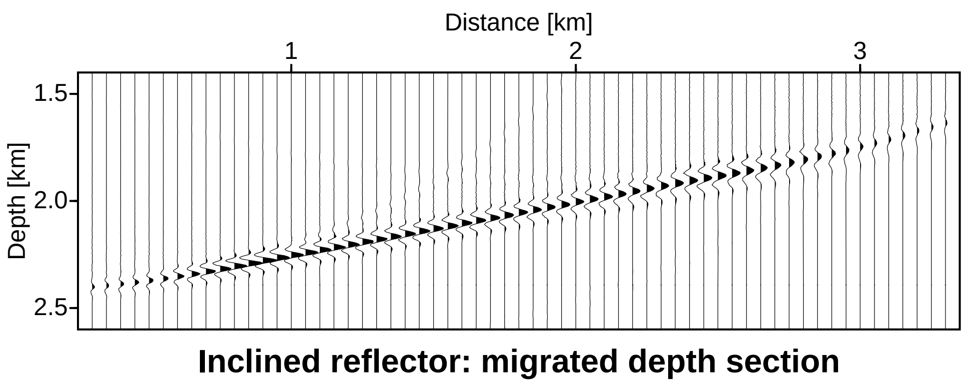

Figure 1: Migrated depth section. The reflector was migrated to the correct depth and inclination. The source pulse was correctly reconstructed.

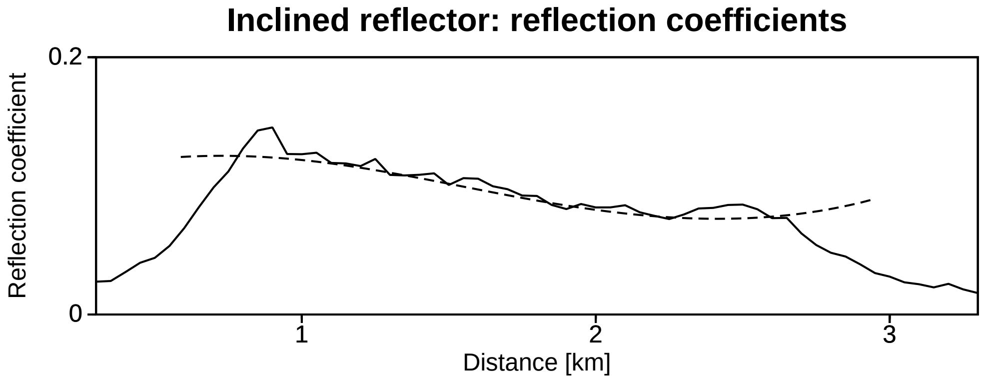

Figure 2: Solid line: picked reflection coefficients from the migrated section in Figure 1, dashed line: analytical values for the reflection coefficients.

CONCLUSIONS

We have presented a method for the determination of weight functions for an amplitude preserving migration. Traveltimes on coarse grids are the only necessary input data. Since every required quantity can be computed instantly from this coarse grid data alone the technique is very efficient in computational time and storage. Dynamic ray tracing is not required. It is particularly suited to be used in connection with techniques for traveltime computation that can directly provide coarse gridded data, like, e.g. the wavefront construction method which does not require a fine grid for sufficient accuracy of traveltime as, e.g., FD eikonal solvers do. The examples show good accordance between the reconstructed reflectors and theoretical values in terms of position as well as in amplitudes. This demonstrates also the applicability of the method to the special situation of 2.5-D symmetry.

ACKNOWLEDGEMENTS

We thank the members of the Applied Geophysics Group in Hamburg for continuous and helpful discussions. Special thanks go to Andrée Leidenfrost for providing an FD eikonal solver and thus the necessary input traveltimes. This work was partially supported by the German Research Society (DFG, Ga 350-10) and the sponsors of the Wave Inversion Technology (WIT) consortium.

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PUBLICATIONS

Previous results concerning true-amplitude migration were published by (Vanelle and Gajewski, 2000b) and (Vanelle and Gajewski, 2000a). A paper containing these results has been submitted (Vanelle and Gajewski, 2000c).

