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Reference ellipsoids for anisotropic media

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ABSTRACT

Perturbation techniques are common tools to describe wave propagation in weakly anisotropic media. The anisotropic medium is replaced by an average isotropic medium were wave propagation can be treated analytically and the correction for the effect of anisotropy is computed by perturbation techniques. This works well for anisotropies of up to 10%. Some materials (e.g., shales), however, can exhibit a much stronger anisotropy. In this case a background medium is required which still can be treated analytically but allows to consider a stronger P-wave anisotropy. In this paper we present a technique to compute a best fitting ellipsoidal medium to an arbitrary anisotropic medium. Elliptical media can still be treated analytically but allow to consider strong P-wave anisotropy. Corrections from the ellipsoidal medium to the anisotropic medium are again obtained by the perturbation approach. The averaging of the arbitrary anisotropic medium can be carried out globally (i.e., for the whole sphere) or sectorially (e.g., for seismic waves propagating prominently in the vertical direction). We derive linear relations for the coefficients of the ellipsoidal medium which depend on the elastic coefficients of the anisotropic medium. Numerical examples for different rocks demonstrate the improved approximation of the anisotropic model using the ellipsoidal medium compared to the average isotropic medium.

INTRODUCTION

One approach to deal with the complexity of anisotropic wave propagation is to use perturbation techniques which are based on the approximation of an anisotropic medium by a simpler, analytically treatable, reference medium. Differences between both media are taken into account by adding corrections to the results obtained for the reference medium. These corrections can be of arbitrary order, theoretically. However, for practical applications mostly a first-order correction is used (see, e.g., Cervený, 1982). To minimize errors which are inherent in the low-order perturbation approach, one should choose the reference medium as close as possible to the true medium –

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with respect to the physical properties to be investigated.

The assumption of weak anisotropy in real subsurface structures is often valid and justifies an approximation by isotropic reference media. Formulas for the best fitting isotropic reference velocity were derived by Fedorov (1968) by minimizing the norm of differences between elastic coefficients of the anisotropic and the isotropic reference model and by Sayers (1994) by expanding slowness surfaces into spherical harmonics. However, media with ellipsoidal shape of the slowness surface and, therefore, of the group velocity surface, are appropriate for analytical calculations too and can be chosen closer to the true medium if a stronger velocity differences for the slow and fast direction are present. Lecomte (1993) computes traveltimes in two dimensions (2D) by approximating arbitrary symmetry of anisotropy by ellipsoids (i.e. ellipses in 2D). Traveltimes computed for this elliptical reference medium are accepted as traveltimes for the original medium. For this approach all quantities needed for fast finite-difference (FD) traveltime computation are analytically available. However, to achieve higher accuracy in case of strong anisotropy Lecomte rather uses orthorhombic reference media, therefore, involving (expensive) numerical calculations. Ettrich (1998) presents a 3D traveltime tool utilizing (analytically treatable) ellipsoidal media but compensating for stronger anisotropy by integrating a perturbation scheme into the finite-difference algorithm.

Recently, Mensch & Farra (1999) presented a scheme for ray tracing in orthorhombic media perturbing from ellipsoidal anisotropy. Their reference medium is an ellipsoid with main axes being parallel to the main axes of orthorhombic symmetry where velocities along these directions are chosen identical in both media.

To optimize the applications mentioned above we here address the problem of finding parameters of a best fitting ellipsoidal reference medium for an arbitrary anisotropic medium. The fit is performed in the sense that the propagation of P-waves is best approximated. Since in arbitrary anisotropic media group velocity is an extremely complicated function of the coefficients of elasticity this aim is synonymous with approximating the phase velocity best. The ellipsoidal model used by Mensch & Farra (1999) is a quite obvious one and turns out to be a reasonable choice. We will call it the “obvious ellipsoid” below. We will, however, demonstrate that more rigorously derived formulas lead to a superior approximation.

Our solution to derive a best-fitting ellipsoidal medium for a general anisotropic medium and its accuracy is described in the next sections. Alternatives to the approach chosen could be based on relations between elastic coefficients that make an orthorhombic medium ellipsoidal. However, utilizing these relations previously published by Burridge et al. (1993) either lead to non-linear relations in elastic coefficients or to intersecting slowness surfaces (i.e., the same wave type could be attributed to different ellipsoids for different directions).

