An Inversion for Fluid Transport Properties of 3-D Heterogeneous Rocks Using Induced Microseismicity

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ABSTRACT

We propose an approach for 3-D mapping the large-scale permeability tensor of heterogeneous reservoirs and aquifers. This approach uses the seismic emission (microseismicity) induced in rocks by fluid injections (e.g., borehole hydraulic tests). The approach is based on the hypothesis that the triggering front of the hydraulic-induced microseismicity propagates like the low-frequency (in respect to the global-flow critical frequency) second-type compressional Biot wave (corresponding to the process of the pore-pressure relaxation) in a heterogeneous anisotropic poroelastic fluid-saturated medium. Assuming that the wavelength of the second-type wave corresponding to the triggering front is shorter than the typical permeability heterogeneity we derive its differential equation. This equation describes kinematical aspects of the propagation of the triggering front in a way similar to the eikonal equation for seismic wavefronts. In the case of isotropic heterogeneous media the inversion for the hydraulic properties of rocks follows from a direct application of this equation. In the case of an anisotropic heterogeneous medium only the magnitude of a global effective hydraulic-diffusivity tensor can be mapped in a 3-D spatial domain.

INTRODUCTION

It is well known that the characterization of fluid-transport properties of rocks is one of the most important and difficult problems of the reservoir geophysics. Seismic methods have some fundamental difficulties in estimating such hydraulic properties of rocks like the fluid mobility or the permeability tensor (see e.g., Shapiro and Mueller, Geophysics, 1999, p.99-103 for references related to this problem).

Recently an approach was proposed to provide in-situ estimates of the permeability tensor characterizing a reservoir on the large spatial scale (of the order of $10^3 m$). This approach (we call it SBRC: Seismicity Based Reservoir Characterization) uses a

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spatio-temporal analysis of fluid-injection induced microseismicity to reconstruct the permeability tensor (see Shapiro et al., 1997, 1998, 1999 and 2000). Such a microseismicity can be activated by perturbations of the pore pressure caused by a fluid injection into rocks (e.g., fluid tests in boreholes). Rather broad experience shows that even small pore-pressure fluctuations seem to be able to modify the effective normal stress and/or the friction coefficients in rocks to an extend, which is enough for spontaneous triggering of microearthquakes. Evidently, the triggering of microearthquakes occur in some locations (which can be just randomly distributed in the medium), where rocks are in a near-failure equilibrium.

The SBRC approach is based on the hypothesis that the triggering front of the hydraulic-induced microseismicity propagates like the low-frequency second-type compressional Biot wave corresponding to the process of the pore-pressure relaxation. Here, under the low frequency is understood a frequency, which is much lower, than the critical frequency of the global flow (critical Biot's frequency). This is usually much larger than $10^4$ Hz in the case of well consolidated rocks.

Until now the SBRC approach has considered real heterogeneous rocks as an effective homogeneous anisotropic poroelastic fluid-saturated medium. The permeability tensor of this effective medium is the permeability tensor of the heterogeneous rock volume upscaled to the characteristic size of the seismically-active region.

In this paper the SBRC approach is further generalized to provide in-situ estimates of the permeability tensor distribution in heterogeneous volumes of rocks.

This paper has the following structure. First, we motivate and generally describe the idea of the SBRC approach to the 3-D hydraulic diffusivity mapping. Then a differential equation is derived which approximately describes kinematical aspects of the propagation of the microseismicity triggering front in the case of quasi harmonic pore pressure perturbation. This approximation is similar to the geometric-optic approach for seismic wave propagation. The triggering-front propagation is considered in the intermediate asymptotic frequency range, where the frequency is assumed to be much smaller than the critical Biot frequency and larger than $2\pi D/\alpha^2$, where $D$ is the upper limit of the hydraulic diffusivity and $\alpha$ is the characteristic size of the medium heterogeneity (in respect to the hydraulic diffusivity). Then we suggest an algorithm of hydraulic diffusivity mapping in 3D.

**THE CONCEPT OF THE 3-D MAPPING OF HYDRAULIC DIFFUSIVITY**

In order to demonstrate the idea of the 3-D mapping of hydraulic diffusivity let us consider an example of the microseismicity cloud collected during the Hot-Dry-Rock Soultz-sous-Forêts experiment in September 1993 (see Dyer et al., 1994).

Figure 1 shows a view of this cloud. For each event the color shows its occurrence
time in respect to the start time of the injection. Evidently, the occurrence times contain much more information than just the large-scale global velocity of the triggering front propagation in an effective homogeneous anisotropic medium. If we subdivide the space to a number of 3-D cells we can then define an arrival time of the triggering front into each of these cells. Under such an arrival time we can understand a minimum occurrence time in a given cell. One can also introduce another formal criterion, e.g., the arrival time in a given cell is defined by the time moment 98 percent of events in this cell occur later of which. In this way triggering fronts can be constructed for given arrival times. Of course, some smoothing is required.

