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Hybrid Method for Traveltime Computation in a Complex 3D Model

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ABSTRACT

We present a hybrid method for computing multi-arrival traveltimes in weakly smoothed 3D velocity models. An efficient computation of multi-valued traveltimes and of the KMAH index in 3D models is important for 3D prestack Kirchhoff migration. The hybrid method is based on the computation of first-arrival traveltimes with a finite-difference eikonal solver and the computation of later arrivals with the wavefront construction method (WFCM). For a faster traveltime computation we implement a WFCM without dynamic ray tracing. The detection of later arrivals is done automatically and permits the estimation of the KMAH index. The hybrid method is a better alternative to WFCM, since it is considerably faster and has a comparable accuracy.

INTRODUCTION

3D traveltime computation has many applications in seismic processing (Sethian and Popovici, 1999). One of them is 3D prestack Kirchhoff migration. 3D traveltime maps are commonly computed with finite-difference eikonal solvers (FDESs) or with ray-tracing techniques (e.g., Vidale, 1990; Cervený, 1985).

FDESs provide fast and robust first-arrival traveltime computations (Vidale, 1990; Sethian and Popovici, 1999). However, in complex velocity structures, first arrivals do not necessarily correspond to the most energetic wave, and later arrivals can be more important for accurate modeling and imaging (e.g., Geoltrain and Brac, 1993).

Multiple arrivals are traditionally computed with ray-tracing methods (e.g., Cervený, 1985). The most suitable implementation of ray tracing for computing a large number of two-point problems is the wavefront construction method (WFCM) (e.g., Vinje et al., 1996; Lambaré et al., 1996; Coman and Gajewski, 2000). Unfortunately, the computational efficiency of WFCM is lower than for FDES (Leidenfrost et al., 1999).

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In this paper, we present a more efficient computation of multi-valued 3D traveltimes in weakly smoothed media. We combine FDES and WFCM to a hybrid method, taking advantage of the computational speed of FDES and using the ability of WFCM to compute multi-valued traveltimes. This idea was also used by Ettrich and Gajewski (1997) for a related 2D hybrid method.

Very important for the hybrid method is an efficient detection of later arrivals. This is necessary to avoid the computation of first arrivals by the WFCM. The detection of later arrivals in a 3D model is more difficult than in a 2D model. For the 3D model we have developed and tested different techniques and finally selected one that uses the detection of caustic regions as an intermediate step.

The WFCM plays a central role in the hybrid method. It not only computes later arrivals but also provides subroutines for detection of later arrivals. The 3D WFCM proposed by Vinje et al. (1996) is based on dynamic ray tracing (DRT). For a faster traveltime computation we have implemented a WFCM without DRT.

We start the next section by a presentation of the hybrid method followed by theoretical and practical aspects of later arrivals. We present different approaches to detect later arrivals and show that by using the distance between two neighbouring rays we get an efficient technique which also permits the estimation of the KMAH index. We show numerical examples on a 3D two-layer velocity model and on a 3D version of the Marmousi model and estimate the CPU time that can be saved by using the hybrid method instead of the WFCM alone.

The hybrid method was developed for an efficient prestack Kirchhoff depth migration. For true-amplitude Kirchhoff migration (Schleicher et al., 1993), we suggest the computation of traveltimes and KMAH index with the hybrid method followed by the computation of the weighting function from traveltimes only (Vanelle and Gajewski, 2000).

HYBRID METHOD

For a more efficient computation of multi-valued 3D traveltimes in weakly smoothed media we present a hybrid method. The method consists of three steps:

1. Computation of first-arrival traveltimes with FDES.
2. Detection of later arrivals.
3. Computation of later arrivals with WFCM.

As arrival we consider the transmission traveltime of a wave propagating from a point source through the velocity model. Multi-valued arrivals are events corresponding to rays with different paths that arrive at the same receiver. A typical multi-valued wavefront (WF) is shown in Figure 1. The points C_1 and C_2 are caustics. The branch between caustics (the reverse branch of the triplication) is built by rays which have passed a caustic and now carry third arrivals. Rays between the caustics and the wavefront crossing point carry second arrivals, while all other rays carry first arrivals.

