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Aspects of the unified approach theory to seismic imaging: theory, implementation, and application

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ABSTRACT

This contribution presents theoretical and practical aspects of the challenge, how to handle 2D and 3D data in a true-amplitude, efficient, and time-saving way for a laterally and vertically inhomogeneous isotropic earth model. The underlying basic imaging processes — Kirchhoff-type true-amplitude migration and demigration — are explained and an implementation of the unified approach theory is shown. Practical aspects such as aliasing, tapering, the migration and demigration apertures or the creation of Green's Function Tables (GFT) are considered and, finally, some synthetic dataset examples are shown.

INTRODUCTION

In the past two decades 3D reflection seismics has become a powerful tool in the world of seismic exploration. This development is driven by the accuracy of images of the subsurface obtained by 3D seismics compared with the images of 2D seismic lines, as only 3D seismics can handle out-of-plane reflections correctly. Many of the basic concepts that have been developed for 2D seismic imaging are still valid for 3D imaging. However, 3D imaging presents several new problems that are caused by higher dimensionality – prestack seismic data is defined in a 5D space (time, two components of the source position, two components of the receiver position), compared with the 3D space (time, one source coordinate, one receiver coordinate) of 2D prestack data. An obvious problem is the amount of data that is recorded in the field and that has to be processed. Thus, an important aspect is the right strategy for reducing the time and resources which are necessary to obtain an image of the subsurface from the original large amount of data without compromising the accuracy of the results.

During the last years, processing methods which do not only account for the

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kinematic (related to traveltimes) but for the dynamic (related to amplitudes) aspects as well have become more and more important. Such methods are able to provide images that can be used to determine geological and quantitative physical properties of the subsurface which are useful for reservoir characterization by means of amplitude-variation-with-offset (AVO) and amplitude-versus-angle (AVA) analyses. An AVO/AVA analysis aims to point out amplitude anomalies which may be caused by hydrocarbon reservoirs. The quantitative physically well-defined amplitude values of images are referred to as 'true amplitudes'. A true amplitude is defined as nothing more than the amplitude of a recorded reflection compensated by its geometrical spreading factor. Moreover, the construction of a true-amplitude reflection implies not only a scaling of the considered reflection amplitude with the geometrical spreading factor but also the reconstruction of the analytic source pulse.

To achieve the above mentioned objectives, the unified approach theory by (Hubral et al., 1996) (basic concepts) and (Tygel et al., 1996) (theory) is presented which is based on ray theory and simple geometrical concepts. This approach is composed of firstly a weighted Kirchhoff-type diffraction stack integral to transform seismic reflection data from the time domain into the depth domain, and secondly a weighted Kirchhoff-type isochrone-stack integral to transform the migrated seismic image from the depth domain back to the time domain. By cascading or chaining both processes, we are able to solve various kinds of seismic reflection imaging problems, e.g. migration to zero offset (MZO) or remigration (RM). Due to the fact that all methods presented here are target-oriented, efficient, and highly parallelizable, they can be used to perform 4D seismic monitoring by means of modelling by demigration. That means time-dependent variations of subsurface properties can be investigated, e.g. the exploitation of an oil field and the effect on recorded seismograms.

THEORY

The task of depth migration is the transformation of seismic data acquired with an arbitrary shot-receiver configuration at a certain measurement surface from the time to the depth domain. If this transformation is performed in such a way that the dynamic as well as the kinematic parts are treated correctly, it is called true-amplitude migration. A true-amplitude trace is a seismic trace where the amplitude is free of geometrical spreading effects, i.e., where the amplitude is corrected by the geometrical spreading factor. Therefore, the amplitude of a true-amplitude migrated reflector image is a measure of the angle-dependent reflectivity and can be used to determine geological and (quantitative) physical properties of the media by means of an AVO/AVA analysis. It is important to recognize that a true-amplitude migration is not able to provide the true reflection coefficient, although the name implies it, because some factors – including for example source and receiver effects (e.g. strength and coupling, geophone sensitivity), transmission, attenuation, thin-layer effects, scattering, and anisotropy – may have falsified the amplitudes. The latter mentioned effects, however, are usually small. Furthermore, we are mainly interested in relative changes of the medium parameters at an interface not at global quantitative values.