Notation

- Wherever it is more convenient we use the Voigt-notation C_{IJ} (capital indices running from 1 to 6) for the density normalized tensor of elasticity rather than the four-indices tensor c_{ijkl} (non capital indices running from 1 to 3) with the usual correspondence: 11-1, 22-2, 33-3, 23-4, 13-5, 12-6.
- Averaging a function f is denoted by $< f >$. For f depending on angle of azimuth ϕ and angle of inclination θ (measured with respect to the vertical) the averaging is defined by $< f >= \frac{1}{A}\int\limits_0^{2\pi}\int\limits_0^{\theta_M} f(\phi,\theta)\sin\theta d\theta d\phi$ with $A = \int\limits_0^{2\pi}\int\limits_0^{\theta_M}\sin\theta d\theta d\phi$ and θ_M the maximum inclination.
- Normal vectors are defined by: $(n_1, n_2, n_3) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$.
- Summation over repeated indices is applied.
- δ_{kl} is the Kronecker-delta.

BEST-FITTING ELLIPSOIDAL REFERENCE MEDIUM

Following ray theory one finds for the phase velocity v of rays propagating in general anisotropic media (Cervený, 1972):

$$v^2 = c_{jklm} n_k n_l g_j g_m \quad \text{with} \quad \begin{array}{ll} c_{jklm} & \text{density normalized tensor of elast. coef.} \\ n_j & \text{phase normal vector} \\ g_j & \text{polarization vector} \end{array} \tag{1}$$

This equation is not appropriate for analytical calculations since v^2 and g_j are eigenvalue and eigenvector, respectively, of an eigenvalue problem. However, for weak anisotropy the phase velocity of the P-wave is well approximated by $\tilde{v}$ if the polarization vector is substituted by phase normal n_j in equation (1):

$$v^2 \approx \tilde{v}^2 = c_{jklm} n_j n_k n_l n_m . \tag{2}$$

Even if we consider strong anisotropy we will use equation (2) to define the anisotropic medium. It has be shown previously (Pšenčík and Gajewski, 1998), that this assumption is justified for a broad class of anisotropic models.

For ellipsoidal media the eikonal equation describing the P-wave slowness surface factorizes into an equation of the form

$$\hat{v}^2 = R_{jk} n_j n_k \,, \quad j,k = 1,2,3. \tag{3}$$

The square of phase velocities $\hat{v}$ is a simple polynomial of second-order in normal vector components.

The problem under consideration is to minimize the difference

$$I =< (\hat{v}^2 - \tilde{v}^2)^2 >=< \tilde{v}^4 > + < \hat{v}^4 > -2 < \hat{v}^2\tilde{v}^2 > \tag{4}$$

between $\tilde{v}^2$ of the general anisotropic medium and $\hat{v}^2$ of the ellipsoidal medium with respect to the unknown parameters $R_{11}, R_{22}, R_{33}, R_{12}, R_{13}, R_{23}$:

$$\frac{\partial}{\partial R_{jk}} < \hat{v}^4 >= 2\frac{\partial}{\partial R_{jk}} < \hat{v}^2\tilde{v}^2 > . \tag{5}$$

Here, brackets denote averaging over the entire sphere of angles θ and ϕ, i.e. maximum inclination $\theta_M = \pi$. We obtain:

$$< \hat{v}^4 >= R_{jk}R_{lm} < n_j n_k n_l n_m > \tag{6}$$

and

$$< \hat{v}^2\tilde{v}^2 >= R_{ij}c_{klmn} < n_i n_j n_k n_l n_m n_n > . \tag{7}$$