APPENDIX A

The weight function in equation (4) contains the Hessian matrix of the difference between diffraction (τ_D) and reflection (τ_R) traveltimes. To write $\mathcal{H}_F$ in terms of second derivatives of traveltimes we will now derive expressions for τ_D and τ_R containing first and second derivatives. Consider an arbitrary velocity model. Let sources be positioned in a reference surface that we will denote the source surface. If the resulting traveltime field for a source at the position $\vec{x}_0$ is single-valued, the traveltime $\tau(\vec{x}, \vec{x}\,')$ from a point $\vec{x}$ in the source surface and in a near vicinity of $\vec{x}_0$ to a subsurface point $\vec{x}\,'$ near $\vec{x}\,'_0$ can be expressed by a Taylor series provided that $\triangle\vec{x} = \vec{x} - \vec{x}_0$ and $\triangle\vec{x}\,' = \vec{x}\,' - \vec{x}\,'_0$ are small, the size of this small vicinity depends on the model under consideration and the required accuracy. For a multi-valued traveltime field the Taylor expansion is valid if the different branches of the traveltime curve are treated separately. As $\vec{x}$ and $\vec{x}_0$ lie in one surface the traveltimes are expanded into the surface in this variable. We also expand τ into a surface in $\vec{x}\,'$, this will be the reflector surface, or, more precisely, the reflector's tangent plane at $\vec{x}\,'_0$ if $\vec{x}\,'$ is on a curved reflector. The traveltime expansion looks as follows:

$$\tau(\vec{x}, \vec{x}\,') = \tau_0 - \vec{p}\triangle\vec{x} + \vec{p}\,'\triangle\vec{x}\,' - \frac{1}{2}\triangle\vec{x}^\top\hat{\mathcal{S}}\triangle\vec{x} + \frac{1}{2}\triangle\vec{x}\,'^\top\hat{\mathcal{G}}\triangle\vec{x}\,' - \triangle\vec{x}^\top\hat{\mathcal{N}}\triangle\vec{x}\,' \qquad \text{(A-1)}$$

where the first order traveltime derivatives

$$p_i = -\left.\frac{\partial\tau}{\partial x_i}\right|_{\vec{x}_0, \vec{x}\,'_0}, \qquad p'_i = \left.\frac{\partial\tau}{\partial x'_i}\right|_{\vec{x}_0, \vec{x}\,'_0} \qquad (i, j = 1, 2) \qquad \text{(A-2)}$$

are the slowness vectors at $\vec{x}$ and $\vec{x}\,'$, respectively. The second order derivatives are given by the matrices $\hat{\mathcal{S}}$, $\hat{\mathcal{G}}$ and $\hat{\mathcal{N}}$ with

$$\hat{\mathcal{S}}_{ij} = -\left.\frac{\partial^2\tau}{\partial x_i \partial x_j}\right|_{\vec{x}_0,\vec{x}\,'_0}, \quad \hat{\mathcal{G}}_{ij} = \left.\frac{\partial^2\tau}{\partial x'_i \partial x'_j}\right|_{\vec{x}_0,\vec{x}\,'_0} \quad \text{and} \quad \hat{\mathcal{N}}_{ij} = -\left.\frac{\partial^2\tau}{\partial x_i \partial x'_j}\right|_{\vec{x}_0,\vec{x}\,'_0} \quad . \tag{A-3}$$

We will now derive expressions in terms of (A-1) for diffraction and reflection traveltimes as needed for the weight functions. Since $\tau(\vec{x}_1,\vec{x}_2) = \tau(\vec{x}_2,\vec{x}_1)$ we can use (A-1) twice for the down- and upgoing branches of the reflection traveltime τ_R and the diffraction traveltime τ_D. Similar as for the sources we assume the receivers to be placed in a receiver surface. Source coordinates will be denoted by $\vec{s}$, receivers by $\vec{g}$ and subsurface points by $\vec{r}$. The traveltimes for the branch from $\vec{s}$ to $\vec{r}$ is