Demigration is the (asymptotic) mathematical inverse to migration and undoes what the migration process has done to the original seismic section, i.e., the aim of demigration is the reconstruction of a seismic time section (primary reflection events only) from a depth migrated section. This transformation can also be performed in consideration of geometrical spreading effects which will then also be called true-amplitude.

The seismic record is expected to consist of analytic traces that contain the selected analytic elementary reflection events $U(\vec{\xi},t)$ where $\vec{\xi}$ is a configuration parameter characterizing the shot-receiver geometry (Schleicher et al., 1993). Analytic traces of reflection events are necessary to correctly account for the phase shift of primary reflections; they are constructed from the real traces by adding their Hilbert transform as an imaginary part. One primary reflection event $U(\vec{\xi},t)$ can be expressed in zero-order ray approximation as

$$U(\vec{\xi},t) = U_0(\vec{\xi})\; W(t-\tau_R) = R_c \frac{\mathcal{A}}{\mathcal{L}}\; W(t-\tau_R)\;, \tag{1}$$

where $W(t)$ represents the analytic point-source wavelet which has to be reproducible for all source points S. The function τ_R provides the traveltime along the ray SM_RR where M_R is an actual reflection point and R is the receiver located at the surface. The parameter R_c is the plane-wave reflection coefficient at the reflection point, $\mathcal{A}$ is the total loss in amplitude due to e.g. transmission across all interfaces along the ray, and $\mathcal{L}$ is the normalized geometrical spreading factor.

True-amplitude migration

The true-amplitude migration procedure is performed by a weighted Kirchhoff-type diffraction stack. The kinematic part of the transformation can easily be performed in the following way: assume a dense rectangular grid of points M in the depth domain in which we want to construct the depth-migrated reflector image (target zone) from the given time section. Moreover, let all depth points M on the grid be treated like diffraction points (which primarily led to the name 'diffraction-stack migration') in the given (or already estimated) macro-velocity model. Their diffraction traveltime surfaces – which would pertain to actual diffraction responses if the points M were actual diffraction points in the given model – can be calculated using the macro-velocity model. Now, a Kirchhoff-type depth migration involves in principle nothing else but performing a summation (also called stack) of all the seismic trace amplitudes encountered along each diffraction-time surface (Huygens surface) in the time section, and placing the summation value into the corresponding point M. The dynamic part which we refer to as true-amplitude can be performed by weighting all seismic trace amplitudes along the diffraction-time surface during the stacking process by a varying true-amplitude weight factor. The summation can mathematically be expressed by

$$V(M) = \frac{-1}{2\pi} \iint\limits_{A} d\xi_1 \; d\xi_2 \; W_{DS}(\vec{\xi}, M) \frac{\partial U(\vec{\xi}, t)}{\partial t}\bigg|_{t=\tau_D(\vec{\xi},M)} . \tag{2}$$

The stacking surface is the Huygens surface given by

$$t = \tau_D(\vec{\xi}, M) = \tau(S(\vec{\xi}), M) + \tau(M, R(\vec{\xi})) \; , \tag{3}$$

with the vector $\vec{\xi}$ varying over the aperture A. Note, that to each subsurface point M, one generally assigns the value Re$\{V(M)\}$ to display the depth migrated image. However, to permit the extraction of complex reflection coefficients in case of overcritical reflections, one has to consider the full complex value $V(M)$ instead. The choice of $\frac{\partial U}{\partial t}$ is needed in order to correctly recover the source pulse. The region of integration A should ideally be the total (ξ_1, ξ_2)-plane. This is, of course, impossible due to the limitation of the aperture of the data acquisition (Tygel et al., 1996).

The true-amplitude weight function, is given by

$$W_{DS}(\vec{\xi}, M) = \frac{h_B \, v_M^2}{2\cos^2 \alpha_M} \, \mathcal{L}_{SM} \, \mathcal{L}_{MR} \; , \tag{4}$$

where $h_B = h_B(\vec{\xi}, M)$ is the Beylkin determinant (Jaramillo et al., 1998). It is important to note that the weight function on its own does not remove the geometrical spreading effect. Only the true-amplitude weight function *and* the summation process along the diffraction surface account for the whole geometrical spreading effect. Surprisingly, the evaluation of the stacking integral by means of the stationary phase method shows that the stack automatically corrects for the Fresnel factor, i.e. reflector curvature effects on the total geometrical spreading factor are automatically taken into account. The chosen weight function (4) simply corrects for the remaining geometrical spreading effects caused by ray segments in the reflector overburden. This is the reason why the weight function is independent of the reflector's curvature.