![Figure 1: A perspective projection of the 3-D distribution of microseismic events registered during the Soultz-Sous-Forets experiment: Borehole GPK1, September 1-22, 1993. The color corresponds to the event occurrence time. The axes X, Y and Z point to the East, the North and the earth surface, respectively. The vertical and horizontal size of the shown spatial domain is 1.5 km roughly](image)

Figures 2 and 3 show such triggering fronts for the arrival times of 50 and 200h. Such surfaces can be constructed for any arrival time presented in microseismic data. Thus, the time evolution of the triggering surface, i.e., the triggering front propagation can be characterized. In a heterogeneous porous medium the propagation of the
triggering front is determined by its heterogeneously distributed velocity. Given the triggering front positions for different arrival times, the 3-D distribution of the propagation velocity can be reconstructed. In turn, the hydraulic diffusivity is directly related to this velocity.

In the following we try to formalize this concept. For this, let us shortly remind the basic concept of the SBRC, which was applied to interpret microseismicity induced by borehole-fluid injections. The pore pressure perturbation at the injection point by a step function (see e.g., Shapiro et al, 2000), which differs from zero till the time $t_0$ of a particular seismic event. The time evolution of the injection signal after this time is of no importance for this event. The power spectrum of this signal (see Figure 4) shows that the dominant part of the injection signal energy is concentrated in the frequency range below $2\pi/t_0$ (note that the choice of this frequency is of partially heuristic character; see the related discussions in Shapiro et al, 1997 and 1999). Thus, the probability, that this event was triggered by signal components from the frequency range $\omega \leq 2\pi/t_0$ is high. This probability for the low energetic higher frequency components is low. However, the propagation velocity of high-frequency components is higher than those of the low frequency components (see Shapiro et al, 1997). Thus, to a given time $t_0$ it is probable that events will occur at distances, which are smaller than the traveldistance of the slow-wave signal with the dominant frequency $2\pi/t_0$. The events are characterized by a significantly lower probability for larger distances. The spatial surface which separates these two spatial domains we call the triggering front.

Therefore, the propagation of the triggering front is approximately defined by kine-
matic features of a slow-wave front of a particular frequency. For the SBRC approach the earliest microseismic events are of importance. It is natural to assume, that for their triggering in a heterogeneous medium a possibly quickest front configuration will be responsible.

From the other hand, in the low-frequency range the slow wave represents the process of the pore pressure relaxation and, therefore, is a kind of a diffuse wave. The real and imaginary parts of its wavevector are equal. Thus, it is a very rapidly attenuating wave. However, recent studies of diffuse waves (these studies were performed for a particular type of diffuse waves called the diffuse photon-density waves: see Youdh and Chance, 1995 and Boas et al., 1997 and further references there ) show that they demonstrate all typical wave phenomena, like scattering, diffraction, refraction, reflection etc..

This argumentation leads to the idea to use a geometrical-optics like description of triggering fronts as an approximative basis of the looked for inversion procedure.

TRIGGERING FRONTS IN THE CASE OF QUASI-HARMONIC PRESSURE PERTURBATION

In the following we approximate a real configuration of a fluid injection in rocks by a point source of pore pressure perturbation in an infinite anisotropic poroelastic saturated medium. We shall assume that the medium is heterogeneous in respect of its hydraulic properties. Then, at the extremely low frequencies the pore-pressure perturbation \( p \) can be approximately described by the following differential equation of
diffusion:

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial x_i} \left[ D_{ij} \frac{\partial}{\partial x_j} p \right], \quad (1)$$

where $D_{ij}$ are components of the tensor of the hydraulic diffusivity heterogeneously distributed in the medium, and $t$ is the time. This equation corresponds to the second-type Biot wave (the slow P-wave) in the limit of the frequency extremely low in comparison with the global-flow critical frequency (Biot 1962).