We are using Fedorov's (1968) technique to average the components of the normal vector (the derivation is not presented here, some details are found in Ettrich et al. (1999)). Equations for $< \hat{v}^4 >$ and for $< \hat{v}^2\tilde{v}^2 >$ are used for the minimization and after inserting into equation (5) we obtain for the coefficients R_{ij} of the best-fitting ellipsoid:

$$\begin{aligned}
R_{11} = & \ \tfrac{1}{35} & (27C_{11} + 8C_{12} + 8C_{13} + 16C_{66} + 16C_{55} \\
& & -4C_{44} - 3C_{33} - 2C_{23} - 3C_{22}) \\
R_{22} = & \ \tfrac{1}{35} & (27C_{22} + 8C_{12} + 8C_{23} + 16C_{66} + 16C_{44} \\
& & -4C_{55} - 3C_{11} - 2C_{13} - 3C_{33}) \\
R_{33} = & \ \tfrac{1}{35} & (27C_{33} + 8C_{13} + 8C_{23} + 16C_{55} + 16C_{44} \\
& & -4C_{66} - 3C_{22} - 2C_{12} - 3C_{11}) \\
R_{12} = & \ \tfrac{2}{7} & (3C_{16} + 3C_{26} + C_{36} + 2C_{45}) \\
R_{13} = & \ \tfrac{2}{7} & (3C_{15} + 3C_{35} + C_{25} + 2C_{46}) \\
R_{23} = & \ \tfrac{2}{7} & (3C_{42} + 3C_{43} + C_{14} + 2C_{56})
\end{aligned} \tag{8}$$

Diagonal elements R_{ii} of the matrix of the best-fitting ellipsoid are influenced by those coefficients of the anisotropic medium which define an orthorhombic medium while out-off diagonal elements account for the deviation from orthorhombic symmetry. Please note that the obvious ellipsoid of Mensch & Farra (1999) has the coefficients $R_{11} = C_{11}$, $R_{22} = C_{22}$, $R_{33} = C_{33}$ and $R_{12} = R_{13} = R_{23} = 0$.

Specifying the C_{IJ} for isotropic media with

$$C_{IJ}^{\mathrm{iso}} = \frac{1}{\rho}\begin{pmatrix}
\lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\
\lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\
\lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\
0 & 0 & 0 & \mu & 0 & 0 \\
0 & 0 & 0 & 0 & \mu & 0 \\
0 & 0 & 0 & 0 & 0 & \mu
\end{pmatrix}$$

formula (8) gives the expected result:

$$R_{11} = R_{22} = R_{33} = \frac{1}{\rho}(\lambda + 2\mu).$$

For transversely isotropic (TI) media defined by

$$C_{IJ}^{\mathrm{TI}} = \frac{1}{\rho}\begin{pmatrix} \lambda+2\mu & \lambda & \lambda-l & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda-l & 0 & 0 & 0 \\ \lambda-l & \lambda-l & \lambda+2\mu-p & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu-m & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu-m & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{pmatrix}$$

one can get

$$R_{11} = R_{22} = \frac{1}{\rho}\left(\lambda + 2\mu - \frac{6}{35}l - \frac{12}{35}m + \frac{3}{35}p\right)$$

$$R_{33} = \frac{1}{\rho}\left(\lambda + 2\mu - \frac{6}{35}l - \frac{12}{35}m - \frac{24}{35}p\right).$$

Perturbation p describes the difference between horizontal and vertical phase velocity of the TI medium and it is, therefore, the main parameter which affects this difference in the approximate formulas too. However, even if we have a TI medium with $l = m = 0$ and $p \neq 0$ we obtain an ellipsoid with horizontal and vertical phase velocity being not equal the original values, i.e., C_{11} and C_{33} . The best-fitting ellipsoid derived here does not equal an ellipsoidal input model. Reason is that we used a different ansatz than the one used by Burridge et al. (1993). In their approach the ellipsoidal approximation of parameters of an orthorhombic medium is explicitly considered.