True-amplitude weight functions for special shot-receiver configurations and a comparison can be found in (Hanitzsch, 1997).

True-amplitude demigration

The true-amplitude demigration procedure is performed by a weighted Kirchhoff-type isochrone stack. The kinematic part of the transformation can easily be performed in a completely analogous way to true-amplitude migration as described in the last section. Rather than defining a grid of points M in the depth domain, we now define a dense grid of points N in the time domain, i.e., in the $(\vec{\xi}, t)$-volume in which we desire to construct the time section from the depth-migrated section. Each grid point N together with the macro-velocity model defines an isochrone in the depth domain. The isochrone determines all subsurface points for which a reflection from a possible true reflector would be recorded at N after traveling along a primary reflected ray from S to R. We now have to perform a stack along each isochrone on the depth-migrated section amplitudes. Then, we place the resulting stack value into the corresponding point N. As in Kirchhoff migration, the isochrone-stack demigration will only result in significant values if the point N is in the near vicinity of an actual reflection-traveltime surface. Elsewhere it will yield negligible results. The dynamic part can once again be performed by a multiplication of the amplitudes with a varying weight function.

The summation can mathematically be expressed by

$$V(N) = \frac{1}{2\pi} \iint_E dx\; dy\; W_{IS}(\vec{r}, N) \frac{\partial \Phi(\vec{r}, z)}{\partial z} \bigg|_{z=\zeta_I(\vec{r}, N)} . \tag{5}$$

The stacking surface is the isochrone $z = \zeta_I(\vec{r}, N)$ given by

$$\tau_D(\vec{\xi}, M) = \tau(S(\vec{\xi}), M) + \tau(M, R(\vec{\xi})) = t \tag{6}$$

for all points M with $\vec{r}$ in E. As we can see here, the isochrone and the Huygens surface are defined by the same traveltime function $\tau_D(\vec{\xi}, M)$. The function $\Phi(\vec{r}, z)$ represents the migrated reflector image to be demigrated and can be expressed as

$$\Phi(\vec{r}, z) = \Phi_0(\vec{r})\; W(m_D(z - \zeta_R)) \;, \tag{7}$$

where $W(t)$ represents the analytic point-source wavelet, $m_D(\vec{r})$ is the vertical stretch factor, and points $M_R = M(\vec{r}, z = \zeta_R(\vec{r}))$ define the actual reflector. W_{IS} is the true-amplitude weight function to be specified. The value Re$\{V(N)\}$ is the demigration result which is generally assigned to the point N. However, if we want to chain true-amplitude migration and demigration, we have to use the complex value $V(N)$ (Tygel et al., 1996).

The true-amplitude weight function for demigration relates to the weight function for migration because the demigration should undo what the migration has done to the original seismic section (provided the same macro-velocity model, the same measurement configuration, and the same ray codes are used; only primary events are considered here). The weight function for demigration can be calculated in an analogous way to the previously described migration weight function, and reads

$$W_{IS}(\vec{r}, N) = \frac{1}{2\mathcal{L}_{SM}\mathcal{L}_{MR}\cos^2\beta_M} \,, \tag{8}$$

where $\mathcal{L}_{SM}$ and $\mathcal{L}_{MR}$ denote once again the point-source geometrical spreading factors for the ray segments SM and MR, respectively; β_M is the angle between the half-angle direction – defined by the two ray segments – and the vertical at M (Jaramillo et al., 1998).

IMPLEMENTATION

As we have shown in the previous section, the knowledge about the Huygens surface and the isochrone (stacking surfaces) are fundamental for Kirchhoff-type migration and demigration. These surfaces can be built by the sum of traveltimes along the ray branches from the source S to the depth point M and from M to the receiver R. Thus, we need a priori information about the traveltimes (for the calculation of the Huygens surface or isochrone) and dynamic parameters (for the calculation of the weight function) to perform a true-amplitude migration or demigration. This is where the macro-velocity model comes into operation. The necessary information, i.e., the Green's function of the medium, is usually obtained by asymptotic dynamic ray tracing for a specified ray-code in the macro-velocity earth model, and storing the parameters in a database which we call Green's Function Table (GFT). This database is the link between the input and output space, see Figure 1. In other words, we need information about the wavefield from every shot and receiver position at the surface to every diffraction point M in the subsurface. Obviously, this is a large amount of information, and the GFT can exceed the input data by several orders of magnitude. In order to avoid computation and storage of redundant information, methods to optimize the creation of the GFT are explained by (Riede et al., 2000) and (Hertweck, 2000).

