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Simultaneous computation of P- and S-wave traveltimes for pre-stack migration of P-S converted waves

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ABSTRACT

In doing converted wave migration we need to compute traveltimes for both P- and S-wave branches. In order to compute these traveltimes given a set of macro velocity models we use the perturbation technique to simultaneously compute P- and S- traveltimes in one run taking into consideration that the v_p/v_s-ratio deviates only slightly from an initial constant average value. To accomplish our goals, we use the raypath of P-wave to compute traveltimes for the S-wave. We can generate a reference S-wave velocity model by scaling the well-determined P-wave velocity with the average v_p/v_s-ratio of the whole model. P-wave traveltimes are computed using Vidale FD-method. S-wave traveltimes in the reference model will then be given by rescaling of the P-wave traveltimes with the average v_p/v_s-ratio. Traveltimes of the perturbed S-velocity models are then computed using a first-order perturbation technique under the assumption that absolute changes in the perturbed models are within the range of the validity of perturbation principles.

For this paper we incorporate the method in a fast and robust finite difference (FD) traveltime computational tool. The technique is evaluated by computing S-traveltimes of a 10% perturbed constant gradient model and the results are compared with the directly computated traveltimes.

INTRODUCTION

The key element of the pre-stack Kirchhoff depth migration is the calculation of traveltime tables, used to parameterize the asymptotic Green's functions at grid points. In order to migrate converted waves one needs to compute traveltimes for the P-and S-wave field to any subsurface point of the discretized model. In the past years much research work have been done on fast computational methods for traveltimes either by using ray tracing implemented as wavefront construction (WFC) (Vinje et al., 1993; Ettrich and Gajewski, 1996a) or finite difference (Vidale, 1988; Podvin

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and Lecomte, 1992; Qin et al., 1992; Van Trier and Symes, 1991). The traveltimes computed with these methods are very accurate if the macro-velocity model used is exactly correct. This requirement is hardly met in practical applications. Thus, it is sometimes recommended to compute traveltimes for various models. (Ettrich and Gajewski, 1998) developed a finite difference method based on Vidale's FD-eikonal solver to compute traveltimes by first-order perturbation but their method was only for single component wave fields (i.e., P-P, S-S). They called this FD-perturbation method.

In this paper, we extend their method to simultaneously compute traveltimes for both P- and S- waves needed for mode converted reflection migration. The simultaneous computation is efficiently realized by computing traveltimes only for a P-wave model and calculating the traveltimes for some perturbed S-wave models by perturbation thereby using the raypaths of the P-waves. We scale the P-wave model with an initial constant v_p/v_s-value of the whole model to obtain a reference S-wave model (assuming that the P/S-wave velocity ratio deviates only slightly from the initial average value). This initial reference model determines the raypaths of the S-wave which is the same as in the P-wave model (see below). The other S-wave models are then considered as perturbed models. The Vidale finite-difference method is used to compute P-wave traveltimes. S-wave traveltimes in the reference model are then obtained by multiplying the P-wave reference traveltimes by the average v_p/v_s-ratio. If the velocity ratio remains truely constant throughout the whole model then P- and S-rays propagate exactly along the same path. In this case, the first-order perturbation method is exact for infinitely large velocity differences. However, in practice we are often confronted with velocity ratios which vary along the ray path. For example, in the shallow unconsolidated subsurface, the v_p/v_s-values are relatively high due in part to extensive microcracks which will lead to low S-velocity (Thurber and Atre, 1993). We apply our technique using an isotropic medium and show that it is valid if the velocity ratios vary up to about 10% of its original value.

Taking into account the range of applicability of any first-order perturbation method, the usuall assumptions made when applying FD-methods for solving the eikonal equation are made. The main advantage of using these perturbation principles lies in the computational speed as compared to a case where the migration is done using P- and S-wave velocity model separately. The method is applicable to all converted reflection modes (P-S, S-P). Direct application to non-converted modes has been implemented into the WFC method (Ettrich and Gajewski, 1996a). Incorporation of this technique to WFC method is much easier since raypaths are computed intrinsically with ray tracing.

