Wave Inversion Technology, Report No. 4, pages 183-191

Effective velocities in fractured media: Intersecting and parallel cracks

E.H. Saenger and S.A. Shapiro[1]

keywords: *finite differences, effective velocities, fractured media*

ABSTRACT

This paper is concerned with a numerical study of effective velocities in two types of fractured media. We apply the so-called rotated staggered finite difference grid technique. Using this modified grid it is possible to simulate the propagation of elastic waves in a 2D or 3D medium containing cracks, pores or free surfaces without hard-coded boundary conditions. Therefore it allows an efficient and precise numerical study of effective velocities in fractured structures. We model the propagation of plane waves through a set of different randomly cracked media. In these numerical experiments we vary the crack density. The synthetic results are compared with several theories that predict the effective P- and S-wave velocities in fractured materials. For randomly distributed and randomly oriented rectilinear intersecting thin dry cracks the numerical simulations of velocities of P-, SV- and SH-waves are in excellent agreement with the results of a new critical crack density (CCD) formulation. On the other hand for randomly distributed rectilinear parallel thin dry cracks three different classical theories are compared with our numerical results.

INTRODUCTION

The problem of effective elastic properties of fractured solids is of considerable interest for geophysics, for material science, and for solid mechanics. In this paper we consider the problem of a fractured medium in two dimensions. With this work some broad generalizations can be elucidated that will help solving problems with more complicated geometries.

Strong scattering caused by many dry cracks can be treated only by numerical techniques because an analytical solution of the wave equation is not available. So-called boundary integral methods are well suited to handle such discrete scatterers in a homogeneous embedding. They allow the study of SV-waves (Davis and Knopoff, 1995; Murai et al., 1995), SH-waves (Dahm and Becker, 1998) and P-waves (Kelner et al.,

[1]**email:** saenger@geophysik.fu-berlin.de, shapiro@geophysik.fu-berlin.de

1999) in multiple fractured media, but they are restricted to non-intersecting cracks. Finite difference (FD) methods discretize the wave equation on a grid. They replace spatial derivatives by FD operators using neighboring points. The wave field is also discretized in time, and the wave field for the next time step is generally calculated by using a Taylor expansion. Since the FD approach is based on the wave equation without physical approximations, the method accounts not only for direct waves, primary reflected waves, and multiply reflected waves, but also for surface waves, head waves, converted reflected waves, and waves observed in ray-theoretical shadow zones (Kelly et al., 1976). Additionally, it automatically accounts for the proper relative amplitudes. Consequently, FD solutions of the wave equation are widely used to study scattering of waves by heterogeneities (e.g. (Saenger et al., 2000; Saenger and Shapiro, 2000; Frankel and Clayton, 1986; Kneib and Kerner, 1993; Andrews and Ben-Zion, 1997; Kusnandi et al., 2000)).
In this paper we present a numerical study of effective velocities of two types of fractured 2D-media. We model the propagation of plane waves through well defined fractured regions. The numerical setup is described in section . Our numerical results for intersecting and parallel cracks are discussed in section and .

EXPERIMENTAL SETUP

As mentioned above, the rotated staggered FD scheme (Saenger et al., 2000) is a powerful tool for testing theories about fractured media. In order to test these formalisms we design some numerical elastic models which include a region with a well known crack density. The cracked region was filled randomly with rectilinear dry cracks. A typical model contains 1000 $\times$ 1910 grid points with an interval of 0.0001m. In the homogeneous region we set $v_p = 5100\ m/s$, $v_s = 2944\ m/s$ and $\rho_g = 2700\ kg/m^3$. Table 1 summarizes the relevant parameters of all the models we use for our experiments. For the dry cracks we set $v_p = 0\ m/s$, $v_s = 0\ m/s$ and $\rho_g = 0.0001\ kg/m^3$ which approximates vacuum. To obtain effective velocities in different models we apply a body force plane source at the top of the model. The plane wave generated in this way propagates through the fractured medium. With two horizontal lines of 1000 geophones at the top and at the bottom, it is possible to measure the time-delay of the mean peak amplitude of the plane wave caused by the inhomogeneous region. With the time-delay one can calculate the effective velocity. The direction of the body force and the source wavelet (i.e. source time function) can vary to generate two types of shear (SH- and SV-) waves and one compressional (P-) wave. The source wavelet in our experiments is always the first derivative of a Gaussian with a dominant frequency of 50kHz and with a time increment of $\Delta t = 5 * 10^{-9} s$. From the modeling point of view it is important to note that all computations are performed with second order spatial FD operators and with a second order time update.