Our imaging tool is written in C++ in an object-oriented way. According to the previously mentioned stacking integrals, we have to calculate the analytical signal and its derivative (3D) or half-derivative (2D). This is done in the frequency or wavenumber domain, respectively. Tapering of the input data is used to avoid boundary effects at the border of the aperture during the summation process. We multiply the input data (along the shot coordinates) by the function $f(x) = 1 - \cos x$ over a small range at the border, usually some traces. No tapering is needed within the shot gathers along the offset dimension. The actual number of traces which are used for tapering may be specified by the user. The complete taper function in 3D is

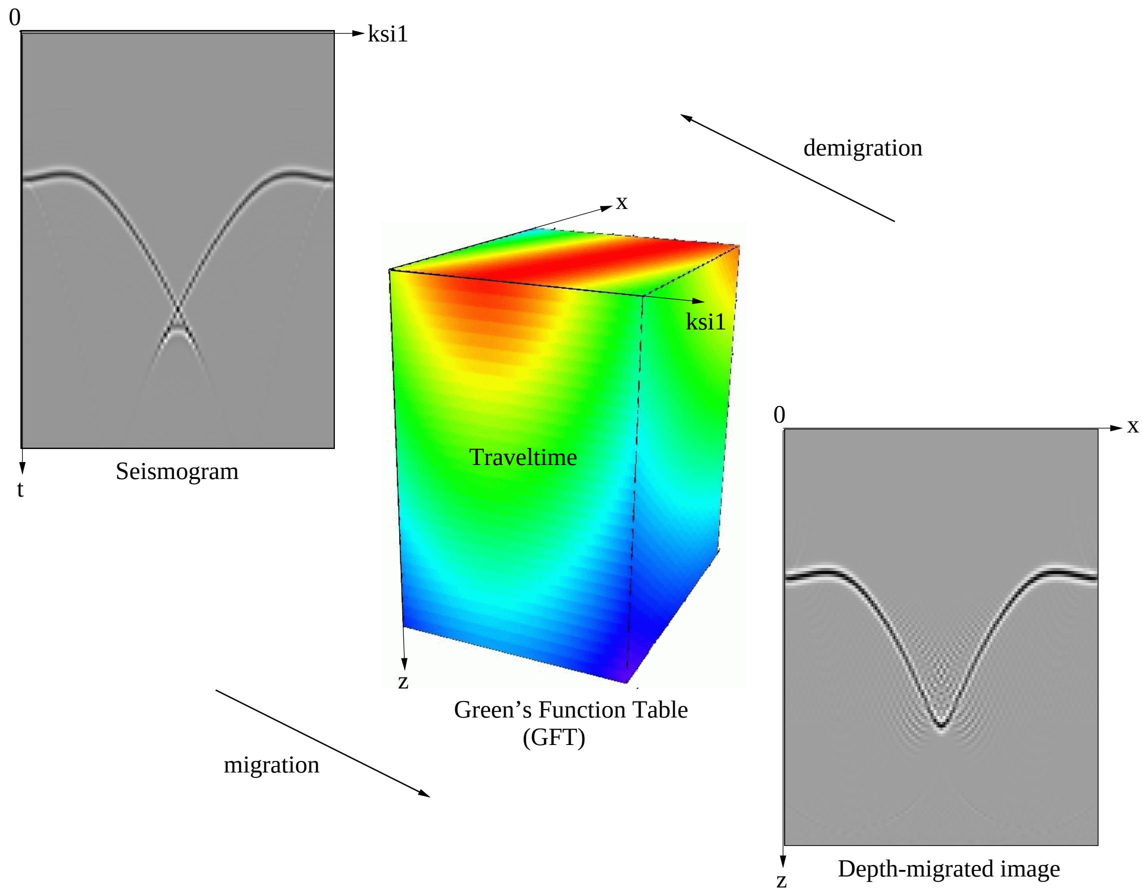

Figure 1: Scheme showing the migration and demigration procedure for 2D data. The input data is transformed to the output domain with the help of the Green's Function Table (GFT) which contains all necessary information to construct the stacking surfaces as well as the weight functions.

plotted in Figure 2.