ACCURACY OF GLOBAL APPROXIMATION

We consider media of two different symmetries to illustrate the accuracy of approximation (8). Firstly, we discuss transversely isotropic (TI) media, a shale and a mudshale, both selected from the table in Thomsen (1986). Secondly, we discuss a triclinic sandstone. Coefficients of elasticity are:

TI shale

$\epsilon = 0.2, \quad \delta = -0.075$

$$\begin{pmatrix} 15.96 & 6.99 & 6.06 & 0.00 & 0.00 & 0.00 \\ & 15.96 & 6.06 & 0.00 & 0.00 & 0.00 \\ & & 11.40 & 0.00 & 0.00 & 0.00 \\ & & & 2.22 & 0.00 & 0.00 \\ & & & & 2.22 & 0.00 \\ & & & & & 4.48 \end{pmatrix}$$

TI mudshale

$$\epsilon = 0.034, \quad \delta = 0.211$$

$$\begin{pmatrix} 15.87 & -1.91 & 15.67 & 0.00 & 0.00 & 0.00 \\ & 15.87 & 15.67 & 0.00 & 0.00 & 0.00 \\ & & 14.86 & 0.00 & 0.00 & 0.00 \\ & & & 8.12 & 0.00 & 0.00 \\ & & & & 8.12 & 0.00 \\ & & & & & 8.87 \end{pmatrix}$$

triclinic sandstone

$$\begin{pmatrix} 4.95 & 0.43 & 0.62 & 0.67 & 0.52 & 0.38 \\ & 5.09 & 1.00 & 0.09 & -0.09 & -0.28 \\ & & 6.77 & 0.00 & -0.24 & -0.48 \\ & & & 2.45 & 0.00 & 0.09 \\ & & & & 2.88 & 0.00 \\ & & & & & 2.35 \end{pmatrix}$$

(Numbers in $(km/s)^2$.)

In terms of Thomsen parameters both TI media are quite different, exhibiting strong P-wave anisotropy and a large positive anellipticity ($\epsilon - \delta$) for the shale while horizontal and vertical velocity are similar for the mudshale but anellipticity is large and negative. For the triclinic sandstone velocity surfaces are more irregular (see figures below) than could be guessed here from the relatively small non-orthorhombic coefficients.

Coefficients for the best-fitting ellipsoids using equation (8) for all examples are listed below:

shale

$$\begin{pmatrix} 15.42 & 0.00 & 0.00 \\ & 15.42 & 0.00 \\ & & 9.95 \end{pmatrix}$$

mudshale

$$\begin{pmatrix} 18.71 & 0.00 & 0.00 \\ & 18.71 & 0.00 \\ & & 22.47 \end{pmatrix}$$

sandstone

$$\begin{pmatrix} 5.10 & -0.05 & 0.27 \\ & 5.08 & 0.27 \\ & & 6.88 \end{pmatrix}$$

Non-diagonal elements accounting for deviation from orthorhombic symmetry occur only for the approximation of triclinic sandstone. Diagonal elements are quite close to C_{11}, C_{22}, and C_{33}, respectively, of the original models for the TI shale and the triclinic sandstone whereas we observe large differences for the TI mudshale. Reason is the strong negative anellipticity which has the effect that velocities are highest around the diagonal directions. To approximate these velocities well the ellipsoid has to be blown up resulting in a severe misfit for vertical and horizontal propagation (see also Figure 1). Figure 1 displays different phase velocities for the shale (left-hand side) and the mudshale (right-hand side) while Figure 2 is the corresponding figure for the sandstone for azimuth 0 deg. (left-hand side) and azimuth 60 deg. (right-hand side).

Black solid lines display phase velocities of the best fitting ellipsoid while the slightly differing dotted curves show phase velocities of the exact, approximate, average isotropic and obvious models. Please, note the proximity of the exact and approximate phase velocities, which justifies the ansatz to fit the ellipsoidal model to the approximate model instead of the exact model even in case of strong anisotropy (see also the remark after Eq. 2). It is obvious from the figures, that the ellipsoidal approximation gives a much better approximation to the exact model than the best isotropic approximation usually applied in perturbation techniques. Without any

