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Combining Fast Marching Level Set Methods with Full Wave Form Modeling

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ABSTRACT

Full wave form modeling techniques in 3D complex heterogeneous elastic media are computationally expensive. In contrast asymptotic techniques only compute a small subset of the wave field, but can be very fast. Using asymptotic methods in combination with full wave form techniques can speed up the overall computation. I use a particular asymptotic method to compute dynamically the currently active computational domain for a full wave form finite difference technique. This leads to an overall decreased computational runtime.

INTRODUCTION

Traditional full wave form seismic simulation techniques are well established such as pseudospectral and high-order finite difference methods and other that can be found in recent SEG abstracts. Although improvements are continuously ongoing, the major limitation so far has been the huge expense associated with realistic 3D prestack computation for complex heterogeneous subsurface models. Full wave form techniques aim at producing exact seismic wave form solutions in complex 3D subsurface models for a prescribed recording period. One of the biggest challenges in 3D full wave form modeling is to make the computational effort more economical. Some attempts promise to be successful, such as fixed-geometry based domain decomposition and the use of unstructured grids.

The method of active domain decomposition which I am presenting here, can be used in conjunction with traditional Finite Difference techniques as well as with the previously mentioned novel approaches. Active domain decomposition techniques as well as hybrid simulation techniques can be implemented using various asymptotic methods. However, one critical issue is efficiency and speed. If the asymptotic method

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is slower than computing the full wave form solution, then the asymptotic methods is not practical, unless certain wave field subsets need to be computed only.

Fast marching level set methods are particularly suitable for the approach presented here, because they can be efficiently implemented to solve 3D problems. Given an equilateral computational grid with n grid points along each axis, full wave form finite-difference solutions show an operational count of the order $O(n^3)$ per time step while fast marching methods, which track the wave front only, have a substantially reduced operational count $O(logn^3)$ for the entire problem.

Several years ago seismic modeling initiatives Aminzadeh et al. (1996); House et al. (1996) were pursued by SEG, EAGE, Industry and National Laboratories to numerically calculate acoustic 3D seismic data. The 3D models were chosen to represent realistic geologic settings. However, due to computer memory limitations and time constraints the seismic data were modeled purely acoustically, and thus lack certain real-world effects. Nevertheless, even up to now, repeating the same simulation for an elastic subsurface model, has been hampered by the availability of computers that can provide enough memory and compute power.

FINITE DIFFERENCE TECHNIQUES

For computing the full wave form solution, I am using high-order optimized finite difference operators to approximate partial derivatives in space and time as described in more detail in Karrenbach (1995); Virieux (1986); Karrenbach (1998). I am solving basic anisotropic wave equations of the form

$$\mathbf{a}(\mathbf{x})\ \boldsymbol{\nabla}\ \mathbf{b}(\mathbf{x})\ \boldsymbol{\nabla}^t\ \mathbf{u}(\mathbf{x},t)\ -\ \frac{\partial^2}{\partial t^2}\mathbf{u}(\mathbf{x},t) = \mathbf{f}(\mathbf{x}_0,t)\ , \tag{1}$$

which can be easily extended to visco-elastic and more complicated cases and where $\mathbf{u}$ is an arbitrary wave field (scalar or vector) and $\mathbf{f}$ is the force applied at source locations $\mathbf{x}_0$. ∇^t is a general gradient operator and $\boldsymbol{\nabla}$ the associated divergence operator applied to the wave field components in three dimensions. $\mathbf{a}$ and $\mathbf{b}$ are medium property fields such as density, velocity or stiffnesses.

I solve equation 1 numerically as set of first order coupled equations. This allows to freely impose boundary and initial conditions and to extract all desired wave field quantities, such as pressure, particle displacement, acceleration, stress, strain and force. Such a flexible simulation method, easily produces wave fields for acoustic, elastic, anisotropic and viscous media, with and without free surface effects and allows large degree of freedom for variable recording geometry and observable. The computational complexity increases to the third power with increasing computational 3D volume.

FAST MARCHING METHODS

Fast marching level set methods Sethian (1996) are relatively recent contributions to numerically solving eikonal equations of the form

$$\left(\frac{\partial t}{\partial x}\right)^2 + \left(\frac{\partial t}{\partial y}\right)^2 + \left(\frac{\partial t}{\partial z}\right)^2 = \frac{1}{v^2(x,y,z)}\ , \tag{2}$$

where x, y, z are Cartesian coordinates and $t(x, y, z)$ is the travel time field at each point is space. In the early 1990s methods were developed and became popular that solved the eikonal equation by using finite-difference approximations and produced travel time maps for fixed source seismic experiments. These traveltime maps were usually used successfully in the context of seismic imaging Vidale (1990); van Trier and Symes (1991). Fast marching methods aim at producing viscosity solutions by using upwind finite-difference schemes. Thus, they compute the continuous first arrivals of the travel time field. By choosing particular finite-difference stencils the method can be made numerically extremely stable. Using a particularly structured algorithm, the method can be made extremely fast. The numerical complexity is orders of magnitudes simpler than for a full wave form finite-difference computation in the same volume.

