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# A new model of scattering attenuation: theory and application

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## ABSTRACT

*We derive a scattering attenuation model based on the statistical wave propagation theory in random media. This new description of scattering attenuation additionally obeys the causality principle. It practically has no restriction in the frequency domain. The presented formulas allow to quantify scattering attenuation in complex geological regions using simple statistical estimates from well-log data. This knowledge is important for further petrophysical interpretations of reservoir rocks. This description of scattering attenuation can be also helpful in order to design and to improve monitoring systems.*
*Furthermore, we apply this model to measured Q-values at the German KTB-site and find that scattering attenuation plays an important role in the upper crust in this area. This is not confirm with previous studies that suggest absorption mechanisms as the main reason for attenuation.*


## INTRODUCTION

Fundamental signatures of seismic waves in rocks are the attenuation and the dispersion. It is of great importance for the interpretation of seismic data to quantify the magnitude as well as the frequency dependence of attenuation. The knowledge of attenuation is needed for a correct estimation of the magnitude of reflection coefficients. From a practical point of view, there are however serious problems connected with the determination of attenuation (Bourbié et al., 1987). One of them is the fact that, in general, attenuation is not only caused by absorption, i.e. due to viscoelastic effects (anelasticity, presence of fluids) but also due to the heterogenous composition of rocks and reservoirs on many length scales. A further difficulty is the frequency dependence of attenuation measurements. That is why laboratory experiments cannot easily be compared with field measurements.

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It is well-known that inhomogeneities of any scale affect the seismic wavefield and characteristics of scattering attenuation and dispersion are observed like e.g. the decrease of seismogram amplitudes with increasing travel-distance and the broadening of seismic pulses. In contrast to attenuation by absorption (sometimes called intrinsic attenuation), where the energy of the wavefield is transferred into heat, scattering attenuation means a spatial re-distribution of wavefield energy. So that at a certain recording position only a part of the wavefield energy can be received within a certain time interval. The signals seem apparently damped (apparent attenuation).

Usually the attenuation of seismic wavefields are characterized with help of the quality factor $Q$ (see Bourbié et al., 1987, for a detailed exposition of the quality factor concept). In general, Q-values inferred from seismic data sets include both kinds of attenuation. In many studies it is assumed that the total reciprocal quality factor can be obtained by superposition of the reciprocal quality factors connected with a certain attenuation mechanism. There are also some efforts to separate the intrinsic part from the scattering attenuation part. This is, however, no easy task, especially in reservoir geophysics, because a priori it is not known which absorption mechanisms are present nor how the geometrical composition of rocks looks like.

Deterministic approaches are not suitable to describe the complex structures of reservoirs. In contrast to this, stochastic models provide an interesting alternative and are ideally used complementary with deterministic (macro-) models. Analytical results are obtained within framework of wave propagation theory in random media. Theoretical methods developed in order to quantify scattering attenuation include the meanfield theory using the Born approximation or the traveltime-corrected meanfield formalism (for an overview see Sato and Fehler, 1998). The meanfield theory over-estimates the scattering attenuation. The traveltime-corrected meanfield formalism excludes large wavenumbers so that scattering on large-scale heterogeneities is not taken into account. It requires a heuristically chosen cut-off wavenumber (or a scattering angle) which can be only determined by numerical tests.

It is the purpose of this study to derive a model of scattering attenuation that is applicable in connection with standard methods in reservoir geophysics. Based on the Rytov approximation and the causality principle, we present tractable expressions for scattering $Q$ in 2-D and 3-D random media (see Müller et al., 2000).
In the second part of this paper we apply this new model to the KTB area, where statistical estimates from the boreholes are used in order to quantify the amount of scattering attenuation. In contrast to previous studies, where the scattering attenuation estimate of the traveltime-corrected meanfield formalism was found to be insignificant (e.g. Holliger, 1997), we show that the measured $Q$-values can be explained to a considerable amount due to scattering on upper-crustal heterogeneities.