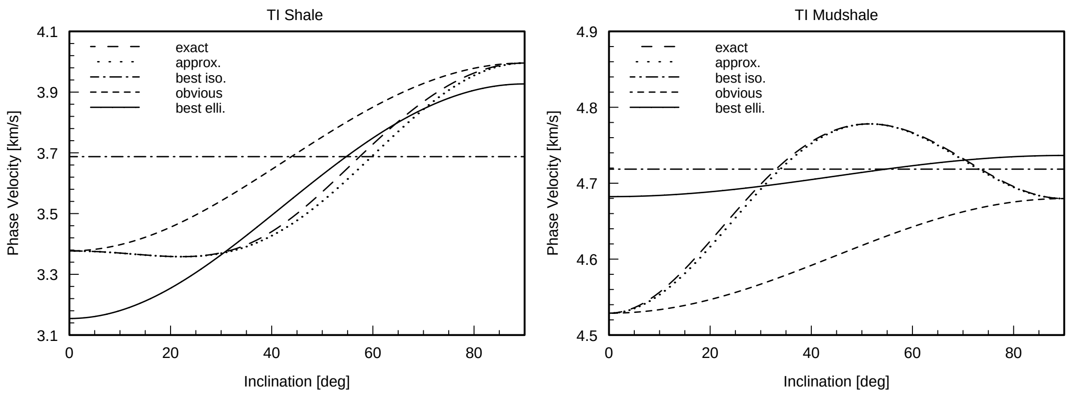

Figure 1: Plot of phase velocities for TI-Shale (left) and Mudshale (right). Exact phase velocity (*exact*), approximate (*approx.*) phase velocity, best isotropic approximation (*best iso*), best ellipsoidal approximation (*best elli.*), obvious ellipsoid (*obvious*).

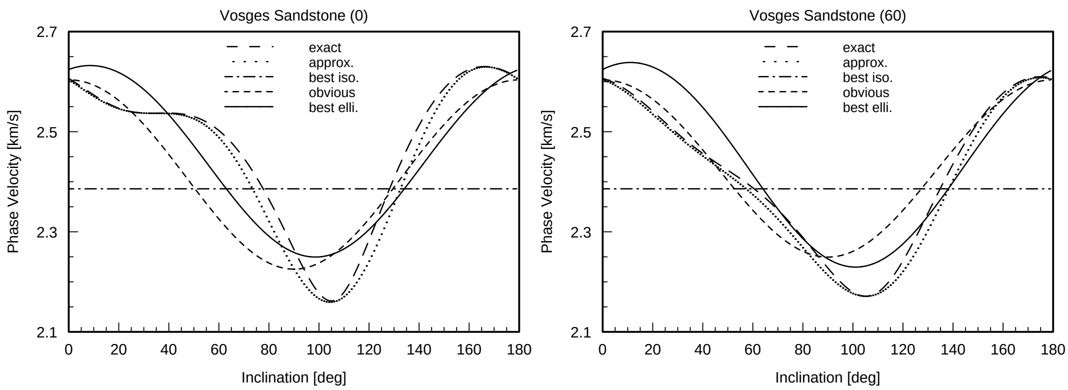

Figure 2: Phase velocities for triclinic sandstone along a 0 (left) and 60 deg. profile (right). For further explanations see Fig. 1.

mathematics the parameters for an ”obvious” ellipsoidal medium are obtained by just setting $R_{11} = C_{11}$, $R_{22} = C_{22}$, and $R_{33} = C_{33}$. For TI media such a choice leads to exact velocities for the horizontal and vertical direction. However, Figures 1 and 2 and the following table show that in average the derived formulas give best results.

	shale	mudshale	sandstone
ref. medium	average rel. err. of phase velocity	average rel. err. of phase velocity	average rel. err. of phase velocity
best fitting ellipsoid	1.4%	5.3%	2.1%
”obvious” ellipsoid	2.8%	12.4%	2.5%
isotropic	5.7%	6.1%	4.7%

In particular for the mudshale the “obvious” ellipsoid is not a good choice where even the best-fitting isotropic medium (abbreviated as “best iso.”) gives better results. The formulas for the best-fitting ellipsoid seem to give an especially good approximation for the TI shale with strong positive anellipticity while for the triclinic medium the better accuracy is less clear from Figure 2 but proven from the previous table. Note, that differences are stronger weighted the closer they are to horizontal direction of propagation. However, this property, inherent in the averaging formula owing to factor $\sin\theta$, explains the disadvantage of Eq. (8). While exhibiting an overall best fit it is near-vertical directions where the largest errors occur and where it is worse compared to the ”obvious” ellipsoid. If we restrict the averaging to a sector or cone around the vertical direction, this disadvantage can be overcome.