In this paper we are not going to give any details on how to implement the perturbation technique into the FD-eikonal solver algorithm since these are given in Ettrich and Gajewski (1998). For highest efficiency and with further respect to 3-D, they implemented the pertubation integrals along rays into the eikonal solver developed by Vidale (1988). They also compared the computing times for 10 slightly deviating 2-D isotropic models and concluded that the computational speed of the FD-perturbation method with respect to the original Vidale method was about 43% faster.

In the next section we will show that for a constant v_p/v_s-value the transmission angles for P- and S-waves are equal. Next we show the basic perturbation method which will be followed by numerical tests.

RAYPATH FOR A CONSTANT P/S-VELOCITY RATIO

Here we show that if v_p/v_s-ratio is constant then P-and S-waves propagate along the same raypaths. For this we use the kinematic ray tracing (KRT) equations. If the raypaths are the same then it should be possible to derive KRT-equations for S-waves from those of P-waves and snell's law. The KRT-equations for P-wave are given as (Aki and Richards, 1980):

$$\frac{d\vec{x}}{d\tau} = v_p^2\vec{p}; \quad \frac{d\vec{p}}{d\tau} = -\frac{1}{v_p}\nabla v_p, \tag{1}$$

where $\vec{p}$ = slowness vector, v_p = P-wave velocity, τ = traveltime and $\vec{x}$ = position vector of the ray. Let us write

$$\frac{v_p}{v_s} = \text{const.} = \gamma. \tag{2}$$

From equation (2), we see that $v_p = \gamma v_s$. Now inserting this into equation (1) we obtain for the first part:

$$\frac{d\vec{x}}{d\tau} = v_s^2\vec{p}\gamma^2, \tag{3}$$

which then gives, apart of the constant, the first part of the KRT-equation for S-waves. For the second part, we have

$$\nabla v_p = \gamma\nabla v_s, \qquad \frac{d\vec{p}}{d\tau} = -\frac{1}{\gamma v_s}\gamma\,\nabla v_s. \tag{4}$$

This implies

$$\frac{d\vec{p}}{d\tau} \;=\; -\frac{1}{v_p}\nabla v_p \;=\; -\frac{1}{v_s}\nabla v_s, \tag{5}$$

which is another form of snell's law.

Thus, from the above derivations we see that when the P/S-velocity ratio is constant the transmission angles for P-wave and S-wave are equal. On the other hand if γ is a function of $\vec{x}$, then equation (4) will be different since we will then have

$$\frac{d\vec{p}}{d\tau} \;=\; -\frac{1}{\gamma v_s}\nabla(\gamma v_s) \;=\; -(\frac{1}{\gamma}\nabla\gamma + \frac{1}{v_s}\nabla v_s). \tag{6}$$

Now we expand the first term on the right hand side of equation (6) and show that this will give second-order terms which can be neglected. To first-order the perturbed velocity ratio γ is written as

$$\gamma \;=\; \gamma_o + \Delta\gamma, \tag{7}$$

such that

$$|\Delta\gamma| \;\ll\; \gamma_o,$$

whereby γ_o is the constant unperturbed ratio and $\Delta\gamma$ is the P/S-wave velocity ratio difference. Inserting equation (7) into equation (6) and retaining only the first term of the right hand side,

$$-\frac{1}{\gamma_o + \Delta\gamma}(\nabla(\gamma_o + \Delta\gamma)) \;=\; -\frac{1}{\gamma_o + \Delta\gamma}(\underbrace{\nabla\gamma_o}_{=0} + \underbrace{\nabla(\Delta\gamma)}_{second-order\ term}). \tag{8}$$

Not that the symbol Δ indicates difference and should not be confused with the Laplace operator. Similar insertion of equation (7) into equation (3) will result also in second-order terms.

THE PERTURBATION METHOD

Following first-order perturbation, the traveltime differences between a reference medium and a slightly deviating perturbed medium are given by integration of the slowness differences.

$$\Delta t \; = \; \int_{ray} (\frac{1}{v_{pert}} - \frac{1}{v_{ref}}) dl, \tag{9}$$

whereby

v_{pert} = perturbed velocity model and v_{ref} = reference velocity model.