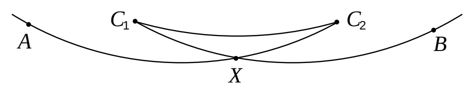


Figure 1: A triplicated WF. Point X is a WF crossing point. At this point two rays with different paths arrive at the same traveltime. At the WF crossing point the first-arrival isochron is not smooth. The points C_1 and C_2 represent caustic points. The branch between these points, the reverse branch, is built by rays which have passed a caustic point. Rays between the caustic point and the wavefront crossing point (WF C_1XC_2) carry second arrivals, while all other rays carry first arrivals.

The consideration of later arrivals improves the quality of prestack Kirchhoff migration (Geoltrain and Brac, 1993). In regions where later arrivals occur they most often carry a higher amount of energy than the first-arrival. Later arrivals commonly are built by rays which passed a low velocity region. In this region the geometrical spreading is low, i.e. a high ray amplitude.

For first-arrival traveltime computation we use the FDES proposed by Vidale (1990), but other methods could be also used (e.g., Sethian and Popovici, 1999). To avoid the computation of first arrivals with the WFCM we detect later arrivals. The detection of later arrivals is the key element of the hybrid method. Theoretical and practical aspects regarding the detection of later arrivals are given in the next section. To compute later arrivals we adapt the 3-D WFCM (Figure 2) proposed by Vinje et al. (1996) to the needs of the hybrid method.

The WFCM proposed by Vinje et al. (1996) is based on DRT. By using DRT, we need a velocity model with smooth second derivatives. DRT also decreases the computational efficiency of ray tracing because the DRT system must be supplimentary solved along each ray.

DRT consists in solving a system of several ordinary differential equations along a ray to get the elements of two matrices ($\mathbf{P}$ and $\mathbf{Q}$). For more details about DRT refer to, e.g, Cervený (1985). $\mathbf{Q}$ is the transformation matrix from ray to Cartesian

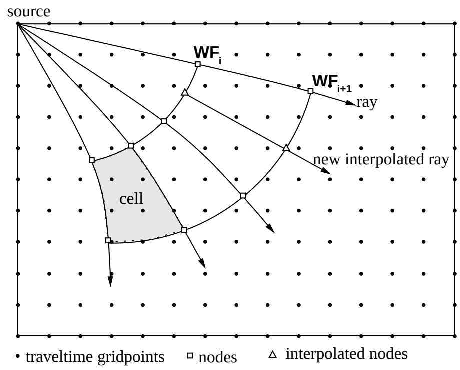


Figure 2: The WFCM method (2D and simplified). The basic concept of the WFCM is to propagate a wavefront as a representation of endpoints (nodes) of a number of rays at a given time and to retain a good representation of the wavefront by interpolation of new rays. Another interpolation is done for traveltimes from nodes to the rectangular grid. For details see Vinje et al. (1996).

coordinates. Its elements are:

$$\mathbf{Q} = \begin{pmatrix} \frac{\partial x_1}{\partial \gamma_1} & \frac{\partial x_2}{\partial \gamma_1} & \frac{\partial x_3}{\partial \gamma_1} \\ \frac{\partial x_1}{\partial \gamma_2} & \frac{\partial x_2}{\partial \gamma_2} & \frac{\partial x_3}{\partial \gamma_2} \\ \frac{\partial x_1}{\partial \gamma_3} & \frac{\partial x_2}{\partial \gamma_3} & \frac{\partial x_3}{\partial \gamma_3} \end{pmatrix} \tag{1}$$

where x_i are Cartesian coordinates. In case of a point source, γ_1 and γ_2 may be the take-off angles at the point source and γ_3 is a parameter along the ray (e.g., τ traveltime or s arclength). At caustic points the ray jacobian ($\mathbf{J} = \det|\mathbf{Q}|$) vanishes ($\mathbf{J} = 0$).