# THEORY

## Scattering attenuation and dispersion in random media

Based on the Rytov approximation and the causality principle, we give a description of scattering attenuation for plane and spherical waves propagating in 2-D and 3-D weakly heterogenous elastic solids. Shapiro et al. (1996) derived analytical expressions for the phase velocity in random media which is practically valid for all frequencies. Applying the Kramers-Kronig relationship (which follows from the causality, passivity and linearity of the medium) to the results for the phase velocity, we calculate the attenuation (for a detailed derivation see Müller et al., 2000). For plane wave propagation in 3-D we obtain

$$\alpha_{plane}^{3D} = 2\pi^2 k^2 \int_0^\infty d\kappa \, \kappa \, \Phi^{3D}(\kappa) \left[ H(\kappa - 2k) - \frac{\sin(2\pi A^2)}{2\pi A^2} \right] , \tag{1}$$

where $A = \sqrt{\frac{\kappa^2 L}{2\pi k}}$ and $k = \frac{\omega}{c_0}$ denotes the wavenumber, $c_0$ is the constant background velocity, $L$ the travel-distance. $\Phi^{3D}(\kappa)$ is the fluctuation spectrum which contains the second-order statistics of the medium's fluctuations, i.e. the variance $\sigma^2$ and the correlation length $a$, and $H$ denotes the Heaviside (unit step) function. Note that the corresponding results in 2-D can be obtained by skipping $\kappa$ in the integral over $\kappa$ and dividing by $\pi$. The validity range of equation (1) in terms of the wave parameter $D = 2L/(ka^2)$ is

$$\frac{1}{\pi L/\lambda} \leq D \leq \left(\frac{L}{a}\right)^2 \tag{2}$$

if $L > \max\{\lambda, a\}$ where $\lambda$ denotes the wavelength.
In the case of a point source excitation in 3-D we find by an analogical treatment

$$\begin{aligned} \alpha_{point}^{3D} \approx \; & 2\pi^2 k^2 \int_0^\infty d\kappa \, \kappa \, \Phi^{3D}(\kappa) \\ & \left[ H(\kappa - 2k) - \frac{\cos(\pi/2A^2)\, C(A) + \sin(\pi/2A^2)\, S(A)}{A} \right] . \end{aligned} \tag{3}$$

Here the functions $C$ and $S$ denote the Fresnel cosine and sine integrals, respectively. Note, however, that in a strict sense the Kramers-Kronig relations can be only applied to the wavenumber of a plane wave. A rigorous derivation of equation (3) should be based on a plane wave decomposition of the used (point source) wavefield attributes This work is still going on.

Figures (1) and (2) depict the phase velocity $v$ divided by $c_0$ and the reciprocal quality factor $1/Q = 2\alpha/k$ as functions of the dimensionless wavenumber $ka$ for plane waves propagating in 2-D and 3-D (exponentially and Gaussian correlated) random media. Additionally we compute $v/c_0$ and $1/Q$ for waves propagating in 1-D random media according to the generalized O'Doherty-Anstey formalism of Shapiro and Hubral (1999). The reciprocal quality factor in Fig. (2) is normalized by $\sqrt{\pi}\sigma^2$.

