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# Coherency Analysis of Seismic Data

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## ABSTRACT

*In general, coherency analysis aims to gain the information about a signal which is recorded on several channels. The kinematical part of the signal is assumed to be defined by a set of parameters which parameterize a theoretical traveltime model. Using a coherence measure one can decide if a parameter combination corresponds to a genuine signal or not, by analyzing the dynamical distribution along a traveltime trajectory for* coherency.
*The properties of the* coherence measures *are crucial for the result of any coherency analysis. These properties are summarized and explained with standard coherence measures. An application of a coherency analysis is shown with a real example of the common-reflection-surface (CRS) method. Some eigenstructure coherence measures which are results from recent investigations in the field of high-resolution coherence measures are also mentioned.*


## INTRODUCTION

Coherency analyses are influenced by many factors and have always to be regarded as combination of *four* integral parts: First, it is mainly the quality of the *input data* affecting the result of a coherency analysis. Second, the *moveout model*. It describes the kinematics of a signal. Third, the *coherence measure*. The theory of coherence measures implies some assumptions about the composition of signal and noise of the input data and assumes ideal alignment of traces through the moveout model. Forth, the *computing capabilities*. The implementation of coherency analysis in a computing environment constitutes an important practical and financial aspect.

### Previous Developments

Just after multiple ground coverage methods had gained acceptance within the petroleum industry multichannel coherence techniques have been employed routinely.

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One of the first methods which has become standard was reported by Schneider and Backus (1968) and was presented at the 36th Annual International SEG Meeting in Houston, 1966. Their coherence measure is based on the *correlation* of traces and constitutes a realization of the familiar stochastic concepts of *correlation analysis* (e.g. Montgomery and Runger (1994)) and *random processes* (e.g. Bendat and Piersol (1986)) for multichannel seismic data.

The most commonly used coherence measure, even today, is the *semblance* criterion which was firstly reported by Taner and Koehler (1967). The criterion was later published by Taner and Koehler (1969) and Neidell and Taner (1971). The measure is also based on trace correlations but is extended by an energy-normalization scheme.

At about the same time the *stacking* result of traces was firstly used as coherence measure. Garotta and Michon (1967) defined the *mean* of the amplitudes along a traveltime trajectory as coherence measure.

The intensified use of statistical criteria to improve the resolution of a coherency analysis has led to some modifications of already existing measures or to independent developments (e.g. Gelchinsky et al. (1985); Katz (1991); Sacchi (1998)). Morozov and Smithson (1996) summarized some analysis methods and developed a hybrid measure combining a phase correlation measure with a statistical hypotheses filter. Fuller and Kirlin (1992) introduced weighting schemes to improve the resolution of conventional measures.

Recent developments are based on the separation of the eigenstructure of the data covariance matrix into signal and noise subspaces (Biondi and Kostov, 1989; Key and Smithson, 1990; Kirlin, 1992). The method exhibits superior resolving power in comparison to conventional stack-based measures. Marfurt et al. (1999) used both, the conventional semblance and crosscorrelation measures as well as eigenstructure measures to generate seismic coherency cubes for structural interpretation.

## COMPARISON OF COHERENCE MEASURES

The decision which coherence measure one should use to analyze the data is supported by some criteria with wich the performance of coherence measures can be assessed by. Along with this criteria the decision depends on the knowledge about quality and signal and noise characteristics of the data, the type of information to be extracted, the assumptions (or validation of the assumptions) the moveout model is based on, the way the information is to be extracted (visual or numerical) and on the time, i.e., the costs, the computation may take.

Besides all these considerations, the coherence measures must proof their applicability in practice, i.e., to a large degree it is still an empirical decision as to which measure to use.

**Coherence Measure Properties**

In any application, coherency analysis is used to extract some information from the given data. Therefore, the main property of any coherence measure is its *resolution capability* determining the accuracy the desired information can be resolved. Resolution is a general attribute for coherence measures and is further influenced by a set of special properties as listed below.