The interpolation of the elements of the matricies $\mathbf{Q}$ and $\mathbf{P}$ at a new ray is a weak point in the WFCM. A third order polynomial interpolation may lead to unstable solutions in some cases (Vinje et al. 1996) and the linear interpolation is not accurate enough and leads to wrong detection of caustic points. Therefore, in the hybrid method we use an implementation of the WFCM without DRT (Coman and Gajewski, 2000).

DETECTION OF LATER ARRIVALS

An efficient detection of later arrivals is essential for the hybrid method. In this section we present different approaches to detect later arrivals and show that by using the distance between two neighbouring rays we get a simple, fast, robust and accurate technique. This technique also permits us to estimate the KMAH index.

In the classical WFCM, Vinje et al. (1996) use the take-off direction from the source, $\mathbf{d}_S$ to distinguish between arrivals. $\mathbf{d}_S$ is defined as the initial tangent vector of the ray that would reach the receiver if the ray actually had been traced. Two arrivals at a receiver cannot have equal $\mathbf{d}_S$. This technique to detect later arrivals is not suitable for the hybrid method because it assumes that the first arrival is also obtained by WFCM. To keep the hybrid method efficient we must avoid double computation of first arrivals. In other words, we must find regions with later arrivals without interpolating of first arrivals with the WFCM.

Next, we present assumptions regarding later arrivals and argue why this assumptions could be made. The assumptions are:

1. Before a ray belongs to the reverse branch, it first passed a WF crossing point and a caustic.
   The support for this assumption is given by Figure 3. The bold ray at t_6 which belongs to the reverse branch passed the WF crossing point at t_3 and the caustic at t_5.

2. The first-arrival traveltime is not smooth in the region around the WF crossing point.
   The derivative of the traveltime is the slowness vector. At the WF crossing point two rays from different directions arrive at the same traveltime. A function with an ambiguous derivative at a point is not smooth at that point. For a graphical support see Figure 1.

3. The distance between two neighbouring rays is very small (local minimum) in the caustic region.
   In the caustic region the wave energy is very high, that means a high ray density, or in other words a small distance between two neighbouring rays. A graphical support is given in Figure 3, where the distance between the rays reaches a local minimum between the traveltimes t_5 and t_6.

4. Caustic points of second order are exceptions.
   At caustics the cross-sectional area of the ray tube shrinks to zero. In a 3D medium there are two types of caustics. At a caustic point of first order, the ray tube shrinks to an elementary arc, perpendicular to the direction of the propagation. For rays which passed this caustic the KMAH index should be increased by one. At the caustic point of second order, the ray tube shrinks to a point. For rays which passed this caustic the KMAH index should be increased by two (e.g., Cervený, 1985). A caustic point of second order is an exceptional case that is unlikly to occur in real geological situations unless is has been imposed by symmetry of the problem (Hanyga, 1988).

We have assumed above (assumption 1) that before carrying later arrivals a ray passed a WF crossing point. Rays which belong to the reverse branch also passed a caustic. Based on this assumption, we present two classes of approaches for detection of later arrivals. The first presumes the detection of the WF crossing point, the second presumes the detection of the caustic regions. Next we discuss this two approaches.

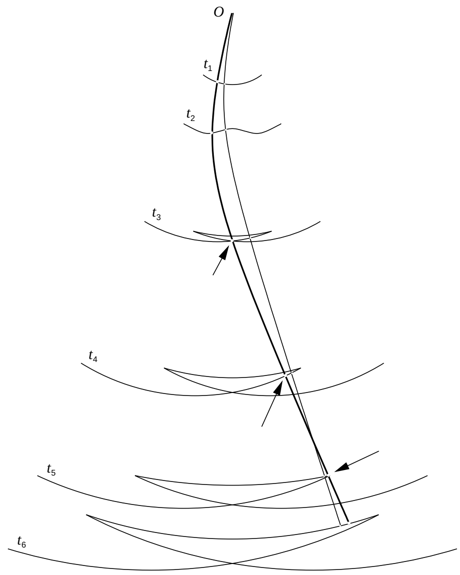


Figure 3: The evolution of a WF that triplicates. Six WFs are represented at different traveltimes. O is the source point and t_i are traveltimes. The position of two rays at given traveltimes is marked with arrows or white dots. At t_3 the bold ray passes a WF crossing point. Up to this traveltime it carries first arrivals. Then it carries second arrivals till t_5 when it passes a caustic. After passing the caustic the ray carries third arrivals. The distance between the rays passes a local minimum in the caustic region.