$$\tau_1(\vec{s},\vec{r}) = \tau_{01} - \vec{p}_1 \Delta\vec{s} + \vec{p}\,'_1 \Delta\vec{r} - \frac{1}{2}\Delta\vec{s}^{\top}\hat{\mathcal{S}}_1\Delta\vec{s} + \frac{1}{2}\Delta\vec{r}^{\top}\hat{\mathcal{G}}_1\Delta\vec{r} - \Delta\vec{s}^{\top}\hat{\mathcal{N}}_1\Delta\vec{r} \tag{A-4}$$

and from $\vec{g}$ to $\vec{r}$:

$$\tau_2(\vec{g},\vec{r}) = \tau_{02} - \vec{p}_2 \Delta\vec{g} + \vec{p}\,'_2 \Delta\vec{r} - \frac{1}{2}\Delta\vec{g}^{\top}\hat{\mathcal{S}}_2\Delta\vec{g} + \frac{1}{2}\Delta\vec{r}^{\top}\hat{\mathcal{G}}_2\Delta\vec{r} - \Delta\vec{g}^{\top}\hat{\mathcal{N}}_2\Delta\vec{r} \quad . \tag{A-5}$$

The sums of equations (A-4) and (A-5) give us τ_D and τ_R. For the diffraction traveltime the diffractor position is fixed at $\vec{r}_0$ and thus with $\Delta\vec{r}$=0 we get ($\tau_0 = \tau_{01} + \tau_{02}$)

$$\tau_D = \tau_0 - \vec{p}_1 \Delta\vec{s} - \vec{p}_2 \Delta\vec{g} - \frac{1}{2}\Delta\vec{s}^{\top}\hat{\mathcal{S}}_1\Delta\vec{s} - \frac{1}{2}\Delta\vec{g}^{\top}\hat{\mathcal{S}}_2\Delta\vec{g} \quad . \tag{A-6}$$

For the reflection traveltime we must take into account that variation of source and/or receiver positions will result in a different reflection point $\vec{r}$. Aiming for an expression containing $\Delta\vec{s}$ and $\Delta\vec{g}$ only, we make use of Snell's law stating that $\vec{p}\,'_1 + \vec{p}\,'_2 = \vec{\nabla}_r\tau_R = \vec{0}$. This we can solve for $\vec{r}$ and eliminate $\Delta\vec{r}$ from the sum of (A-4) and (A-5) resulting in

$$\tau_R = \tau_0 - \vec{p}_1 \Delta\vec{s} - \vec{p}_2 \Delta\vec{g} - \frac{1}{2}\Delta\vec{s}^{\top}\hat{\mathcal{S}}\Delta\vec{s} + \frac{1}{2}\Delta\vec{g}^{\top}\hat{\mathcal{G}}\Delta\vec{g} - \Delta\vec{s}^{\top}\hat{\mathcal{N}}\Delta\vec{g} \quad , \tag{A-7}$$

where we introduced the following matrices to bring (A-7) into the same form as (A-1):

$$\begin{aligned} \hat{\mathcal{S}} &= \hat{\mathcal{S}}_1 + \hat{\mathcal{N}}_1(\hat{\mathcal{G}}_1 + \hat{\mathcal{G}}_2)^{-1}\hat{\mathcal{N}}_1^{\top} \\ \hat{\mathcal{G}} &= -\hat{\mathcal{S}}_2 - \hat{\mathcal{N}}_2(\hat{\mathcal{G}}_1 + \hat{\mathcal{G}}_2)^{-1}\hat{\mathcal{N}}_2^{\top} \\ \hat{\mathcal{N}} &= \hat{\mathcal{N}}_1(\hat{\mathcal{G}}_1 + \hat{\mathcal{G}}_2)^{-1}\hat{\mathcal{N}}_2^{\top} \quad . \end{aligned} \tag{A-8}$$