Properties of coherence measures:

- Resolving power
- Discrimination threshold
- Separation of events with equal zero-offset time
- Sensitivity to amplitude changes versus offset
- Sensitivity to sign/phase changes versus offset
- Noise level dependency (statistical significance)
- Enhancement of weak signals
- Dynamic range of display
- Stability
- Computational effort

Some of these attributes are interrelated. The discrimination threshold, for example, can be a visual attribute and depends then on the dynamic range of values a coherence measure needs for display; however, the visual discrimination threshold may differ from thresholds numerical search algorithms need to discriminate events.

The separation of events with equal zero-offset time is a challenging task for coherence measures. Theoretical, separation of events is better the wider the spatial range is chosen. In practice, however, a wider spatial range often collides with the small spread approximation of theoretical models. One method to improve this compromise is to introduce offset dependent weighting factors.

The sensitivity of a coherence measure to amplitude, sign or phase changes versus offset depends on the theoretical properties of each measure.

The noise level dependency rather is based on the practical properties of a coherence measure. For example, the *normalized crosscorrelation* measure is a statistical estimate of the normalized crosscorrelation function. The estimate is defined only for a limited time range while the crosscorrelation function is defined for infinite records. The variance of the estimate varies with the length of the time gate, the bandwidth of signal and noise and with the signal-to-noise ratio (see Bendat and Piersol (1986)). It is evident that the statistical significance of the estimate shrinks with shorter time gates. In coherency analysis the operation windows are commonly short, thus the statistical significance is low. That is, coherence measures must proof their applicability in practice (see in addition Schneider and Backus, 1968).

The enhancement of weak reflections is affected by the properties of the coherence measure and is normally supported by the normalization of measures. *Stacked amplitude*, for example, yields its highest sum for the reflection with the highest amplitudes, no destructive summation provided. *Semblance*, however, theoretically yields the same result for any scaled version of a signal. It may only yield lower values for weaker signals because of the reduced statistical significance, i.e., the lower S/N ratio for weaker reflections.

Normalization reduces the dynamic range for display while with unnormalized measures the dynamic range is mainly occupied by the largest coherence value obtained for the reflection with the strongest amplitutes.

To extract weak reflections by visual means it may be desirable to reduce the distance between the coherence values for different events. However, for numerical algorithms which search for local maxima it may be easier to find these maxima if the separation between the values is as great as possible.

The possible resolution of coherence measures is limited by the discrete parameter space. If the discretization of the space is too coarse in combination with the resolving power of a measure an aliasing problem will occur (see Claerbout (1992)). That is, it is possible that a measure would yield a local maximum for a parameter value that lies right between two values from the discrete range of values.

The computational effort of a coherence measure becomes a crucial point when moveout models with three or even more parameter dependencies are used. Any additional parameter increases the computation time with a factor equal to the number of its elements. Therefore, effective measures are needed. Also, one can reduce the computation time if only selected parameter combinations which yield local maxima are calculated. This requires numerical search algorithms which must be optimized to be effective. Although it is generally profitable to interpret the structure of the whole parameter space, visual examinations are less practicable the more parameters are involved. With modern technologies, however, such as virtual reality cubes, it is possible to visual investigate three or four-dimensional parameter sets in a reasonable effective way. In any case, since sophisticated moveout models becomes rather more complex and recent coherence measures also, the employment of massive parallel computer systems with highly efficient parallel computer algorithms is a requirement to manage forthcoming tasks.

