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Analytic Moveout formulas for a curved 2D measurement surface and near-zero-offset primary reflections

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ABSTRACT

Analytic moveout formulas for primary near-zero-offset reflections in various types of gathers (e.g. common shot, common midpoint) play a significant role in the seismic reflection method. They are required in stacking methods like the common-midpoint (CMP) or the Common-Reflection Surface (CRS) stack. They also play a role in Dix-type traveltime inversions. Analytic moveout formulas are particularly attractive, if they can be given a "physical" or "quasi-physical" interpretation, involving for instance the wavefront curvatures of specific waves. The formulas presented here have such a form. They give particular attention to the influence that a curved measurement surface has on the moveout or normal-moveout (NMO) velocity. This influence should be accounted for in the CMP or CRS stack and in the computation of interval velocities.

INTRODUCTION

Analytic moveout formulas for isotropic media have a long tradition of being applied in the seismic reflection method. Just to mention a few papers that may be considered as some milestone contributions we would like to refer to the works of (Dürbaum, 1954; Dix, 1955; Shah, 1973; Fomel and Grechka, 1998). Particularly in the light of macro-model-independent reflection imaging (Hubral, 1999) analytic moveout formulas in midpoint(m)-half-offset(h) coordinates like Polystack (de Bazelaire, 1988; de Bazelaire and Viallix, 1994; Thore et al., 1994) and Multifocusing (Gelchinsky et al., 1999a,b) have gained a new relevance and importance. Here we generalize the so-called Common-Reflection-Surface (CRS) stack formula (Müller et al., 1998), which is used to simulate zero-offset sections from prestack data in a macro-velocity-model independent way (Jäger et al., 2001). The generalized CRS stack moveout formula

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is formulated in such a way that the influence of the curved surface can be clearly recognized.

THEORY

According to (Schleicher et al., 1993), the hyperbolic traveltime approximation for a ray from S' to R' and from R' to G' both on a curved surface, in the vicinity of a normal (zero-offset) ray from SG to R and from R to SG (Figure A-1) is given by

$$t^2_{hyp}(m,h) = (t_0 - 2\,p_0\,m)^2 + 2\,t_0\,[(B^{-1}\,A - B^{-1})\,m^2 + (\,B^{-1}\,A + B^{-1})\,h^2] \quad (1)$$

with

$$p_0 = \frac{-\sin{\beta_S}^*}{v}\,, \qquad (2)$$

$$m = \frac{x_s + x_g}{2}\,, \quad h = \frac{x_g - x_s}{2}\,. \qquad (3)$$

This formula is valid for a 2D laterally inhomogeneus medium. v is the velocity at the ZO location SG, t_o is zero-offset (two-way) traveltime. x_s and x_g are the coordinates of the source S' and receiver G' measured along the tangent to the curved surface at SG.

The components $A\,, B\,, C\,, D$ of the so-called 2×2 surface-to-surface propagator matrix for the two-way normal ray (see Appendix) are given by

$$A \;=\; \frac{1}{K_{NIP} - K_N}\left(K_{NIP} + K_N - \frac{2\,K_S}{\cos{\beta_S}^*}\right)\,, \qquad (4)$$

$$B \;=\; \frac{1}{K_{NIP} - K_N}\left(\frac{2\,v}{\cos\beta_S^{2\,*}}\right)\,, \qquad (5)$$

$$C \;=\; \frac{1}{K_{NIP} - K_N}\left(-2\,(K_{NIP} + K_N)\,\frac{\cos^*\beta_S\;K_S}{v} + \frac{2\,K_{NIP}\,K_N\,\cos^*\beta_S^2}{v} + \frac{2\,K_S^2}{v}\right) \qquad (6)$$

$$D \;=\; \frac{1}{K_{NIP} - K_N}\left(K_{NIP} + K_N - \frac{2\,K_S}{\cos{\beta_S}^*}\right)\,, \qquad (7)$$

where ${\beta_S}^*$ is the angle of incidence of the normal ray (emerging on the curved measurement surface at SG) with the normal of the tangent to the curved surface at SG. Let us define K_S as the surface curvature at the point SG. K_{NIP} and K_N are the curvatures of the NIP-wave and the N-wave observed at SG, respectively (Hubral, 1983; Jäger et al., 2001).