In this paper I am using a fast marching algorithm to compute the currently active computational domain for the full wave form finite-difference method. The fast marching algorithm tracks very efficiently the limiting wave fronts and thus decomposes the computational domain dynamically into subdomains. Within these smaller subdomains the full wave form solutions can be computed faster while maintaining identical accuracy.

APPLICATION TO THE SEG/EAGE SALT MODEL

In the following I apply the active domain decomposition by a fast marching method to the computational grid of a high-order finite difference technique.

In the 3D model, Fig. 1, I employed a surface recording geometry. The model consists of a smooth background velocity, with a shallow soft sea floor, that exhibits gentle slopes. There are several interfaces and faults that present some structural complexity. A anomalous geopressure zone is incorporated in the model. Details can be found in O'Brien and Gray (1996) and in workshops Versteeg and Grau (1990) held during the SEG. Previously I generated Karrenbach (1998) an elastic model out of the original purely acoustic subsurface model and compared synthetic seismic data in form of snapshots and seismograms for these two scenarios. In this paper I improve on the computational efficiency.

Figure 1 shows the original 3D subsurface model The dynamic domain decomposition was computed by a fast marching method and Figure 2 shows the outlines of individual subdomains. The dark contour lines represent the boundaries of the subdomains at given instances in time. The shaded colors within those subdomains illustrate the progression of the active domain over time. The domain boundaries are heavily influenced by the 3D subsurface p-wave velocity model. The salt body in particular dramatically increases the computational domain. Figure 3 shows a snapshot of the Z component of the elastic wave field in this model. An explosive source was used at the surface and we see the complicated wave field generated in the layers on the top of the salt and less energetic penetration through the salt. P-wave to S-wave conversions play a major role in this energy partitioning. The full wave form solution computed using the fixed entire 3D volume is identical to the solution computed with the dynamic domain decomposition, except for numerical round-off errors.