No.	crack density ρ	length of cracks [0.0001m]	number of cracks	porosity ϕ of the crack region	number of model realizations
1x	0.050	56	63	0.0045	1
2x	0.100	56	126	0.0091	1
3x.1-3x.6	0.200	56	252	0.0181	6
4x	0.300	56	378	0.0270	1
5x.1-5x.6	0.401	56	504	0.0360	6
6x	0.601	56	756	0.0539	1
7x	0.801	56	1007	0.0720	1
8p	0.025	56	32	0.0018	1
9p	0.050	56	64	0.0036	1
10p	0.100	56	128	0.0073	1
11p	0.200	56	255	0.0145	1

Table 1: Crack models for numerical calculations. The models with an x attached to its number have intersection of cracks. The models with a p attached to its number have parallel cracks (see Figure 3). Note that 0.0001m is the size of grid spacing and the size of the crack region is always 1000*1000 grid points.

INTERSECTING CRACKS

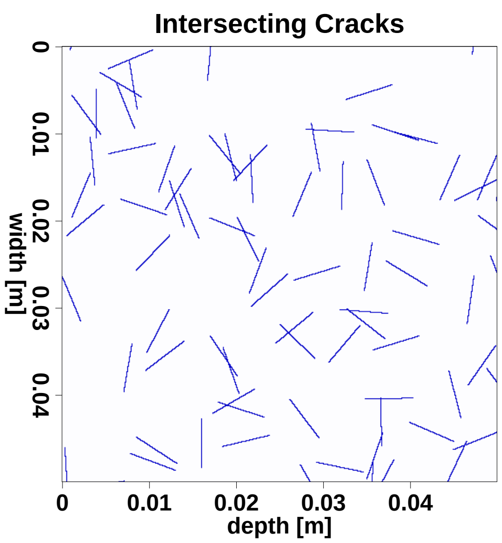

Figure 1: Randomly distributed and randomly oriented rectilinear intersecting thin dry cracks in homogeneous 2D-media. A part of Model 3x.1 is shown.

In this section we consider randomly distributed and randomly oriented rectilinear intersecting thin dry cracks in 2D-media (see Figure 1). Our goal is to compare the numerical results of the present study with the predicted effective velocities of a new critical crack density (CCD) formalism (Saenger, 2000). The CCD formulation is valid for 2D (i.e. 3D transversely isotropic) fracturing configurations with intersecting cracks. We introduce an additional factor into results of a modified self-consistent theory (described e.g. in (Davis and Knopoff, 1995)) to include the physical behavior at the critical crack density (similar to the critical porosity described in (Mukerji et al., 1995)). We propose the following formulae for the effective elastic moduli:

$$< \mu_{CCD} >= \mu_0 e^{-\pi(\rho/2)\left(\frac{\rho_c}{\rho_c-\rho}\right)^{1/2}},$$

$$< E_{CCD} >= E_0 e^{-\pi\rho\left(\frac{\rho_c}{\rho_c-\rho}\right)^{1/2}},$$

$$< \nu_{CCD} >= \nu_0 e^{-\pi\rho\left(\frac{\rho_c}{\rho_c-\rho}\right)^{1/2}},$$

$$\text{with: } \rho = \frac{1}{A}\sum_{k=1}^{n} l_k^2,$$

where μ_0 is the shear modulus, E_0 is the Young's modulus, ν_0 is the Poisson's ratio of the unfractured medium, ρ is the crack density (as in (Kachanov, 1992)), ρ_c is the critical crack density (can be found in (Robinson, 1983) for our models), $2l_k$ is the rectilinear length of a crack and A is the representative area.

Now, we discuss the numerical results on effective wave velocities. They are depicted with dots in Figure 2. For comparison, the predictions of the CCD formulation described above are shown in the same Figure with lines. We show the normalized effective velocities for three types of waves. The relative decrease of the effective velocity for one given crack density is in the following succession: For SH-waves we obtain the smallest decrease followed by SV-waves. For P-waves it is largest. For each wave type we perform numerical FD-calculations with different crack densities to obtain the effective velocity. For these measurements depicted in Figure 2 we use the models No. 1x,2x,3x.1-3x.6,4x,5x.1-5x.6,6x,7x (see Table 1).

A final result is that our numerical simulations of P-, SV- and SH-wave velocities are in excellent agreement with the predictions of the CCD formulation.