APPENDIX B

To compute the weight functions in equation (5) we need the matrices $\hat{\mathcal{S}}, \hat{\mathcal{G}}$ and $\hat{\mathcal{N}}$. In (Vanelle and Gajewski, 2000a) we presented a hyperbolic traveltime expansion similar to the parabolic expansion in (A-1) but with respect to three spatial coordinates instead of an expansion into reference surfaces. The procedure of determining the corresponding 3-D slowness vectors $\hat{p}$ and $\hat{p}'$ and matrices $\hat{\hat{\mathcal{S}}}$, $\hat{\hat{\mathcal{G}}}$ and $\hat{\hat{\mathcal{N}}}$ from multi-fold traveltime data sampled on a coarse cartesian grid is described in detail in (Vanelle and Gajewski, 2000a) as well as in (Gajewski, 1998). Once $\hat{\hat{\mathcal{S}}}$, $\hat{\hat{\mathcal{G}}}$ and $\hat{\hat{\mathcal{N}}}$ are known, the desired 2×2 matrices $\hat{\mathcal{S}}$, $\hat{\mathcal{G}}$ and $\hat{\mathcal{N}}$ can be computed by projecting them from the cartesian coordinates into the reference surfaces. Since we assume a velocity model to compute traveltimes, we can also make use of this to extract the reflector position and geometry from it.

In a situation with a 2.5-D symmetry as considered in the numerical examples the components of the slowness vectors $\vec{p}$, $\vec{p}'$ and second order derivative matrices $\hat{\mathcal{S}}, \hat{\mathcal{G}}$ and $\hat{\mathcal{N}}$ simplify. Let the out-of-plane direction have index 2 – coinciding with the y-axis of the cartesian system used for the determination of $\hat{\hat{\mathcal{S}}}$, $\hat{\hat{\mathcal{G}}}$ and $\hat{\hat{\mathcal{N}}}$ – with $y_0 = s_{20} = g_{20} = r_{20}$ the y-position of the sources and receiver line. Then we have

$$\hat{\mathcal{N}}_{22}|_{y_0} = \hat{\hat{\mathcal{N}}}_{yy}|_{y_0}, \quad \hat{\mathcal{G}}_{22}|_{y_0} = \hat{\hat{\mathcal{G}}}_{yy}|_{y_0} \quad \text{and} \quad \hat{\mathcal{S}}_{22}|_{y_0} = \hat{\hat{\mathcal{S}}}_{yy}|_{y_0} \quad . \tag{B-1}$$

From the symmetry we can easily see that the y-components of the slownesses vanish at y_0:

$$\left.\frac{\partial \tau}{\partial y_s}\right|_{y_0} = \left.\frac{\partial \tau}{\partial y_g}\right|_{y_0} = \left.\frac{\partial \tau}{\partial y_r}\right|_{y_0} \quad . \tag{B-2}$$

From this follows that the matrices $\hat{\mathcal{S}}, \hat{\mathcal{G}}$ and $\hat{\mathcal{N}}$ consist only of diagonal elements. Furthermore, for the yy- or 22-components we get

$$\left.\hat{\mathcal{N}}_{22}\right|_{y_0} = \left.\hat{\mathcal{G}}_{22}\right|_{y_0} = -\left.\hat{\mathcal{S}}_{22}\right|_{y_0} \tag{B-3}$$

and the sign of $\left.\hat{\mathcal{N}}_{22}\right|_{y_0}$ is positive; i.e., $\mathrm{sgn}(\hat{\mathcal{N}}_{22}|_{y_0}) = +1$.

If the source-receiver line is equal to the x-direction of the cartesian system from the input traveltimes, the 11-components are computed as follows:

$$\begin{aligned}
\hat{\mathcal{N}}_{11} &= \hat{\hat{\mathcal{N}}}_{xx} \cos\varphi - \hat{\hat{\mathcal{N}}}_{xz} \sin\varphi \\
\hat{\mathcal{G}}_{11} &= \hat{\hat{\mathcal{G}}}_{xx} \cos^2\varphi + \hat{\hat{\mathcal{G}}}_{zz} \sin^2\varphi - 2\,\hat{\hat{\mathcal{G}}}_{xz} \sin\varphi \cos\varphi \\
\hat{\mathcal{S}}_{11} &= \hat{\hat{\mathcal{S}}}_{xx} \quad ,
\end{aligned} \tag{B-4}$$

where φ is the inclination angle of the reflector's tangent plane against the source-receiver line (x-coordinate).