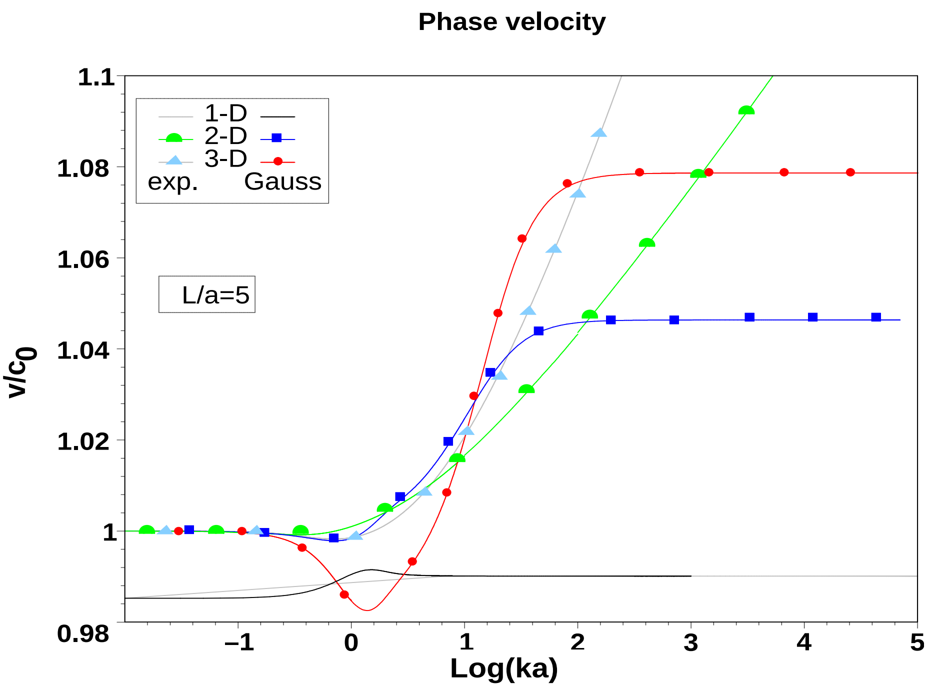


Figure 1: Phase velocity normalized by the background velocity $c_0$ as a function of $ka$ for Gaussian and exponentially correlated random media in 1-D, 2-D and 3-D .

## APPLICATION

### Attenuation measurements at the KTB-site

Within the framework of the KTB (the German continental deep drilling project) a new insight in the structure of the continental crust was obtained. The KTB-site is located in Southeast Germany and consists of two super-deep boreholes. The rocks are of crystalline type (for a thorough review of this project we refer to Harjes et al. 1997). Several techniques (including analysis of VSP-Data and lab measurements) have been applied in order to estimate the attenuation (see also Pujol et al, 1998). Figure (3) displays the measured values in the frequency range from 5 to 85Hz and indicates the corresponding references and depth intervals. In summary, all studies show surprisingly low $Q$-values ($Q < 100$). Note that reported $Q$-values of the Lithosphere are of the order $Q = 100..1000$ (see e.g. Figure 5.2 in Sato and Fehler, 1998).

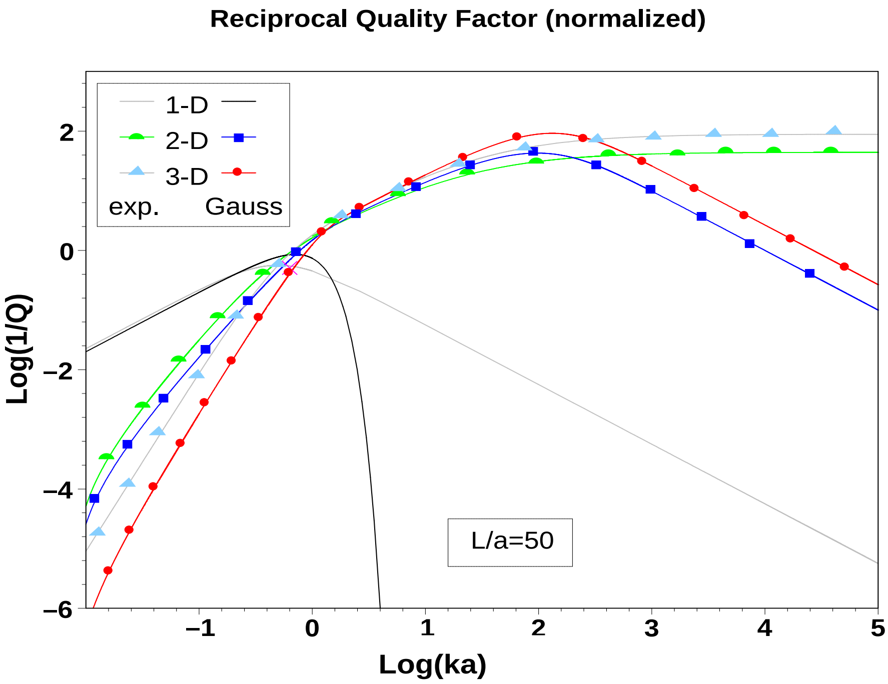


Figure 2: The reciprocal quality factor $1/Q = 2\alpha/k$ as a function of $ka$ for Gaussian and exponentially correlated random media in 1-D, 2-D and 3-D.

In order to interpret these low $Q$-values several attenuation mechanisms have been assumed to play a major role at the KTB-site. Lüschen et al. (1996) argued that lithology contrasts do not cause strong reflections (so that scattering attenuation is small?). Strong reflections are connected with fluid-filled fracture systems. Pujol et al. (1998) argued that scattering due to thin layering is not a dominant effect of attenuation. Holliger (1997) used the single scattering model in 2-D in order to estimate upper crustal scattering $Q$-values ranging between 600 and 1500 and concluded that the attenuation is dominated by absorption rather than by scattering.