The weight functions for true-amplitude migration or demigration are calculated during runtime. The necessary information is obtained from the GFT.

To avoid operator aliasing, the operator itself is tapered or, in other words, the possible reflector dip which can be imaged is limited. The limit can be defined by the user according to his needs and the input and output grid spacings.

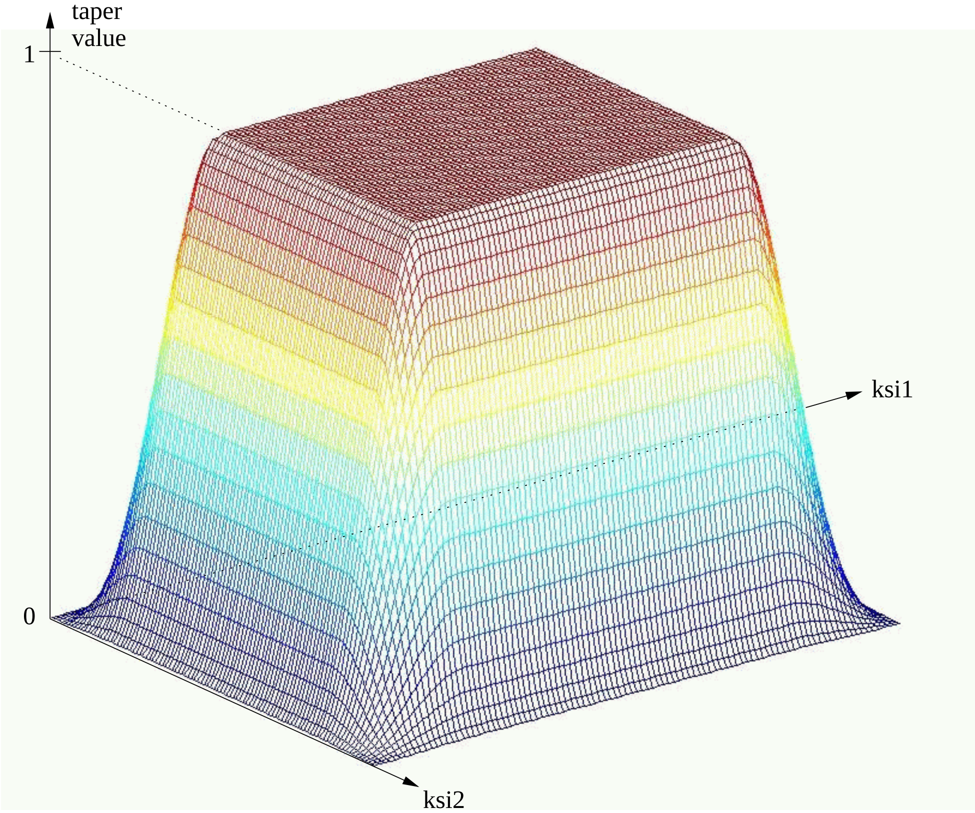

Figure 2: Taper function in 3D. In this plot, the number of traces which are tapered is enlarged compared to the overall number of traces to show the effect more clearly.

APPLICATION

We have applied our imaging tool to several synthetic datasets to show how true-amplitude migration and demigration work. The development and the application on synthetic and real datasets are going on. The current version of the program is able to handle 2D and 3D true-amplitude poststack migration and demigration, and 2D true-amplitude prestack migration. The 3D prestack case is not considered at the moment due to limitations in available computing facilities. We cannot really expect that the theory presented here and the respective weight functions and algorithms work well for all earth models. However, the following results show that our imaging tool is able to handle various input datasets in a flexible and accurate way.

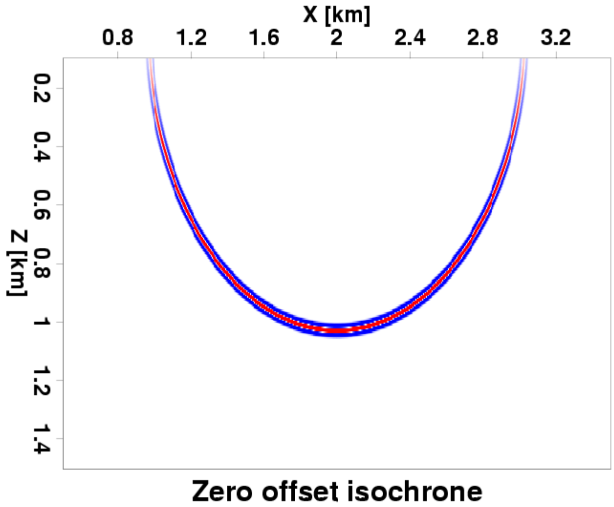

Figure 3: Isochrone. This curve is obtained by migrating a single event, here for zero offset (ZO) configuration and (for simplicity) constant velocity. In a homogeneous medium, the ZO isochrone is a semicircle (2D) or a hemisphere (3D).