MODIFICATION OF THE METHOD

Motivated by the results of the previous section we now modify the algorithm to allow to select the angular section of interest where the fit should be best, e.g., within a cone or sector around the vertical axis. We, therefore, carry out the averaging in the form $< \hat{v}^4 >=< R_{jk}R_{\alpha\beta}n_jn_kn_\alpha n_\beta >$ and $< \hat{v}^2\tilde{v}^2 >=< R_{\alpha\beta}c_{jklm}n_\alpha n_\beta n_jn_kn_ln_m >$ rather than to extract coefficients of elasticity from the averaging as done in equations (6) and (7). We can change the order of first differentiating with respect to R_{ij} and then integrating over the angular section for the averaging. The minimization (see equation 5) becomes:

$$\left\langle R_{\alpha\beta}n_\alpha n_\beta \frac{\partial R_{kl}n_kn_l}{\partial R_{ij}} \right\rangle = \left\langle c_{\alpha\beta kl}n_\alpha n_\beta n_k n_l \frac{\partial R_{kl}n_kn_l}{\partial R_{ij}} \right\rangle . \qquad (9)$$

With the definition $B_{ij} := (\partial R_{kl}n_kn_l)/\partial R_{ij}$ equation (9) reads for each of the six combinations $i = (1,2,3), j = (1,2,3)$:

$$R_{\alpha\beta}\int_0^{2\pi}\int_0^{\theta_M} n_\alpha n_\beta B_{ij}\sin\theta d\theta d\Phi = c_{\alpha\beta kl}\int_0^{2\pi}\int_0^{\theta_M} n_\alpha n_\beta n_k n_l B_{ij}\sin\theta d\theta d\Phi . \qquad (10)$$

θ_M is the maximum angle of inclination, i.e. the opening angle of the conical section within the anisotropic medium should be approximated by the ellipsoid. It is not too cumbersome to carry out all the integrations by hand. With the recursive definition

$$\begin{aligned} S_1 &= 1-\cos\theta_M, \quad S_3 = \tfrac{2}{3}S_1 - \frac{\sin^2\theta_M\cos\theta_M}{3} \\ S_5 &= \tfrac{4}{5}S_3 - \frac{\sin^4\theta_M\cos\theta_M}{5}, \quad S_7 = \tfrac{6}{7}S_5 - \frac{\sin^6\theta_M\cos\theta_M}{7} \end{aligned} \tag{11}$$

we finally obtain equations for determining the R_{ij} for the sector or cone of interest:

$$\begin{pmatrix} \frac{3}{4}S_5 & \frac{1}{4}S_5 & S_3-S_5 \\ \frac{1}{4}S_5 & \frac{3}{4}S_5 & S_3-S_5 \\ S_3-S_5 & S_3-S_5 & 2S_1-4S_3+2S_5 \end{pmatrix} \begin{pmatrix} R_{11} \\ R_{22} \\ R_{33} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$\begin{aligned} R_{12} &= \frac{(4C_{45}+2C_{36})(S_5-S_7)+(C_{16}+C_{26})S_7}{S_5} \\ R_{13} &= \frac{(C_{46}+\frac{1}{2}C_{25}+\frac{3}{2}C_{15})(S_5-S_7)+2C_{35}(S_3-2S_5+S_7)}{S_3-S_5} \\ R_{23} &= \frac{(C_{56}+\frac{1}{2}C_{14}+\frac{3}{2}C_{24})(S_5-S_7)+2C_{34}(S_3-2S_5+S_7)}{S_3-S_5} \end{aligned} \tag{12}$$