# REAL DATA EXAMPLE

## Common-Reflection-Surface (CRS) Method

The *common-reflection-surface* (CRS) stacking method is applied to utilize as much traces as possible from a pre-stack data-set (Tygel et al., 1997; Höcht, 1998). For 2-D seismic, the stacking result is a well illuminated simulated zero-offset section. The method may also be implemented for 3-D. A traveltime model used within this method is based on the paraxial ray theory (Bortfeld, 1989; Cerveny, 1987). In the 2-D case, the moveout function with respect to half-offset ($h$) and midpoint coordinate ($x_m$) for inhomogeneous earth models with arbitrary smooth interfaces can be approximated by a hyperbolic second order Taylor expansion (Schleicher et al., 1993):

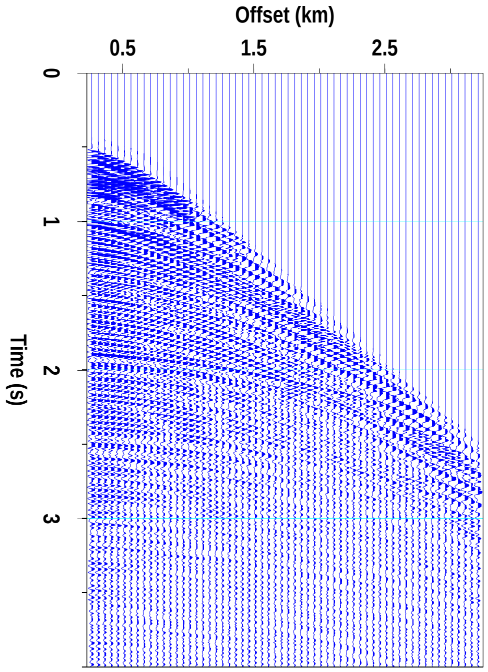
$$t^2(x_m, h) = \left[t_0 + \frac{2\sin\alpha}{v_0}(x_m - x_0)\right]^2 + \frac{2\,t_0\cos^2\alpha}{v_0}\left[\frac{(x_m - x_0)^2}{R_N} + \frac{h^2}{R_{NIP}}\right] \quad (1)$$

This moveout model is parameterized by the radius of curvature of the *normal* wave ($R_N$), the radius of curvature of the *normal-incidence-point* wave ($R_{NIP}$) and the emergence angle of both waves ($\alpha$). This description implies the approximation of the two emerging wavefronts by circles in the vicinity of surface location $x_0$ (see Hubral (1983)). Since only the near-surface velocity $v_0$ need to be known, expression (1) constitutes a *macro-velocity model independent* formulation of the traveltime.

With model (1) the coherency analysis of a pre-stack data-set $(t, x_m, h)$ from a 2-D survey is in principle performed in the same way as described in the previous chapters. The only differences to the analysis of a CMP gather with the velocity-parameterized CMP hyperbola are that we now have three parameters (instead of one) and two spatial dependencies (instead of one) in the model. Furthermore, if a parameter combination is fixed, not only for each zero-offset time but for each point in the zero-offset time – midpoint space $(t_0, x_0)$ one coherence value is computed.

I want to show a result of a coherency analysis with 31 real CMP gathers which are from a marine data-set from Mobil Corp. Figure 1 shows one CMP gather of this record.

Coherency analysis has been performed with moveout model (1) and the *semblance* coherence measure at a specified point $(t_0, x_0)$ of the zero-offset section. The near surface velocity was set to $v_0 = 1.48$ km/s. The midpoint is $x_0 = 7.556$ km and the zero-offset time is $t_0 = 3.62$ s. The ranges for the parameters have been chosen as follows: emergence angle $\alpha$ reached from -15 to +15° with 0.5° increment and the radius of the *NIP*-wave ($R_{NIP}$) reached from 2.0 to 6.0 km with 0.05 km increment. Since the radius of the *N*-wave ($R_N$) may approach values of plus or minus infinity, a special range and increment was specified for this parameter: the range of values, $-1.25 \leq \chi \leq +1.25$, of the function $R_N = f(\chi)$ with $f(\chi) = -\tan(\chi - \pi/2)$




(a) Real CMP gather (wiggle format)

Figure 1: Real CMP gather of a marine data-set.

was divided into $500$ equal parts, i.e., $\chi$ reached from $-1.25$ to $+1.25$ with a linear increment of 0.005. Thus, the corresponding values of $R_N$ reached from $-0.33\ldots-\infty/+\infty\ldots+0.33$ km. The function $f(\chi)$ has at $\chi=0$ an infinite jump discontinuity. For each parameter combination a coherence value was calculated.