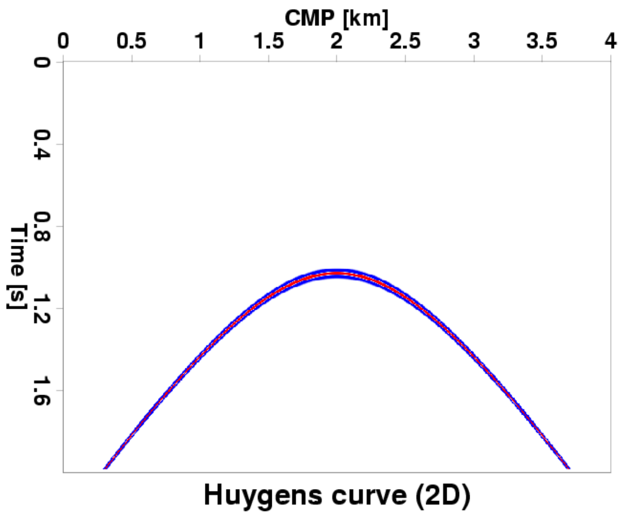

Figure 4: Huygens curve. This curve is obtained by demigrating a single diffraction point, here for zero offset (ZO) configuration and (for simplicity) constant velocity. In a homogeneous medium, the ZO Huygens curve is a hyperbola (2D) or a hyperboloid (3D).

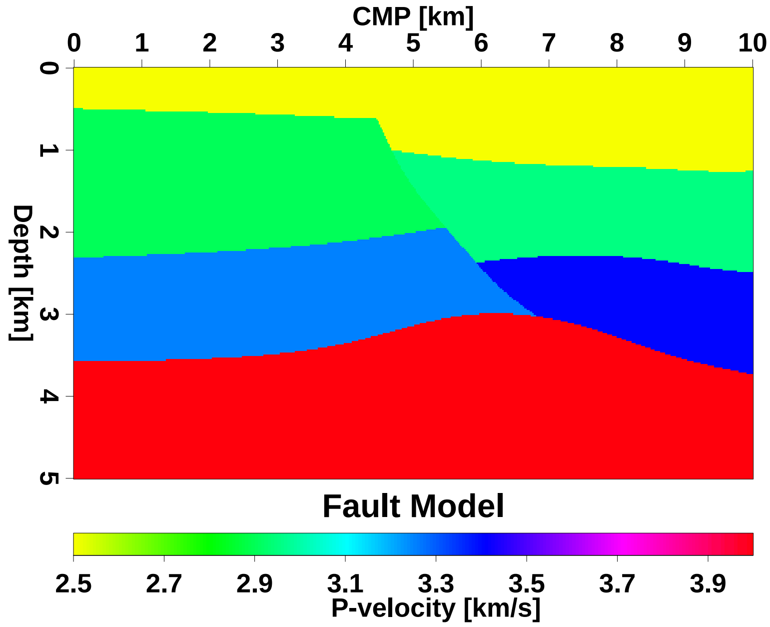

Figure 5: A synthetic model used for testing migration and demigration in 2D. The colors denote P-wave velocities. For simplicity, the density of all layers is constant.

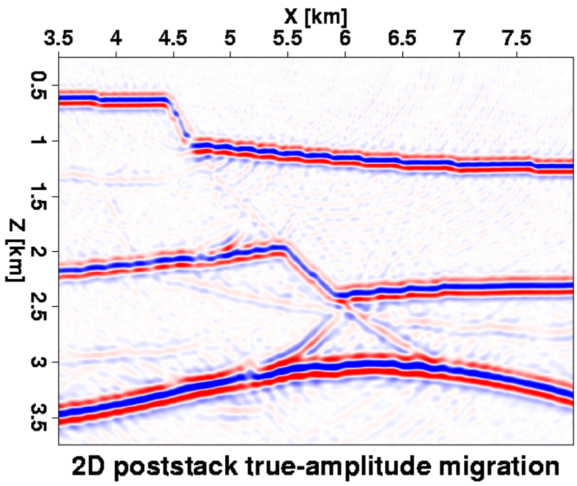

Figure 6: 2D poststack true-amplitude depth migration. The wavelets are correctly mapped from the time to the depth domain, their amplitudes correspond to the impedance contrast at each interface.