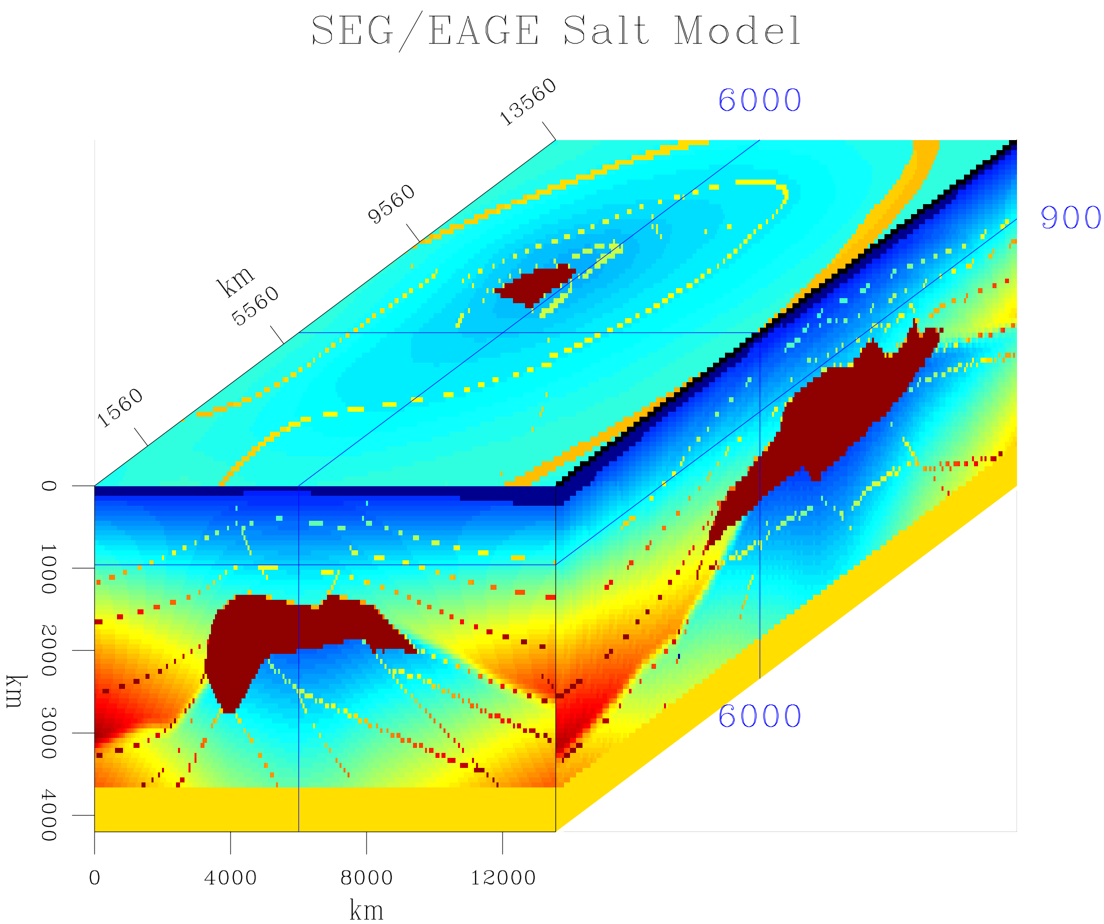

Figure 1: The compressional wave velocity model of the SEG/EAGE salt model.

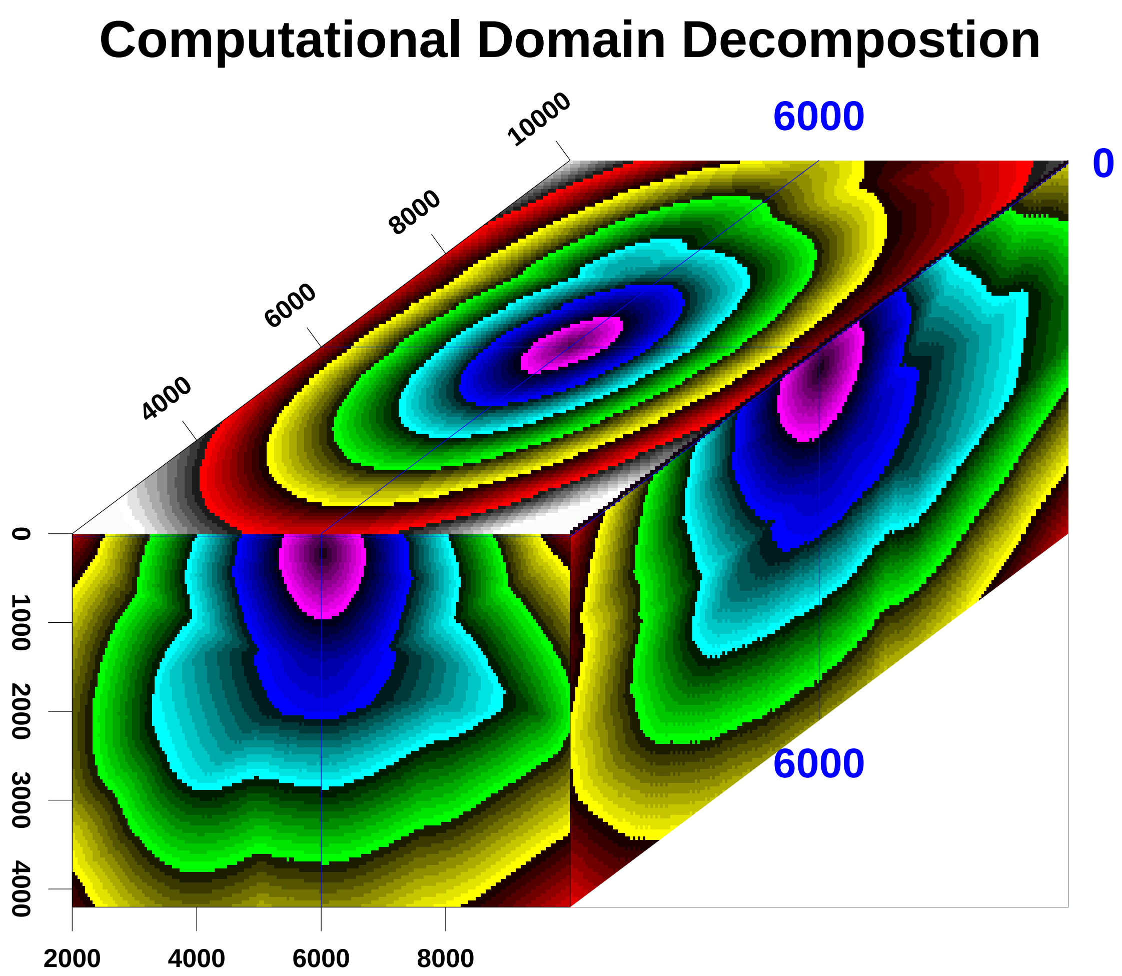

Figure 2: In the SEG/EAGE salt model Fast Marching Level Set Methods can efficiently estimate the active computational domains dynamically. The determination of the active areas in the 3D model can be followed by computing full wave form solutions only in subdomains of reduced size.