APPENDIX C

Equation (5) contains the geometrical spreading $\mathcal{L}$ which we will now express in terms of traveltime derivatives. Equation (A-1) is equivalent to the paraxial ray approximation introduced by (Bortfeld, 1989). (Hubral et al., 1992) give the normalized geometrical spreading in terms of the matrices of second order derivatives of traveltimes. This is

$$\mathcal{L} = \frac{1}{v_s}\sqrt{\frac{\cos\alpha_s\cos\alpha_g}{|\mathrm{det}\hat{\mathcal{N}}|}}\ \mathrm{e}^{-i\frac{\pi}{2}\kappa} \quad , \tag{C-1}$$

where the angles α_s, α_g are the emergence angle at $\vec{s}_0$ and the incidence angle at $\vec{g}_0$ and can be determined from the slownesses at these positions. κ is the KMAH-index of the ray connecting $\vec{s}$ and $\vec{g}$.
For a 2.5-D situation we find that $\mathrm{det}\hat{\mathcal{N}} = \hat{\mathcal{N}}_{11} \cdot \hat{\mathcal{N}}_{22}$ (see Appendix B) and therefore expression (C-1) can be reduced to the simple form

$$\mathcal{L} = \frac{1}{v_s}\sqrt{\frac{\cos\alpha_s\cos\alpha_g}{|\hat{\mathcal{N}}_{11}|}}\ \frac{1}{\sqrt{\hat{\mathcal{N}}_{22}}}\ \mathrm{e}^{-i\frac{\pi}{2}\kappa} \quad . \tag{C-2}$$

This result is no surprise since (Bleistein, 1986) found the relationship between the out-of-plane spreading commonly denoted by σ and the second order traveltime derivative in out-of-plane direction. Thus we have

$$\sigma = \frac{1}{\hat{\mathcal{N}}_{22}} \quad . \tag{C-3}$$

So far, however, it was common practice to determine the out-of-plane spreading from the integral along the ray from $\vec{x}$ to $\vec{x}\,'$

$$\sigma = \int \mathrm{d}s\, v \tag{C-4}$$

with s being the arclength and v the velocity. We compute this quantity from traveltimes and do not have to trace rays to determine σ.

APPENDIX D

Equations (4) and (5) are expressions for weight functions if the diffraction stack is carried out over the aperture in ξ_1 and ξ_2. Since in the 2.5-D case we have only data from a single acquisition line (assumed to coincide with the ξ_1 coordinate), we only integrate over ξ_1. In this case we have $U(\xi_1, \xi_2, t) = U(\xi_1, \xi_2^*, t)$ where the asterisk denotes the stationary point. Inserting the according expression for the input traces (2) into the stack integral (1) then leads to

$$V(M) = -\frac{1}{2\pi}\int_A \mathrm{d}\xi_1 \int_{-\infty}^{\infty} \mathrm{d}\xi_2\, W_{3\mathrm{D}}(\xi_1, \xi_2, M)\frac{\mathcal{R}A}{\mathcal{L}}\left.\frac{\partial F(t)}{\partial t}\right|_{\tau_F(\xi_1,\xi_2,M)} \quad . \tag{D-1}$$