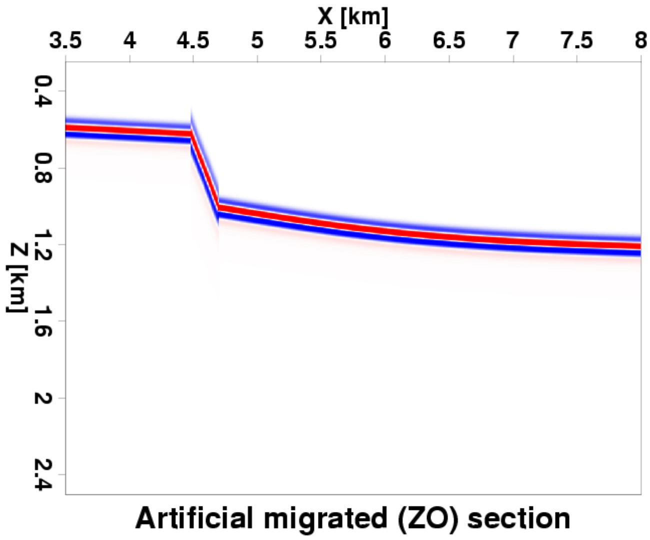

Figure 7: 2D artificial migrated image of the upper three layers (model Fig. 5). This section is created by attaching to each reflector element a correctly stretched wavelet.

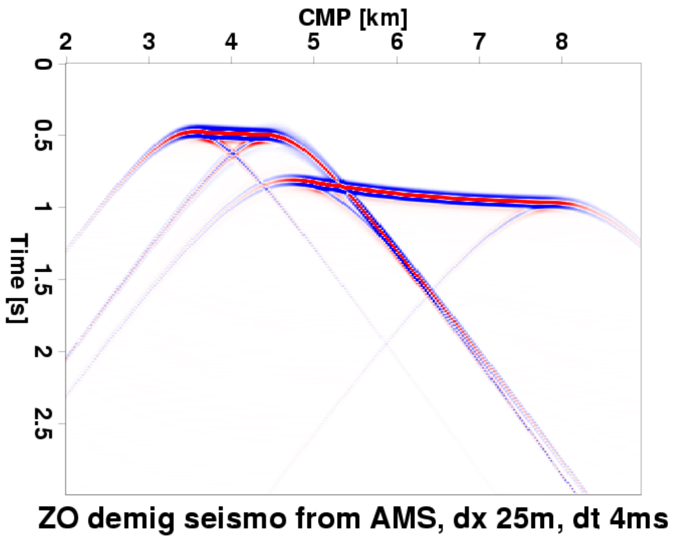

Figure 8: 2D demigrated seismogram of the artificial migrated section shown in Fig. 7. Due to the limited demigration aperture, the seismogram can only be recovered roughly above the target zone.

CONCLUSION

As a generalization of classical kinematic seismic reflection mapping procedures, we have presented a complete wave-equation based, high frequency, amplitude-preserving theory for seismic reflection imaging. The basic tools are two weighted stacking integrals, namely the diffraction and isochrone stacks. Both stacking integrals can be applied in sequence using different macro-models, measurement configurations or ray codes, thus allowing us to solve various kinds of imaging problems. All transformations can be carried out in a true-amplitude sense, that means, the geometrical spreading factors are correctly transformed from the input to the output domain. This is a feature of the Kirchhoff-type stacking structure of the integrals and the adequately determined stacking surfaces. The theory does not depend on any assumption about the medium and the reflector curvature besides the obvious condition that all quantities vary smoothly for the zero-order ray theory and the asymptotic evaluations to be valid. The implementation is straightforward and shows that we are able to handle true-amplitude migration and demigration in a flexible way. Further results are presented by Goertz et al. (this issue).

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PUBLICATIONS

Detailed results were published by (Hertweck, 2000) and (Riede et al., 2000).