The line integral is performed along the raypath computed in the reference S-wave model (Note that this is the scaled P-wave model). How these raypaths are computed in the case of FD-perturbation method is given in Ettrich and Gajewski (1998). For this paper v_{ref} is given as:

$$v_{ref} \; = \; v_p \frac{1}{\gamma_o}, \tag{10}$$

while v_{pert} are the perturbed S-wave velocity models.

The FD-perturbation method is then used to compute S-wave traveltime differences from equation (9). The perturbed S-wave traveltimes are then given as

$$t_{pert} \; = \; t_{ref} \; + \; \Delta t. \tag{11}$$

S-wave traveltimes in the reference model are obtained by multiplying P-wave traveltimes by γ_o, i.e., $t^s_{ref} = \gamma_o t_p$. From equation (9), we see that the traveltime difference is given by summation along the common P- and S-wave raypath determined in the reference model. Differences in P- and S-wave raypath due to v_p/v_s changes are not considered. As in tomographic inversion (Thurber and Atre, 1993), we have shown that these raypath differences would have only a second-order effect on traveltime difference and velocity perturbation estimation (see equation (8)). The Fermat's principle is invoked to justify the use of the P-wave path to compute S-wave traveltimes, so that the variation of the perturbed S-wave path from the initial (P-wave) path (which is zero for truely constant v_p/v_s) is negligible small. Indeed, if spatial variations of v_p/v_s are modest in the range of applicability of first-order perturbation then one would expect that the curved raypath would be one that makes traveltimes an extremal with respect to path perturbation (Raypath stationarity due to Fermat's principle). Ben-Menahem and Singh (1981) (see pp. 738 - 739) also showed that perturbation in raypaths induced by velocity heterogeneities will give rise to second-order effects in the corresponding perturbation in the traveltime. To first-order these effects can well be neglected.

NUMERICAL TESTS

The figures below show some numerical tests of the method. A constant gradient 2-D P-wave velocity model with a parabolic lens is multiplied by an average v_s/v_p-ratio of $\frac{1}{\sqrt{3}}$ to give a reference S-model. The velocity in the lens has a negative gradient. We perturb the S-wave velocity in the parabolic lens by 10% of its original value. The S-velocities around the lens are kept unchanged. With this we have a vertical and lateral change of v_p/v_s-ratio in the model. The model dimension is 0.6 km x 0.6 km and consists of 301 x 301 samples arranged in a rectangular grid with 2 m grid spacing. The source is located at 0.3 km on the surface of the model. In the unperturbed lens v_p= 1.0 km/s and v_s= 0.58 km/s, while in the perturbed lens v_s= 0.52 km/s and the surrounding S-velocities are unchanged such that v_s^0= 1.15 km/s. We use the FD-perturbation method to compute S-traveltimes in the perturbed model. We also computed directly S-traveltimes in the perturbed model using the Vidale's method and compared the relative errors. Figure 2 shows the absolute value of the errors in [%]. Overlayed on the error plot are directly computed traveltime isochrons (white lines) and isochrons computed by perturbation (dashed lines). We see that the relative error is less than 1%.