Carrying out the integration over ξ_2 following (Martins et al., 1997) we get

$$V(M) = \frac{1}{\sqrt{2\pi}}\int_A \mathrm{d}\xi_1\, W_{2.5\mathrm{D}}(\xi_1, \xi_2^*, M)\, f[U(\xi_1, \xi_2^*, t + \tau_D(\xi_1, \xi_2^*, M))] \tag{D-2}$$

with the function $f[U(t)]$ corresponding to a $\sqrt{i\omega}$ filter operation in the frequency domain (commonly called *half derivative*). The 2.5-D weight function

$$W_{2.5\mathrm{D}}(\xi_1, \xi_2^*, M) = W_{3\mathrm{D}}(\xi_1, \xi_2^*, M) \left(\left. \frac{\partial^2 \tau_D}{\partial \xi_2^2} \right|_{\xi_2^*} \right)^{-\frac{1}{2}} \mathrm{e}^{-i\frac{\pi}{4}} \tag{D-3}$$

can be evaluated using the results from the previous sections. We will now express the involved quantities in terms of traveltime derivatives and apply the simplifications from the 2.5-D symmetry. We find

$$\det \mathcal{H}_F = \frac{(\det[\hat{\mathcal{N}}_1^\top \Sigma + \hat{\mathcal{N}}_2^\top \Gamma])^2}{\det[\hat{\mathcal{G}}_1 + \hat{\mathcal{G}}_2]} = \frac{(\hat{\mathcal{N}}_{1_{11}} \Sigma + \hat{\mathcal{N}}_{2_{11}} \Gamma)^2}{\hat{\mathcal{G}}_{1_{11}} + \hat{\mathcal{G}}_{2_{11}}} (\hat{\mathcal{N}}_{1_{22}} + \hat{\mathcal{N}}_{2_{22}}) \quad , \tag{D-4}$$

where the first index denotes the first or second branch of the traveltime curve and the second double index labels the corresponding matrix element. For the out-of-plane spreading we have

$$\left. \frac{\partial^2 \tau_D}{\partial \xi_2^2} \right|_{\xi_2^*} = -\hat{\mathcal{S}}_{1_{22}} - \hat{\mathcal{S}}_{2_{22}} = \hat{\mathcal{N}}_{1_{22}} + \hat{\mathcal{N}}_{2_{22}} = \frac{1}{\sigma} \tag{D-5}$$

For the spreading of the reflected ray we get

$$\mathcal{L} = \frac{\sqrt{\cos\alpha_s \cos\alpha_g}}{v_s} \sqrt{\frac{\hat{\mathcal{G}}_{1_{11}} + \hat{\mathcal{G}}_{2_{11}}}{\hat{\mathcal{N}}_{1_{11}} \hat{\mathcal{N}}_{2_{11}}}} \sqrt{\frac{\hat{\mathcal{N}}_{1_{22}} + \hat{\mathcal{N}}_{2_{22}}}{\hat{\mathcal{N}}_{1_{22}} \hat{\mathcal{N}}_{2_{22}}}} \, \mathrm{e}^{-i\frac{\pi}{2}\kappa} \quad . \tag{D-6}$$

The resulting expression for the 2.5-D weight function is then

$$W_{2.5\mathrm{D}}(\xi_1, \xi_2^*, M) = \frac{\sqrt{\cos\alpha_s \cos\alpha_g}}{v_s} \frac{\left| \hat{\mathcal{N}}_{1_{11}} \Sigma + \hat{\mathcal{N}}_{2_{11}} \Gamma \right|}{\sqrt{\hat{\mathcal{N}}_{1_{11}} \hat{\mathcal{N}}_{2_{11}}}} \sqrt{\frac{\hat{\mathcal{N}}_{1_{22}} + \hat{\mathcal{N}}_{2_{22}}}{\hat{\mathcal{N}}_{1_{22}} \hat{\mathcal{N}}_{2_{22}}}} \mathrm{e}^{-i\frac{\pi}{4}(2\kappa + \mathrm{sgn}\mathcal{H}_F - 1)} . \tag{D-7}$$

We can further insert the KMAH indices κ_1 and κ_2 of the two ray branches with (Schleicher et al., 1993)

$$\kappa_1 + \kappa_2 = \kappa - \left(1 - \frac{\mathrm{sgn}\mathcal{H}_F}{2} \right) \quad . \tag{D-8}$$

The κ_i are not determined from the traveltimes themselves but we can use a suitable algorithm for computing traveltimes, as for example the wavefront construction method in the implementation introduced by (Coman and Gajewski, 2000) that outputs multi-valued traveltimes sorted for the KMAH index.