Inserting eqs. (2), (4), and (5) into eq. (1) we obtain

$$t_{hyp}^2(m,h) \;=\; \left(t_0 + 2\,\frac{\sin\beta_S{}^*}{v}\,m\right)^2 + \frac{2\,t_0}{v}\,\left(K_N\ \cos^2\beta_S^* - \cos\beta_S^*\,K_S\right)\,m^2 + \frac{2\,t_0}{v}\,\left(K_{NIP}\ \cos^2\beta_S^* - \cos\beta_S^*\,K_S\right)\,h^2\,. \tag{8}$$

Here $(\beta_S^*\,, K_{NIP}\,, K_N)$ are wanted parameters that help solve a variety of stacking and inversion problems (Hubral, 1999). They can be obtained by modifying the presently existing 2D CRS stack formula (Jäger et al., 2001) that is obtained from eq. (8) by substituting $K_S = 0$. In the following, we discuss 3 particular reductions of formula (8) which are of practical relevance.

Particular cases

Common-mid-point (CMP) gather

For this case, $m = 0$, the equation (8) reduces to

$$t_{CMP}^2(h) = t_0^2 + \frac{2\,t_0}{v}\,\left(K_{NIP}\ \cos^2\beta_S^* - \cos\beta_S^*\,K_S\right)\,h^2\,. \tag{9}$$

This expression is commonly written as (Shah, 1973)

$$t_{CMP}^2(h) = t_0^2 + \frac{4\,h^2}{v_{NMO}^2}\,, \tag{10}$$

where the normal moveout (NMO) velocity v_{NMO} is now given by

$$v_{NMO}^2 = \frac{2\,v}{t_0\ (K_{NIP}\ \cos^2\beta_S^* - \cos\beta_S^*\,K_S)}\,. \tag{11}$$

For $K_S = 0$ this reduces to Shah's formula. For a 1-D model with a planar surface, straight normal ray and incidence angle $\beta_S^* = 0$ this expression reduces to $V_{NMO} = V_{RMS}$, where V_{RMS} is the familiar root-mean-square velocity.

Common - shot gather

For this case, $x_g = x_s + x$, equation (8) reduces to

$$t_{CS}^2(x,h) = \left(t_0 + \frac{\sin\beta_S{}^*\,x}{v}\right)^2 + \frac{t_0}{2\,v}\,\left(K_N\ \cos^2\beta_S^* + K_{NIP}\ \cos^2\beta_S^* - 2\cos\beta_S^*\,K_S\right)\,x^2\,, \tag{12}$$

where according to eq. (A-9) and (Hubral, 1983), is demonstrated that $K_{NIP} + K_N = 2\,K_o$, with K_o being the wavefront curvature of the reflected wave at SG.

Zero-Offset section

For this case, $h = 0$, equation (8) reduces to

$$t_{ZO}^2(m) = \left(t_0 + 2\,\frac{\sin\beta_S{}^*}{v}\;m\right)^2 + \frac{2\,t_0}{v}\;\left(K_N\;\cos^2\beta_S^* - \cos\beta_S^*\;K_S\right)\;m^2\;. \qquad (13)$$

All three formulas should be considered in the 2D CRS stack (Jäger et al., 2001) for a curved measurement surface.

CONCLUSION

In this paper we have formulated a new analytic moveout formula (8) for a 2D curved measurement surface. It may find application in a number of modeling, inversion and stacking problems. The formula is independent of the 2D laterally inhomogeneous velocity model. For that matter it is also valid for 3D earth models, provided all parameters in the 2D formula represent those in the plane defined by the seismic line and emerging normal ray at SG.