## Application of the new scattering attenuation model

We study whether the relatively low $Q$-values can be explained due to scattering. That is to say, we apply the scattering attenuation model as derived in the previous section to the KTB-site. Fortunately, a statistical analysis of the well-log (Vp, Vs and density logs) and VSP data yielding estimates of characteristic sizes of heterogeneities (correlation lengths $a$) and contrasts (relative standard deviations $\sigma$ of the background P-wave velocity $c_0$) have been already performed. The results are briefly summarized in the following table:

| Statistical parameter estimates from log-data at the KTB-site | | | |
|---|---|---|---|
| reference | depth range ($[z] = m$) | type of correlation function | parameters ($[c_0] = km/s$, $[a] = m$) |
| Wu et al., 1994 | 286-6000 | von Karman (2-D), anisotropic $\frac{a_x}{a_z} \approx 1.8$ | $\nu_z = 0.05$, $\nu_x = 0.5$ $a_z = 2000$, $a_x = 3600$ $\sigma = 6.1..6.5\%$, $P : c_0 = 6.1$ |
| Kneib, 1995 | 285-6000 | superposition of 2 exponential, isotropic | $a_1 = 1$, $a_2 = 20$ $\sigma = 2.9..3.4\%$ $P : c_0 = 6.4$, S: $c_0 = 3.53$ |
| Holliger, 1996, 1997, Jones & Holliger, 1997 | 285-7200 | von Karman (3-D) isotropic $\frac{a_x}{a_z} \approx 1$ | $\nu = 0.1..0.2$ $a = 60..160$ $\sigma = 3.2..6.3\%$, $P : c_0 = 6.3$ |

Moreover, Jones and Holliger (1997) investigate the inter-log coherence and find no significant correlation between the sonic logs from the pilot and main hole, which are located 200m apart. They conclude that the lateral correlation length is smaller than 200m and is approximately the same as the vertical correlation length. These studies

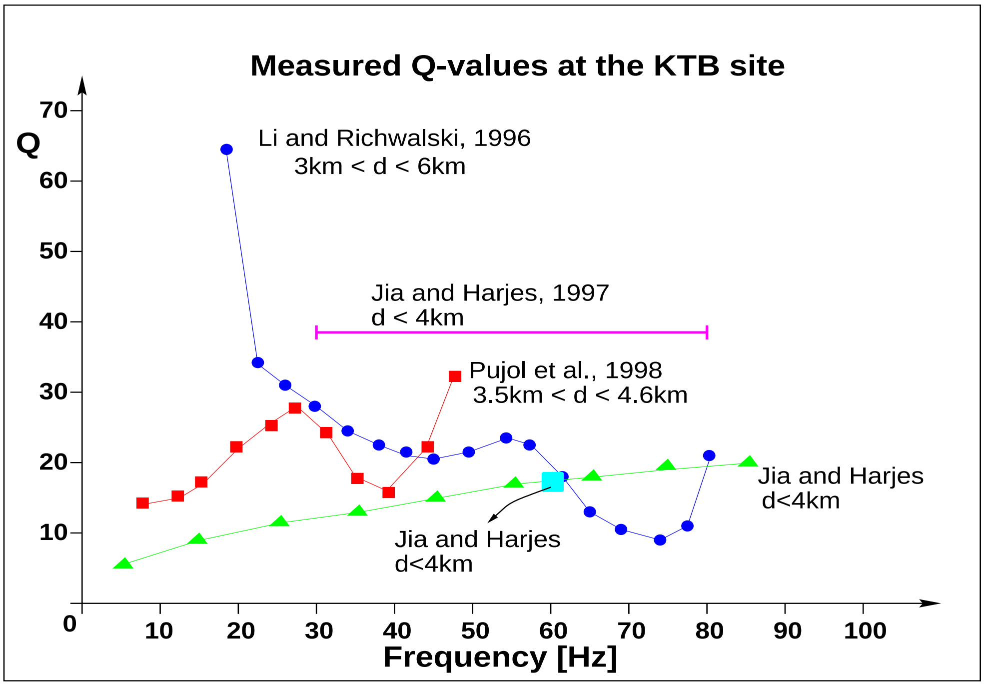


Figure 3: Measured Q values at the KTB-site and the corresponding reference.