Detection of WF crossing points

This approach consists of two steps. The first step is the detection of WF crossing points by using the first arrival traveltime map. The second step is the initialisation of the WFCM.

One possibility to detect a WF crossing point is to use the fact that the slowness vector changes its direction discontinuously at this point. This fact was also used by Ettrich and Gajewski (1997) to detect WF crossing points in the 2D hybrid method. Another possibility is to use the assumption that the traveltime is smooth except at the WF crossing points (assumption 2). To detect these points we estimate the difference between a smoothed and the original first arrival traveltime map. In regions with WF crossing points this difference show high negative values.

After detecting the WF crossing point we have theoretically three possibilities to initialise the WFCM: (1) by using the slowness vectors to construct a WF in the region before the triplication; (2) by performing backward ray tracing from the WF crossing points to the source point and compute the take-off angles; (3) by sending rays into the whole model starting from the source and interpolate traveltimes and other parameters *only* between rays that passed a region with WF crossing points.

A drawback of all these techniques is that they bound the region with later arrivals with poor accuracy and that they cannot estimate the KMAH index. The first two techniques are also difficult to implement.

Detection of caustic regions

This is a one step approach, which is easy to implement as most of the computer code is shared with the WFCM. It works without initialisation of the WFCM. Actually, we only propagate the WF without any parameter interpolation at gridpoints. For the WF propagation we need less than $5\,\%$ of the CPU time of the whole WFCM. Therefore, this approach is very fast.

We have developed and tested two techniques to detect caustic regions. Both techniques are accurate, robust and allow to estimate the KMAH index. The first technique uses the WF curvature sign and was presented in Coman and Gajewski (2000). The second technique uses the distance between two neighbouring rays. This distance is already computed in the WFCM as it is needed for the interpolation of new rays (e.g., Vinje et. al., 1996). Therefore, we select the second technique as it is easier to implement and faster.

We have assumed above (assumption 3) that the distance between two neighbouring rays passes a local minimum in the caustic region. In some cases this happens but there is no caustic. However, these cases are very seldom and easily detected by comparing the interpolated traveltime with the first arrival traveltime. We have also assumed (assumption 4) that caustic points of second order are exceptions. As a consequence, if we detect a caustic we consider it as a caustic of first order and increase the KMAH index by one.

Tests of this technique have shown that it bounds the regions where later arrivals occur with good accuracy and properly estimates the KMAH index.

COMPUTATIONAL SPEED AND ACCURACY

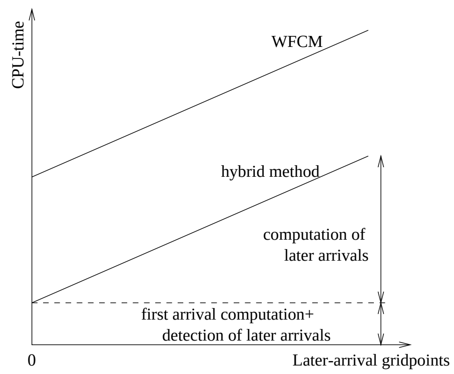
In this section we compare the computational speed by a given accuracy between the hybrid method and the WFCM.

Commonly, the accuracy of a method is defined as the absolute value of the relative error of the computed traveltime compared to the expected traveltime. In complex models the expected traveltime cannot be computed analytically, so we compute a reference traveltime map by running the WFCM in a high accuracy mode.

The accuracy of the WFCM basically depends on the accuracy of ray tracing (what traveltime step and what method is used to propagate the ray), the number of rays that are propagated through the model (mainly they are function of the predefined maximum distance between two neighbouring rays) and the accuracy of the parameter interpolation at new rays and at gridpoints. For more details about accuracy see e.g., Lambaré et al., (1996). To compare the hybrid method with the WFCM we select for the WFCM a comparable accuracy as for the FDES.