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APPENDIX A

Elements of the surface-to-surface propagator matrix $\mathbf{T}$

Following (Hubral et al., 1992a; Cervený, 1999; Schleicher et al., 2001), we can express for the 2-D case the 2×2 submatrices $\mathbf{A_1},\ \mathbf{B_1},\ \mathbf{C_1},\ \mathbf{D_1}$ of the 4×4 surface-to-surface propagator matrix $\mathbf{T}$ (Bortfeld, 1989; Hubral et al., 1992a; Schleicher et al., 1993) form by

$$
\begin{aligned}
A &= \frac{Q_1\, g}{g'} - \frac{Q_2\, X}{g'\, g}\,, && \text{(A-1)}\\
B &= \frac{Q_2}{g'\, g}\,, && \text{(A-2)}\\
C &= g'\, P_1\, g - \frac{g'\, P_2\, X}{g} + \frac{X'\, Q_1\, g}{g'} - \frac{X'\, Q_2\, X}{g'\, g}\,, && \text{(A-3)}\\
D &= \frac{g'\, P_2}{g} + \frac{X'\, Q_2}{g'\, g}\,, && \text{(A-4)}
\end{aligned}
$$

where $P_1,\ P_2,\ Q_1$ and Q_2 are the scalar elements of the so-called $\mathbf{\Pi}$ propagator matrix (Popov and Psenc´ k,1978; Cervený, 1985; Hubral et al., 1992a). The matrix $\mathbf{T}$ is defined in a Cartesian coordinate system and the propagator matrix Π is defined in a ray centered coordinate system (Cervený, 1987).

We assume a 2-D model (Figures 6.1 and 6.2) with a curved measurement surface. We have a source point S coinciding with the receiver point G. In the paraxial vicinity of the normal ray surface location SG, where the medium velocity changes only gradually, we have two points S' and G'. This source-receiver pair is linked by a reflected paraxial ray $S'R'G'$. According to (Cervený, 1987; Schleicher et al., 2001), we can express for the 2-D case the transformation from the local 2-D Cartesian coordinate system $\vec{x}(x_1, x_3)$ at SG to the 2-D ray-centered coordiante system $\vec{q}$ (Figure A-2) by

$$
\begin{aligned}
q &= g\, x\,, \quad q' = g'\, x'\,, && \text{(A-5)}\\
g &= \cos\beta_S^*\,, \quad g' = \cos\beta_S^*\,, && \text{(A-6)}\\
X &= \frac{\cos\beta_S^*}{v_S}\, K_S\,, \quad X' = -\frac{\cos\beta_S^*}{v_S}\, K_S\,, && \text{(A-7)}
\end{aligned}
$$

In paraxial approximation, the curved surface can be expressed as a parabola. In the local 2-D coordinate system $x_1\, x_3$, this is representable as

$$
x_3 = \frac{1}{2}\, K_S\, x_1^2\,, \qquad \text{(A-8)}
$$

where K_S is the surface's curvature at SG (Schleicher et al., 2001).
The matrix Π is expressed in terms of the NIP and N wavefront curvatures K_{NIP} and K_N (Hubral, 1983) by

$$\mathbf{\Pi} = \begin{pmatrix} Q_1 & Q_2 \\ P_1 & P_2 \end{pmatrix} = \frac{1}{K_{NIP} - K_N} \begin{pmatrix} K_{NIP} + K_N & 2\, v_S \\ \frac{2}{v_S}\, K_{NIP}\, K_N & K_{NIP} + K_N \end{pmatrix}; \qquad \text{(A-9)}$$

By substituting eqs. (A-6), (A-7) and the components of matrix Π (eq. A-9) into eqs. (A-1) we obtain the components A, B, C and D of the matrix $\mathbf{T}$ given by eqs. (7). The matrix $\mathbf{T}$ also can now be defined in terms of the matrix Π by

$$\mathbf{T} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \frac{1}{g} & 0 \\ \frac{-X}{g} & g \end{pmatrix} \Pi \begin{pmatrix} g & 0 \\ \frac{-X}{g} & \frac{1}{g} \end{pmatrix}. \qquad \text{(A-10)}$$

Inserting the elements of the propagator matrix Π, we obtain

$$\mathbf{T} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \frac{1}{g} & 0 \\ \frac{-X}{g} & g \end{pmatrix} \begin{pmatrix} Q_1 & Q_2 \\ P_1 & P_2 \end{pmatrix} \begin{pmatrix} g & 0 \\ \frac{-X}{g} & \frac{1}{g} \end{pmatrix}, \qquad \text{(A-11)}$$

thus proving the validity of equations (A-1).