justify the assumption that the upper crust heterogeneities can be well characterized as a realization of an isotropic random medium using spatial correlation functions. This assumption has been made not only for the KTB-site but also for many regions. Especially the fractal-like character of upper crustal heterogeneities may be accommodated by the use of a von Karman correlation function (see Sato and Fehler, 1998). Accordingly, we choose a von Karman correlation function which is additionally characterized by the so-called Hurst coefficient $\nu$. Its fluctuation spectrum in 3-D is given by

$$\Phi^{3D}(\kappa) = \frac{\sigma_n^2 \, a^3 \, \Gamma(\nu + 1.5)}{\pi^{1.5} \, \Gamma(\nu) \, (1 + \kappa^2 a^2)^{\nu+1.5}} \, . \tag{4}$$

For $\nu = 1/2$ one obtains an exponential correlation function. Note that equation (4) differs from somewhere used definitions of the fluctuation spectrum (factor $8\pi^3$ due to different definition of the Fourier transform).

As the Q-measurements result from VSP-data we must further make sure that the weak scattering regime, where our scattering model is valid, is a reasonable assumption. The scattering regime is characterized by 3 parameters, namely the dimensionless wavenumber $ka$, the normalized travel-distance $L/a$ and the strength of the velocity perturbations (i.e., the relative standard deviation $\sigma$). Kneib (1995) shows that the relevant frequencies in the VSP experiment ($f = 30Hz$) and the inferred correlation length lead to $ka > 0.6$. Taking into account the geometry of the experiment one obtains $L/a = 15..400$. Together with the estimated relative standard deviations ($\sigma < 7\%$), we conclude that the weak scattering assumption is fully justified. The regime, where the wavefield fluctuations become saturated, is not realizable within the given VSP experiment (this becomes intuitively clear when looking at the VSP seismograms like in Figure 17 of Kneib (1995), where the wavefield amplitudes after the first arrivals are small and the wavefront is not strongly distorted).

We compute next the quality factor as a function of frequency for waves excited by a point source in 3-D using formulas (3) and (4) The impact of the parameters Hurst coefficient, standard deviation, correlation length and travel-distance are studied (Fig. (4)). The parameters are chosen according to the inferred values from the well log analysis. First we let vary the Hurst coefficient from $\nu = 0.1..0.5$ for fixed values of $L = 7km$, $c_0 = 6350m/s$, $\sigma_v = 5.5\%$ and $a = 160m$ (upper left-sided plot in Fig. (4)). An increased scattering attenuation can be observed for increasing Hurst coefficients. If $\nu = 0.5$ the complete measured attenuation could be explained due to scattering attenuation. This value of $\nu$, however, is according to the statistical estimates too high.