Let us analyze next the computational speed. Figure 4 shows the qualitative variation of the CPU-time as a function of the number of the later arrival gridpoints. In a model without later arrivals the hybrid method is up to four times faster than the WFCM. This is a direct consequence that the FDES is faster than the WFCM and that the detection of later arrivals is also very fast (see above).

Figure 4: CPU-time for the hybrid method and the WFCM. The CPU-time for the hybrid method is the sum of the CPU-time for computating first arrivals, for detecting of later arrivals and for computing later arrivals. In a model without triplication (point 0 at the axis with later-arrival gridpoints) the hybrid method is about four times faster than the WFCM. The CPU-time difference between the WFCM and the hybrid method is not function of the number of gridpoints with later arrivals.



As the computation of later arrivals is the same in both methods the CPU-time difference between both methods stays constant and is not a function of the number of gridpoints with later arrivals.

COMPUTATIONAL EXAMPLES

The first numerical example is in a 3D two layer model (Figure 5), where the traveltime map is single-valued. The computation of traveltime in this model was done only with the FDES and the hybrid method was about four times faster than the WFCM.

The second example represents a 3D version of the Marmousi model. To get this model we extend a 2D smooth Marmousi model in the third dimension to 101 gridpoints. The Marmousi model (Versteeg and Grau, 1990) has been widely used as a reference model to validate new methods. We use a resampled ($12.5\,m$ in each direction) and smooth model (Figure 6). The source is located at coordinates $x = 612.5\,m$, $y = 6000\,m$ and $z = 62.5\,m$. The first-arrival traveltime at gridplane $x = 612.5\,m$ is displayed in Figure 7. The region with later-arrival traveltimes is displayed in Figure 8. By using the hybrid method we compute the traveltimes two times faster than with the WFCM alone.

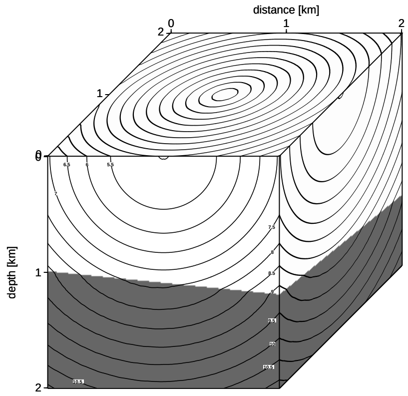


Figure 5: Two layers model and isochrones. The upper layer has a velocity of $2.0\,km/s$. The layer below has a velocity of $2.5\,km/s$. The model has 201 gridpoints in each direction. The distance between gridpoints is $0.01\,km$. The source position is at $x = 1\,km$, $y = 1\,km$ and $z = 0.01\,km$.

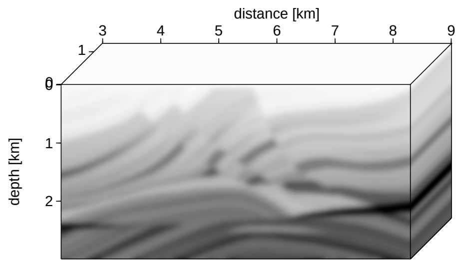


Figure 6: Smoothed 3D Marmousi model.

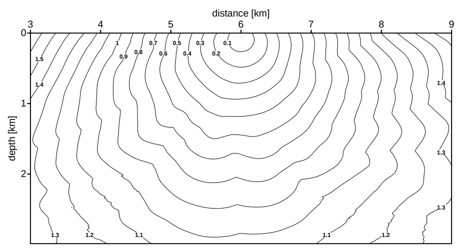


Figure 7: First arrival isochrons in the Marmousi model at $x = 612.5\,m$. The traveltime was computed with Vidale's FDES (Vidale, 1990)