Figure 2 shows the resulting coherence values with colors in the $[\alpha, \chi(R_N), R_{NIP}]$ space. An absolute maximum of $S_c = 0.40$ was obtained at $\alpha = 0°$, $\chi = 0.05$ ($R_N = 20.0$ km) and $R_{NIP} = 4.15$ km. For this values the slices through the coherency cube are shown. We can examine that the maximum (indicated by red color) is rather well resolved. The corresponding wavefield attributes may subsequently be used for stacking or inversion.

## EIGENSTRUCTURE COHERENCE MEASURES

Besides *correlation* or *stack-based* coherence measures, *eigenstructure* measures have reached a considerable use. They are based on the analysis of the eigenstructure of the data covariance matrix (Biondi and Kostov, 1989; Key and Smithson, 1990; Kirlin, 1992).

In principle, the covariance matrix of the data enclosed by a time gate about the moveout trajectory of interest is assumed to be an estimator for the model of a covariance matrix which has a particular eigenstructure. The eigenstructure of the model is

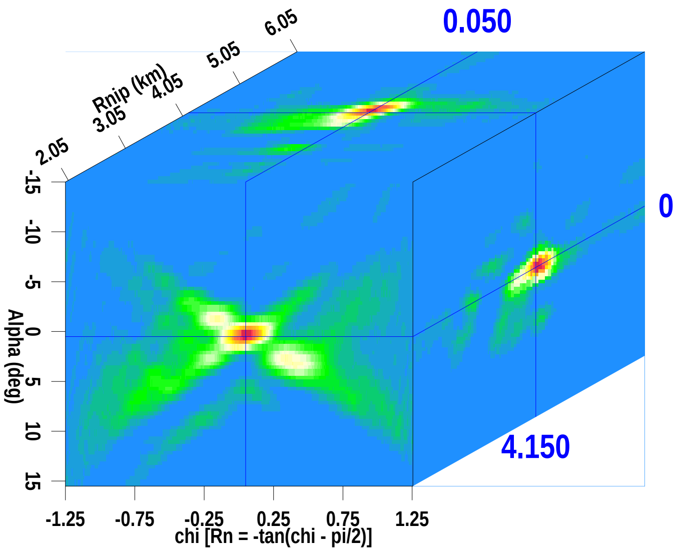



Figure 2: *Semblance* values (colors) in the $[\alpha, \chi(R_N), R_{NIP}]$ space resulted from a coherency analysis of 31 real CMP gathers with moveout model (1) at $x_0 = 7.556$ km and $t_0 = 3.62$ s. The plot shows three slices through the coherency cube at $\alpha = 0°$, $\chi = 0.05$ ($R_N = 20.0$ km) and $R_{NIP} = 4.15$ km. The range of $\chi$ was mapped on $R_N$ with the function $f : R_N = -\tan(\chi - \pi/2)$. The high *semblance* value (red) corresponds to a primary reflection.

decomposable into signal and noise subspaces. The signal wavefronts of an observed wavefield are associated to eigenvectors of the signal subspace and the additive noise components of the wavefield are associated to eigenvectors of the noise subspace. The discrimination of the two subspaces is possible since the largest eigenvalues of the covariance matrix belong to the signal eigenvectors. Hence, coherence measures based on the eigenstructure method include eigenvalues or eigenvectors in their formulations.

Eigenstructure measures are also known as *high-resolution wavefront* measures. Due to the separation of the noise subspace they are able to account for the noise associated with nearby and interfering events which, in contrast, reduces the resolution of stack-based measures (Key and Smithson, 1990).

### Signal Space Semblance

Kirlin (1992) used the concept of signal and noise subspace to enhance the established *semblance* measure. The new *signal space semblance* coefficient exhibits a better resolution than the conventional *semblance* measure.