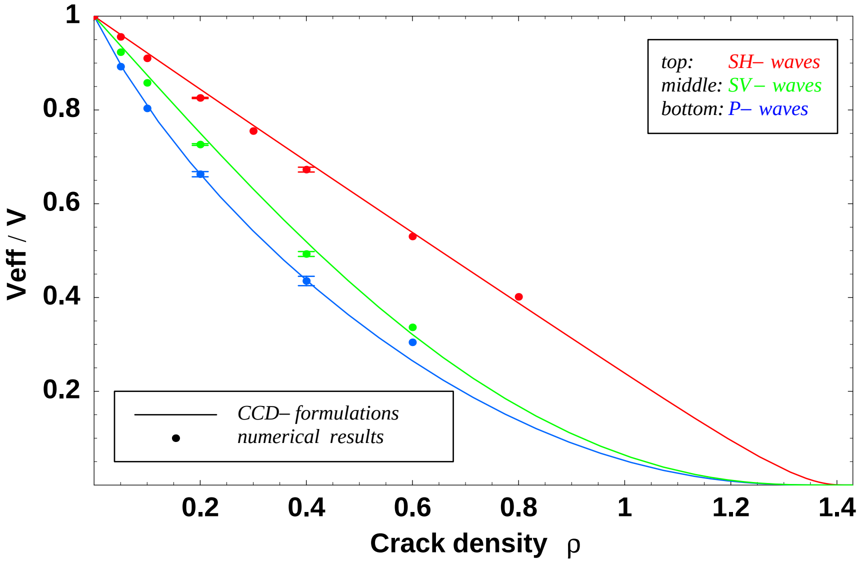

Figure 2: Normalized effective velocity versus crack density. Dots: Numerical results of this study, Lines: Theoretical predictions of the CCD-formulations. The error bar denotes the standard deviation for different model realizations.

PARALLEL CRACKS

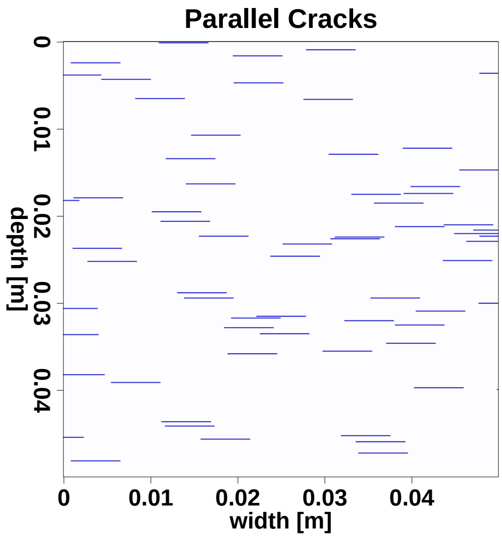

Figure 3: Randomly distributed rectilinear parallel non-intersecting thin dry cracks in homogeneous 2D-media. A part of Model 11p is shown.

This section considers randomly distributed rectilinear parallel non-intersecting thin dry cracks in 2D-media (see Figure 3). The paper of Kachanov (1992) discusses three different theoretical descriptions of effective velocities in such a case. Namely, they are the “theory for non-interacting cracks (NIC)”, the “modified (or differential) self-consistent theory (MSC)” and a extension of the modified self-consistent theory (EMSC) by Sayers and Kachanov (1991). In order to give an overview we present here the effective Young's modulus $< E >$ for the three theories:

$$< E_{NIC} >= E_0 \ \frac{1}{1+2\pi\rho},$$
$$< E_{MSC} >= E_0 \ e^{-2\pi\rho},$$
$$< E_{EMSC} >= E_0 \ \frac{1}{1+2\pi\rho * e^{\pi\rho}},$$

where E_0 is the Young's modulus of the unfractured medium and ρ is the crack density. The effective velocities of SV- and P- waves predicted by this three theories for parallel cracks are plotted in Figure 4 using lines. Our numerical results for parallel cracks can be seen in the same Figure using dots. Details of the models No. 8p-11p used for the experiments are displayed in Table 1. We show the calculations for two types of waves. The relative decrease of the effective velocity for one given crack density is in the following order: For SV-waves we obtain the smallest decrease followed by P-waves. The main result of the investigations in this section is the fact, that the three theories for parallel cracks can only be applied successfully to a dilute crack density. For large crack densities further research is necessary.