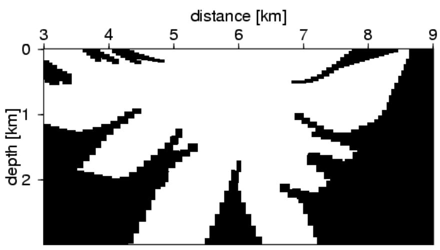


Figure 8: The black regions show domains where later-arrival traveltime were computed

CONCLUSIONS

We have presented a 3D hybrid method for fast multi-valued traveltime computation in weakly smoothed velocity models. The hybrid method computes the first-arrival traveltime with a fast FDES and uses WFCM only in regions where later arrivals occur. The detection of later arrival traveltimes is done automatically, without user intervention. For this we use a fast, simple and robust technique which is based on the distance between two neighbouring rays. This technique also provides the KMAH index. We improved the computational efficiency of the WFCM by avoiding dynamic ray tracing.

If later-arrivals are absent then the hybrid method is roughly four times faster than the WFCM. In models with triplications the hybrid method is 1.5 – 3 times faster than the WFCM. E.g., in a 3D Marmousi model the HM was two times faster than the WFCM. The accuracy of traveltimes computed by the hybrid method is given by the accuracy of the applied FD eikonal solver and by the ray tracing parameters used in the WFCM.

We have optimised the hybrid method to compute multi-valued traveltimes for the 3D prestack Kirchhoff migration. However, since the migration weights can be computed from traveltimes alone (Vanelle and Gajewski, 2000), it is also possible to perform true-amplitude prestack Kirchhoff migration for first and later arrivals with the hybrid method.

ACKNOWLEDGEMENTS

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REFERENCES

Cervený, V., 1985, The application of ray tracing to the numerical modeling of seismic wavefield in complex structure *in* Dohr, G., Ed., Seismic Shear Waves:: Geophysical Press, 1–124.

Coman, R., and Gajewski, D., 2000, 3D wavefront construction method with spherical interpolation: 62th Mtg., Eur. Assn. Geosci. Eng., Extended Abstracts, C43.

Ettrich, N., and Gajewski, D., 1997, A fast hybrid wavefront construction - FD eikonal solver method: 59th Mtg., Eur. Assn. Geosci. Eng., Extended Abstracts, E014.

Geoltrain, S., and Brac, J., 1993, Can we image complex structures with first-arrival traveltime?: Geophysics, **58**, 564–575.

Hanyga, A., 1988, Numerical methods for tracing rays and wavefronts *in* Doornbos, D., Ed., Seismological Algorithms:: Academic Press, 169–233.

Lambaré, G., Lucio, P., and Hanyga, A., 1996, Two-dimensional multivalued traveltime and amplitude maps by uniform sampling of a ray field: Geophys. J. Int., **125**, 584–598.

Leidenfrost, A., Ettrich, N., Gajewski, D., and Kosloff, D., 1999, Comparison of six different methods for calculating traveltimes: Geophys. Prosp., **47**, 269–297.

Schleicher, J., Tygel, M., and Hubral, P., 1993, 3D true-amplitude finite-offset migration: Geophysics, **58**, 1112–1126.

Sethian, J. A., and Popovici, M. A., 1999, 3-D traveltime computation using the fast marching method: Geophysics, **64**, 516–523.

Vanelle, C., and Gajewski, D., 2000, True-amplitude migration weights from traveltimes: 62th Mtg., Eur. Assn. Geosci. Eng., Extended Abstracts, B10.

Versteeg, R., and Grau, G., 1990, Practical aspects of seismic data inversion, the Marmousi experince: 52th Mtg., Eur. Assoc. Expl Geophys., Proc. 1990 EAEG Workshop, 1–194.

Vidale, J. E., 1990, Finite-difference calculation of traveltimes in three dimensions: Geophysics, **55**, 521–526.

Vinje, V., Iversen, E., Åstebøl, K., and Gjøystdal, H., 1996, Estimation of multivalued arrivals in 3D models using wavefront construction–Part I: Geophys. Prosp., **44**, 819–842.

PUBLICATIONS

A paper contating these results has been submitted to Pageoph.