In the following we shall consider relaxation of a harmonic component of a pressure perturbation. Let us firstly recall the form of the solution of (1) in the case of a homogeneous isotropic poroelastic medium. If a harmonic time-harmonic perturbation $p_0 \exp(-i\omega t)$ of the pore-pressure perturbation is given on a small spherical surface of the radius $a$ with the center at the injection point, then the solution is

$$p(r, t) = p_0 e^{-i\omega t} \frac{a}{r} \exp \left( (i-1)(r-a) \sqrt{\frac{\omega}{2D}} \right), \quad (2)$$

where $\omega$ is the angular frequency and $r$ is the distance from the injection point to the point, where solution is looked for. From equation (2) we note that the solution can be considered as a spherical wave (it is the slow compressional wave in the Biot theory) with the attenuation coefficient equal to $\sqrt{\omega/2D}$ (it is the reciprocal diffusion length) and the slowness equal to $1/\sqrt{\omega 2D}$ (it is the reciprocal velocity of the relaxation).
By analogy with (2) we will look for the solution of (1) in a similar form:

\[ p(\mathbf{r}, t) = p_0(\mathbf{r}) e^{-i\omega t} \exp \left[ \sqrt{\omega} \tau(\mathbf{r}) \right], \]  

(3)

We also will assume that \( p_0(\mathbf{r}) \), \( \tau(\mathbf{r}) \) and \( D_{ij}(\mathbf{r}) \) are functions slowly changing with \( \mathbf{r} \).

Substituting (3) into (1), accepting \( \omega \) as a large parameter and keeping only terms with largest powers of \( \omega \) (these are terms of the order \( O(\omega) \)) we obtain the following equation:

\[ -i = D_{ij} \frac{\partial}{\partial x_i} \tau \frac{\partial}{\partial x_j} \tau. \]  

(4)

Considering again the homogeneous-medium solution (2) we conclude that the frequency-independent quantity \( \tau \) is related to the frequency-dependent phase travel time \( T \) as follows:

\[ \tau = (i - 1) \sqrt{\omega} T. \]  

(5)

Substituting this equation in the previous one we obtain:

\[ 1 = 2\omega D_{ij} \frac{\partial}{\partial x_i} T \frac{\partial}{\partial x_j} T. \]  

(6)

In the case of an isotropic poroelastic medium this equation is reduced to the following one:

\[ |\nabla T|^2 = \frac{1}{2\omega D}. \]  

(7)

Thus, we have obtained a standard eikonal equation. The right hand part of this equation is the squared slowness of the slow wave. One can show (Cerveny, 1985) that equation (7) is equivalent to the Fermat's principle which ensures the minimum time (stationary time) signal propagation between two points of the medium. Due to equation (5) the minimum travel time corresponds to the minimum attenuation of the signal. Thus, equation (7) describes the minimum-time maximum-energy front configuration.

**INVERSION FOR THE PERMEABILITY**

In the case of an isotropic poroelastic medium equation (7) can be directly used to reconstruct the 3-D heterogeneous field of the hydraulic diffusivity. In contrast to this, in the case of an anisotropic medium it is impossible to reconstruct a 3-D distribution of the diffusivity tensor. The only possibility is the following. Let us
assume that the orientation and the principal components proportion is constant in the medium. Then, the tensor of hydraulic diffusivity can be expressed as

\[
D_{ij}(\mathbf{r}) = d(\mathbf{r})\xi_{ij},
\]

(8)

where \( \xi_{ij} \) is a constant undimensionalized tensor of the same orientation and principal-component proportion as the diffusivity tensor, and \( d \) is the heterogeneously distributed magnitude of this tensor. Then, the following equation is helpful for the inversion:

\[
1 = 2\omega d\xi_{ij}\frac{\partial}{\partial x_i}T\frac{\partial}{\partial x_j}T.
\]

(9)

Figure 5 shows an example of the reconstructed hydraulic diffusivity for Soultz-1993 data set.

Figure 5: An example of the hydraulic diffusivity reconstruction in 3-D using Soultz-1993 data set. The diffusivity is given in the logarithmic scale. It changes between 0.001 and 1.5 m/s².
CONCLUSIONS

We have developed a technique (SBRC) for reconstructing the permeability distribution in 3-D heterogeneous poroelastic media. For this we use the seismic emission (microseismicity) induced by a borehole-fluid injection.

Usually, global estimates of permeabilities obtained by SBRC agree well with permeability estimates from independent hydraulic observations. Moreover, the global-estimation version of the SBRC provides the permeability tensor characterizing the reservoir-scale hydraulic properties of rocks. It can be also used for monitoring the process of fracturing and for characterizing tectonic fault systems. The processing of SBRC data for global estimates is based on the fact that the triggering front of a hydraulic-induced microseismicity has the form of the group-velocity surface of anisotropic Biot slow waves.

Now we have further generalized the SBRC approach by using a geometrical-optic approximation for propagation of triggering fronts in heterogeneous media. We think, that results of such inversion for hydraulic properties of reservoirs can be used at least semi-quantitatively to characterize reservoirs. They definitely can be helpful as important constrains to reservoir modeling.

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