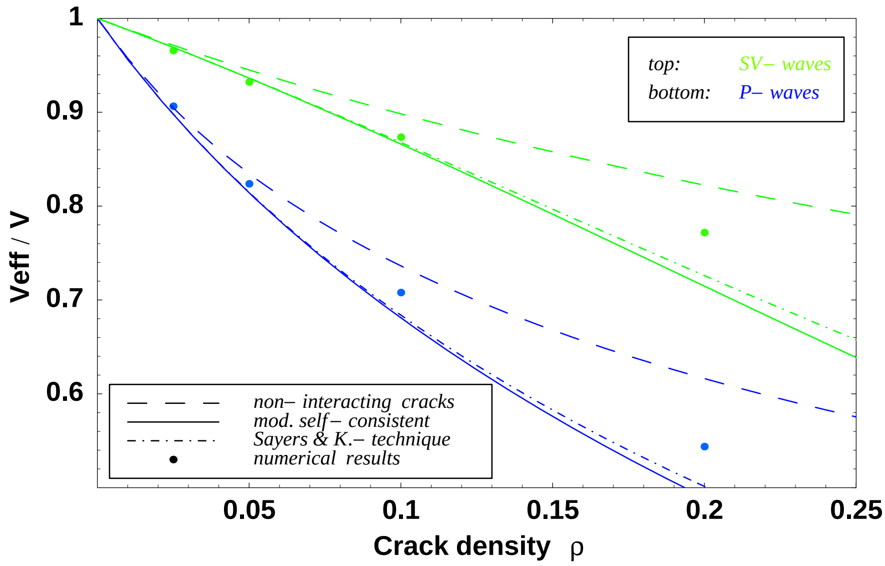

Figure 4: Normalized effective velocity for parallel cracks versus crack density. Dots: Numerical results, Lines: Different theoretical predictions.

CONCLUSIONS

We present a new numerical tool to calculate effective velocities in fractured media. Finite-difference modeling of the elastodynamic wave equation is very fast and accurate. In contrast to a standard staggered grid, high-contrast inclusions do not cause instability difficulties for our rotated staggered grid. Thus, our numerical modeling of elastic properties of dry rock skeletons can be considered as an efficient and well controlled computer experiment.
We propose a heuristic approach called the critical crack density (CCD) formulation. This formulation introduces the critical crack density into the modified (differential) self consistent media theory. The CCD formulation predicts effective velocities for SV-, SH- and P- waves in fractured 2D-media with intersecting rectilinear thin dry cracks. The numerical results support predictions of this new formulation.
Moreover, we show that different theories for effective velocities for parallel cracks can only be applied successfully to a dilute crack density.

REFERENCES

Andrews, D. J., and Ben-Zion, Y., 1997, Wrinkle-like slip pulse on a fault between different materials: Journal of Geophysical Research, **102, No. B1**, 553–571.

Dahm, T., and Becker, T., 1998, On the elastic and viscous properties of media containing strongly interacting in-plane cracks: Pure Appl. Geophys., **151**, 1–16.

Davis, P. M., and Knopoff, L., 1995, The elastic modulus of media containing strongly interacting antiplane cracks: J. Geophys. Res., **100**, 18.253–18.258.

Frankel, A., and Clayton, R. W., 1986, Finite difference simulations of seismic scattering: Implications for the propagation of short-period seismic waves in the crust and models of crustal heterogeneity: Journal of Geophysical Research, **91, No. B6**, 6465–6489.

Kachanov, M., 1992, The elastic modulus of media containing strongly interacting antiplane cracks: Appl. Mech. Rev., **45(8)**, 304–335.

Kelly, K. R., Ward, R. W., Treitel, S., and Alford, R. M., 1976, Synthetic seismograms: A finite-difference approach: Geophysics, **41**, 2–27.

Kelner, S., Bouchon, M., and Coutant, O., 1999, Numerical simulation of the propagation of p waves in fractured media: Geophys. J. Int., **137**, 197–206.

Kneib, G., and Kerner, C., 1993, Accurate and efficient seismic modelling in random media: Geophysics, **58**, 576–588.

Kusnandi, van Baren, G., Mulder, W., Herman, G., and van Antwerpen, V., 2000, Subgrid finite-difference modeling of wave propagation and diffusion in cracked media: , 70th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, ST P1.2.

Mukerji, T., Berryman, J., Mavko, G., and Berge, P., 1995, Differential effective medium modeling of rock elastic moduli with critical porosity constraints: Geophysical Research Letters, **22(5)**, 555–558.

Murai, Y., Kawahara, J., and Yamashita, T., 1995, Multiple scaterring of sh waves in 2-d elastic media with distributed cracks: Geophys. J. Int., **122**, 925–937.

Robinson, P. C., 1983, Connectivity of fracture systems - a percolation theory approach: J. Phys. A: Math. Gen., **16**, 605–614.

Saenger, E. H., and Shapiro, S. A., 2000, Calculation of effective velocities in fractured media using the rotated staggered grid: , European Association Of Geophysical Exploration, 62st Mtg. Eur. Assoc. Expl Geophys., Extended Abstracts, Session:D–34.

Saenger, E. H., Gold, N., and Shapiro, S. A., 2000, Modeling the propagation of elastic waves using a modified finite-difference grid: Wave Motion, **31(1)**, 77–92.

Saenger, E. H., 2000, Wave propagation in fractured media: Theory and applications of the rotated staggered grid: Ph.D. thesis, Karlsruhe University.

Sayers, C., and Kachanov, M., 1991, Single-scattering approximations for coefficients in biot's equations of poroelasticity: Int. J. Solids & Struct., **7(6)**, 671–680.