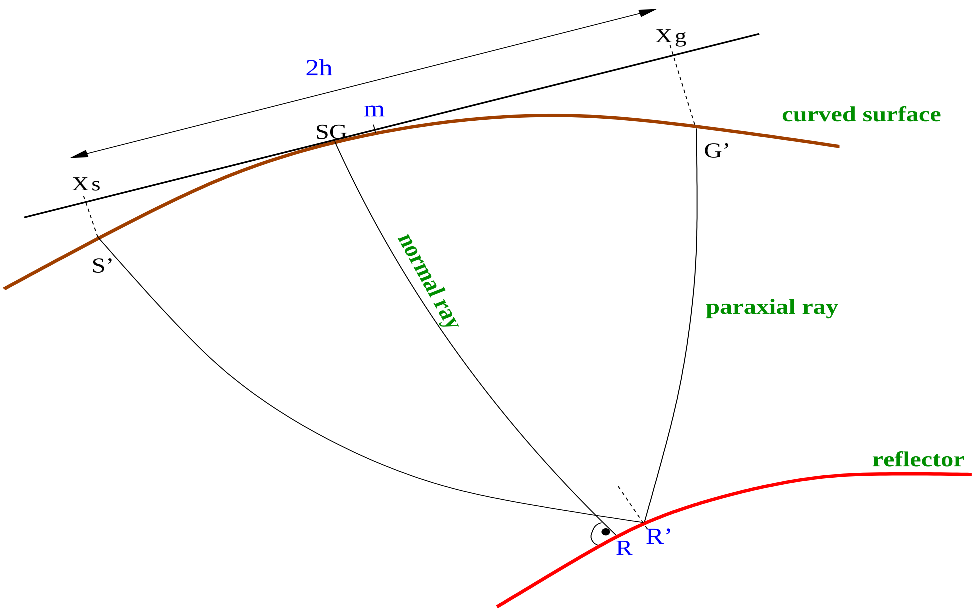

Figure A-1: Ray diagram for a paraxial ray in the vicinity of a normal ray in a 2D laterally inhomogeneous medium.

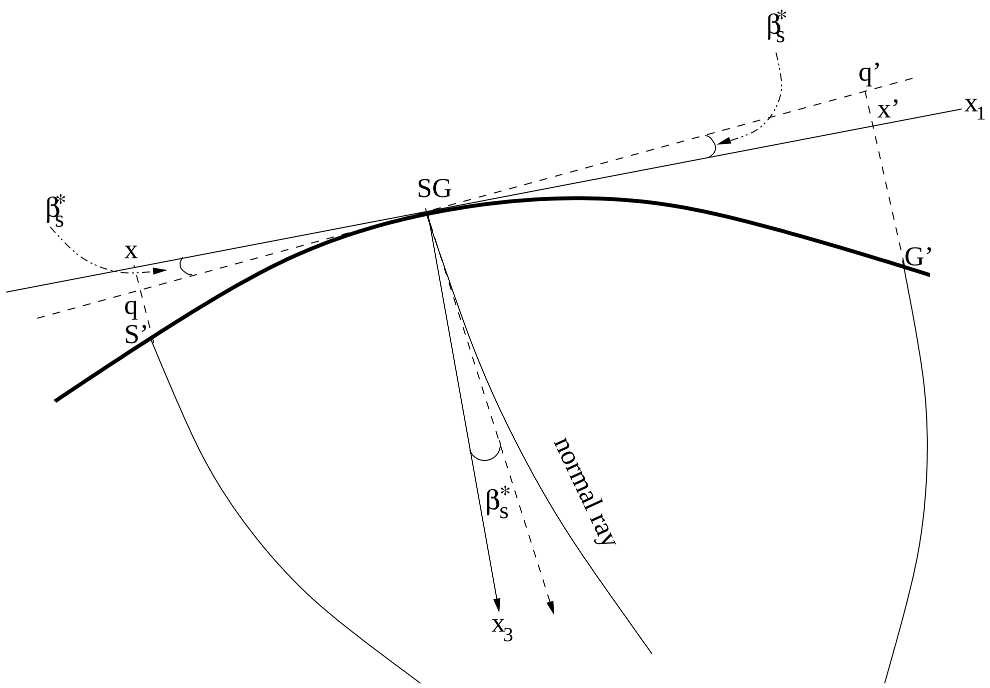

Figure A-2: Blow-up of Figure A-1 showing the 2D local and ray centered coordinate system at SG.