In order to express the semblance of the signal subspace, I first give a definition of the sample covariance matrix and the model covariance matrix which separates the eigenstructure into signal and noise subspaces. The data within a time gate, $N+1$ time samples long, is assigned to vectors $\mathbf{u}_j$, $k - N/2 \leq j \leq k + N/2$, such that each vector contains the data along trajectory $t_j(\mathbf{x}_i)$, $i = 1, \ldots, M$. The sample *covariance matrix* then reads

$$\mathbf{C}_k = \frac{1}{N+1} \sum_{j=k-\frac{N}{2}}^{k+\frac{N}{2}} \mathbf{u}_j \mathbf{u}_j' \tag{2}$$

where $\mathbf{u}'_j$ is the conjugate transposed of $\mathbf{u}_j$.

The *semblance* criterion can be written as

$$S_c = \frac{\mathbf{1}'\mathbf{C}_k\mathbf{1}}{M\mathrm{Tr}[\mathbf{C}_k]} \tag{3}$$

where $\mathrm{Tr}[\mathbf{C}_k]$ denotes the trace of the covariance matrix and $\mathbf{1}$ is a $M$ element vector with ones. This is just an additional formulation of the conventional *semblance* measure. To define an eigenstructure measure, we must make some assumptions about the composition of the eigenstructure of the covariance matrix.

In general, the eigenstructure of a $(M \times M)$ matrix is described by eigenvectors $\mathbf{v}_i$, $i = 1, \ldots, M$ and eigenvalues $\lambda_i$, $i = 1, \ldots, M$:

$$\mathbf{R}_k = \sum_{i=1}^{M} \lambda_i \mathbf{v}_i \mathbf{v}'_i$$

If we assume that the sample data of one analysis window contain one transient signal wavefront, $\mathbf{C}_k$ should have one large eigenvalue $\lambda_1$ and the corresponding eigenvector $\mathbf{v}_1$ would constitute the basis of the signal subspace. There would be $M - 1$ eigenvalues left which would all be equal to the noise power $\sigma^2$ (Kirlin, 1992). Under these assumptions we can define the model of the data covariance matrix as

$$\mathbf{R}_k = (\lambda_1 - \sigma^2)\mathbf{v}_1\mathbf{v}'_1 + \sigma^2\mathbf{I} \tag{4}$$

where $\mathbf{I}$ is the $(M \times M)$ identity matrix. We expect $\mathbf{C}_k$ to be an estimator of $\mathbf{R}_k$, i.e., $E\{\mathbf{C}_k\} = \mathbf{R}_k$. With this definition we can describe the conventional *semblance* measure with a further expression:

$$S_c = \frac{\lambda_1 \mathbf{1}'\mathbf{v}_1\mathbf{v}'_1\mathbf{1} + \sigma^2 \sum_{i=2}^{M} \mathbf{1}'\mathbf{v}_i\mathbf{v}'_i\mathbf{1}}{M \sum_{i=1}^{M} \lambda_i} \tag{5}$$

Kirlin (1992) suggested a modification of Equation (5) to enhance the *semblance* measure. He excluded the noise-space energy from expression (5) to achieve a better resolution and introduced a scaling factor. After that, his *signal space semblance* criterion takes a very simple form:

$$S_c^{\mathrm{K}} = \frac{|\mathbf{1}'\mathbf{v}_1|^2 - 1}{M - 1} \qquad 0 \leq S_c^{\mathrm{K}} \leq 1 \tag{6}$$

Thus, for each data sample, we have to compute the eigenvector $\mathbf{v}_1$ which is associated with the largest eigenvalue of the sample covariance matrix.

Kirlin (1992) compared the conventional *semblance* criterion with measure $S_c^{\mathrm{K}}$ with a synthetic example. The enhanced version exhibited a better resolving power than the conventional one.

**Covariance Measure**

The *covariance* measure proposed by Key and Smithson (1990) constitutes a signal-to-noise estimate based on the simultaneous estimation of the noise and signal energy within a data window. The ability of the continuous adaption of the noise and signal estimates improves the event detection and resolving power of the measure and makes the measure less sensitive to residual statics and deviations from the chosen moveout model.