Then we study the influence of the relative standard deviation of the velocity

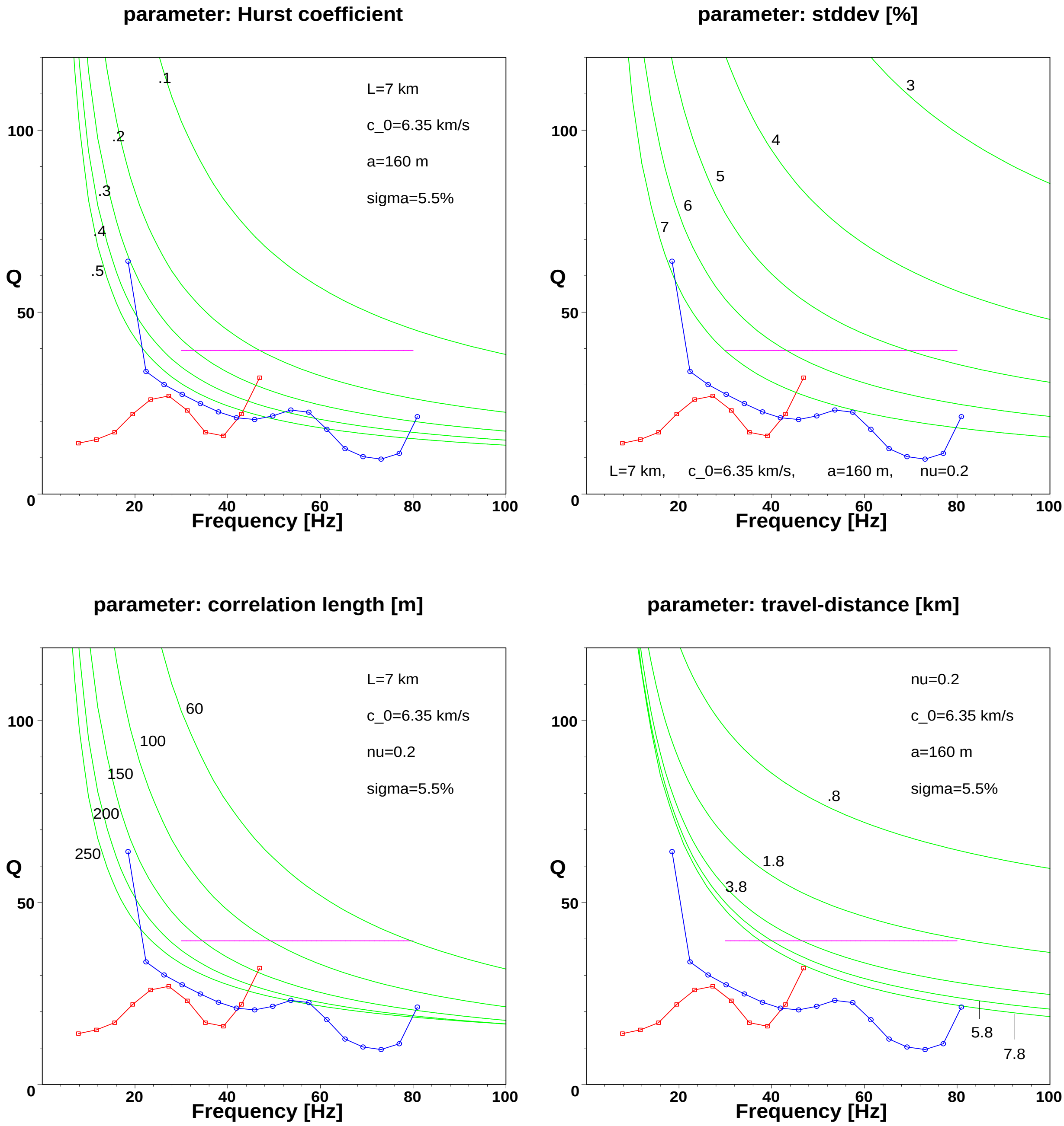


Figure 4: Parameter study of the scattering attenuation model using statistical estimates from well-log data at the KTB-site. The measured Q-values from Fig. (3) are displayed for comparison.

fluctuations (upper right-sided plot in Fig. (4)), where we use the same fixed values as above and choose $\nu = 0.2$. The relative standard deviation of the velocity fluctuations shows a strong influence on the magnitude of attenuation. For large standard deviations ($\sigma > 7\%$) we observe again that the complete measured attenuation is due to scattering (this is of course an unrealistic situation and defines an upper bound of $\sigma$).

We are further interested in the dependency on the correlation length (lower left-sided plot in Fig. (4)). As the estimates from the KTB boreholes suggest, we consider realistic values of $a$ ranging from 60 to 250m. Finally the lower plot on the right side of Fig. (4) displays the dependency on the travel-distance.

In summary, Fig. (4) clearly demonstrates that within the given (realistic) parameter ranges, a considerable amount of the attenuation can be explained in terms of scattering attenuation. Moreover, for frequencies higher than 30 Hz, all relevant parameter combinations yield a quality factor smaller than 100. In contrast to previous interpretations (see above), this indicates that scattering on upper-crustal heterogeneities plays at least a considerable role at the KTB-site. We do not see a contradiction to the fluid-filled crack systems as a possible source of strong attenuation since there would be also strong scattering attenuation on these crack systems. Nevertheless, it is clear that there is strong intrinsic attenuation in this area. Especially for frequencies smaller than 30 Hz, the measured $Q$-values can not be explained in terms of scattering attenuation.

## CONCLUSIONS

We derive a scattering attenuation model based on the statistical wave propagation theory in random media. This new description of scattering attenuation additionally obeys the causality principle. It has practically no restriction in the frequency domain. The presented formulas allow to quantify scattering attenuation in complex geological regions using simple statistical estimates from well-log data. To demonstrate this, we apply the model to measured Q-values at the KTB-site and find that scattering attenuation plays an important role in the upper crust in this area.
Estimates of scattering attenuation as derived from our formulas can be important for further petrophysical interpretations of reservoir rocks. This description of scattering attenuation can be also helpful in order to design enhanced monitoring systems.

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