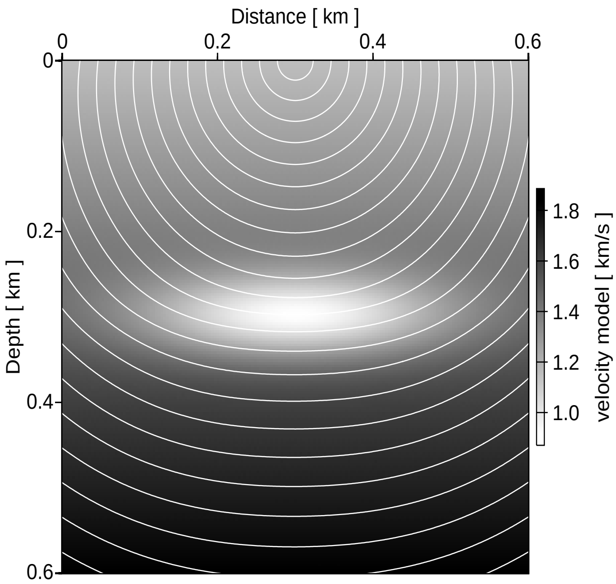

Figure 1: A 2-D constant gradient S-wave velocity model with a parabolic lens at the center (used as reference model). Overlayed are traveltime isochrons directly computed using 2-D-Vidale FD-algorithm.

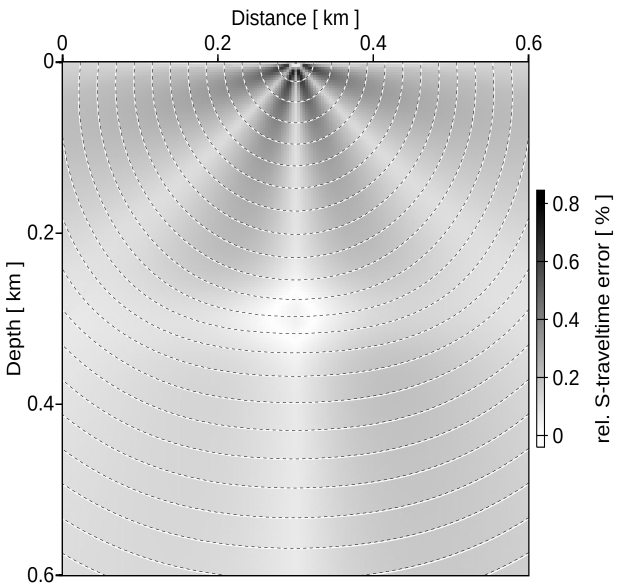

Figure 2: Absolute values of the relative error [%] of directly computed traveltimes using the FD-method and the FD-perturbation method for a perturbed S-wave velocity model. Overlayed are the directly computed traveltime isochrons (white lines) and the isochrons computed by perturbation (dashed lines).

SUMMARY

The following give the various steps done.

- First we compute traveltimes for P-waves using FD-eikonal solver,
- then compute initial traveltimes for S-waves by scaling those of P-wave,
- compute S-wave traveltime differences using equation (9),
- and finally we compute perturbed S-wave traveltimes using equation (11).

LIMITATIONS

The limitations given by Ettrich and Gajewski (1998) are also valid here. The effect of considering a linear relation i.e., first-order for the velocity ratio variation rather than the more accurate nonlinear form will generally result in a slight decrease of accuracy. Furthermore, if the velocity ratio variation is substantial then raypaths will deviate and the perturbation principle will break down.

CONCLUSION

We have demonstrated how to use initial raypaths for S-waves computed from a P-wave reference model to simultaneously compute S- and P-wave traveltimes, thereby assuming an initial constant v_p/v_p-ratio. The accuracy of the method is dependent on the initial P- and S-models, respectively, since the computed traveltime difference is subjected to the assumption of similar path geometries for the P- and S-waves in the initial model. We showed in the examples that if the velocity ratio deviates only slightly (10%) from the initial value, the first-order perturbation technique can be used to correctly compute P- and S-wave traveltimes simultaneously. The relative errors are less than 1%. We also showed that for a truely constant velocity ratio, the raypath of the P- and S-waves will coincide. Thus, FD-perturbation method can advantageously be applied to do at least correct kinematical pre-stack Kirchhoff migration as well as pre-stack velocity estimation (for both P- and S-wave models) and estimation of v_p/v_s variations. It can also be applied in tomography. For migration purposes the reflection traveltimes are computed by combining information from source and geophone traveltime tables. After the calculation of the source (P-wave) and receiver (S-wave) traveltime tables, reflection times are determined by the summation of P- and S-wave times at locations along interfaces, followed by the application of Fermat's principle, which involves finding the interface locations where the traveltime is stationary.

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