Separating the signal and noise subspace of the covariance matrix as in Equation (4), the eigenvalue of the signal wavefront can be expressed by

$$\lambda_1 = [M/(N+1)]E_s + \sigma^2$$

where $E_s$ is the energy of the signal in one channel. Since all minor eigenvalues are estimates of the noise power $\sigma^2$, we get an better estimate of the noise power by averaging the minor eigenvalues:

$$\overline{\lambda} = \overline{\sigma^2} = \sum_{i=2}^{M} \frac{\lambda_i}{M-1}$$

Subtracting $\overline{\lambda}$ from $\lambda_1$ leaves an estimate of the signal energy. Hence, the *covariance* measure from Key and Smithson,

$$C_c = \alpha \, \frac{\lambda_1 - \overline{\lambda}}{\overline{\lambda}} \,, \tag{7}$$

constitutes a weighted signal-to-noise ratio estimate. Note that only $\lambda_1$ needs to be determined since $\overline{\lambda} = \mathrm{Tr}[\mathbf{C}_k] - \lambda_1$. Key and Smithson proposed a log-generalized likelihood ratio for the equality of the eigenvalues as weighting function:

$$\alpha = (N+1)\ln^M \left[ \frac{\left(\sum_{i=1}^{M} \frac{1}{M}\lambda_i\right)^M}{\prod_{i=1}^{M} \lambda_i} \right]$$

Its value reaches zero if no signal is present and infinity if a noise-free signal is present.

Tests of the *covariance* method show significant improvements in time and parameter resolution relative to the *semblance* criterion (Key and Smithson, 1990). The computation costs of $C_c$ can be reduced by a partial stacking of the traces of a data window before calculating the covariance matrix.

With additional effort, high-resolution coherence measures can be further improved with statistical procedures assessing the accuracy of statistical estimates. For instance, Sacchi (1998) combined the *covariance* measure (Equation 7) with a bootstrap procedure to achieve additional attenuation of spurious events which results in a further improvement of high-resolution spectra.

## CONCLUSION

Coherency analysis of seismic data is a common processing step. One important feature of this transformation, the coherence measure, has been investigated.

Since the beginnings of multichannel coherency analysis, the concepts of correlation and trace stacking have built two classes of standard coherence measures. Different aspects of judging a coherence measure have been mentioned. The rating depends on the area of application. As a rule, best visual interpretable results are obtained by normalized coherence measures since the dynamic range is critical for display.

However, coherence measures employed in numerical search algorithms should rather be judged by their ability to determine a local maximum where a signal is present. The conditions of noise and signal variations have to be considered first, since they are most critical for the performance of coherence measures.

The resolving power is an important criterion of coherence measures. Some newer developments of coherence measures aim to improve this aspect. *Eigenstructure* methods are based on the simultaneous analysis of the signal and noise subspaces of the data covariance matrix. They achieve better resolutions, but are more costly to compute. Further methods to improve the resolution are based on the combination of conventional coherence measures with weighting schemes, statistical filters or advanced statistical decision criteria. The problem of the additional computational effort can be managed by an efficient algorithmic and hardware-specific implementation of these methods.

It has been made clear that the demands on coherency analysis techniques will grow enormously in the future. The coherence measures have to be rapidly computable, because of the increasing number of parameters and dimensions in sophisticated moveout models, and they must meet the requirements on accuracy and smoothness of optimized search algorithms.

Where the results of coherency analyses have to be visual interpreted, the most efficient visualization techniques are needed. That includes effective color mapping of coherence and attribute values and immersive visualization techniques, such as virtual reality cubes.

However, an increasing number of techniques are implemented which shall reduce the number of interpretation steps. For this automation, reliable and fast search algorithms and high-resolution coherence measures are nedded.

Both implementations of coherency analysis techniques, visual or automatic, demand the massive use of computational power. Thus, not least the developments in the hardware industry will influence the direction of coherency analysis applications in seismic industry.

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## PUBLICATIONS

Detailed results were published by Mauch (1999).