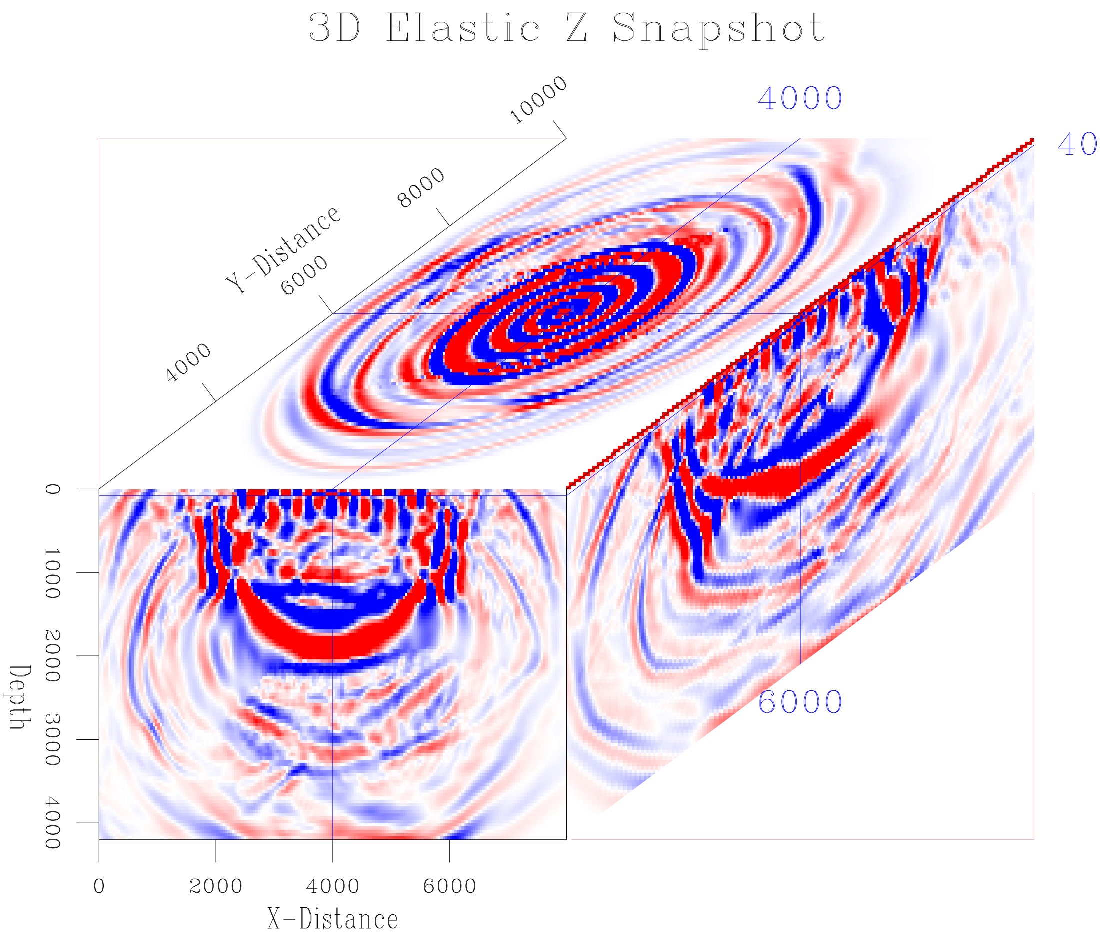

Figure 3: Snapshot of the elastic full wave form solution computed through the SEG/EAGE salt model using the a Fast Marching Level Set Method to bound the active computational domain for each time step. The result is identical to the one computed on the full 3D grid.

CONCLUSIONS

3D elastic (or visco elastic, anisotropic) simulation of full wave fields is useful for survey design, processing and interpretation. However, there are still challenges, that need to be overcome to make them practical. I show here the combination of two computational techniques, that allows to change dynamically the active grid size of the 3D computational grid and thus reduces the overall runtime. The dynamic decomposition is extremely model, source and receiver geometry dependent. This method can be used together with other hybridization techniques to simulate within a reasonable amount of time realistic sized 3D surveys and to produce 3D full wave fields in complex heterogeneous reservoirs.

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