with

$$\begin{aligned} b_1 = {} & \tfrac{5}{8}C_{11}S_7 + \tfrac{1}{8}C_{22}S_7 + C_{33}(S_3-2S_5+S_7) + \\ & \tfrac{1}{8}(2C_{12}+4C_{66})S_7 + \tfrac{3}{4}(2C_{13}+4C_{55})(S_5-S_7) \\ & \tfrac{1}{4}(2C_{23}+4C_{44})(S_5-S_7) \\ b_2 = {} & \tfrac{1}{8}C_{11}S_7 + \tfrac{5}{8}C_{22}S_7 + C_{33}(S_3-2S_5+S_7) + \\ & \tfrac{1}{8}(2C_{12}+4C_{66})S_7 + \tfrac{1}{4}(2C_{13}+4C_{55})(S_5-S_7) + \\ & \tfrac{3}{4}(2C_{23}+4C_{44})(S_5-S_7) \\ b_3 = {} & \tfrac{3}{4}C_{11}(S_5-S_7) + \tfrac{3}{4}C_{22}(S_5-S_7) + 2C_{33}\,(S_1-3S_3+3S_5-S_7) + \\ & \tfrac{1}{4}(2C_{12}+4C_{66})(S_5-S_7) + (2C_{13}+4C_{55})(S_3-2S_5+S_7) + \\ & (2C_{23}+4C_{44})(S_3-2S_5+S_7) \end{aligned} \tag{13}$$

Note that the coefficients of the best-fitting ellipsoid defined by equations (11), (12), and (13) depend on the same elastic coefficients of the anisotropic medium as in equation (8). Now, the weights become θ_M-dependent. For $\theta_M = \pi$ we obtain the same solution as with Eqs. (8).

ACCURACY OF SECTORIAL APPROXIMATION

We now apply equations (11)-(13). Compared to Figures 1 and 2, Figures 3 and 4, respectively, clearly demonstrate the improvement within a cone of 30 or 45 deg. around the vertical when restricting the averaging to the corresponding interval of angles. For applications where a 30 deg. cone of propagation is sufficient even the strongly

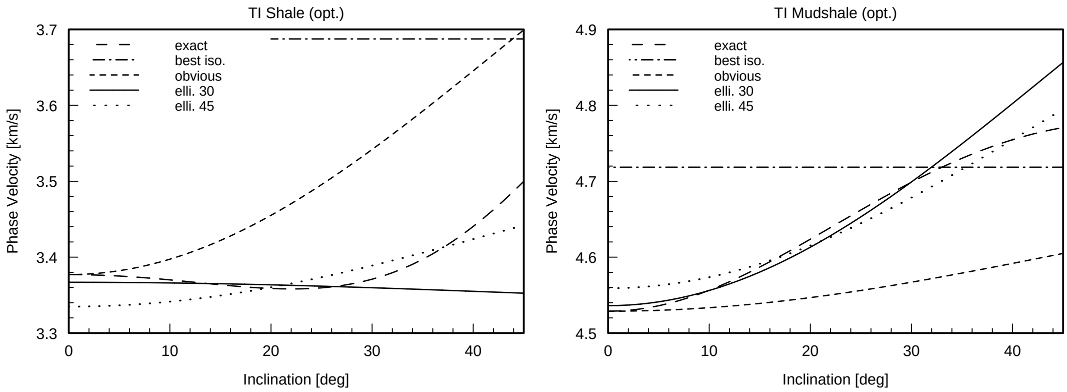

Figure 3: Phase velocities for TI-Shale (left) and Mudshale (right) optimized for a 30 deg. cone (*elli.* 30) and a 45 deg. cone (*elli.* 45). For further explanations see Fig. 1.

non-orthorhombic triclinic sandstone is well approximated (Figure 4). However, it is important that in case of an angular restriction no propagation outside the limited range of directions should be included in possible applications because the approximation degrades here. Again, it is even more obvious for the sectorial approximation that the ellipsoidal model is a much better approximation to the exact medium than the average isotropic model. Thus, perturbation techniques using the ellipsoidal approximation as a background medium will perform superior to the isotropic background medium, particularly if a strong P-wave anisotropy is present.

CONCLUSIONS

Coefficients for the approximation of an arbitrary anisotropic medium by an ellipsoidal medium were derived. Numerical examples have demonstrated the better accuracy of the ellipsoidal model when compared with the average isotropic model. The approximation is particularly superior if the averaging is carried out in the sector of interest (i.e., for seismic wave propagating prominently in the vertical direction). The ellipsoidal approximation allows the use of perturbation techniques even in situations of strong P-wave anisotropy (e.g., for shales). Using the ellipsoidal approximation as a background medium allows the application of the perturbation approach to a wider class of anisotropic models without nearly the same computational efficiency.

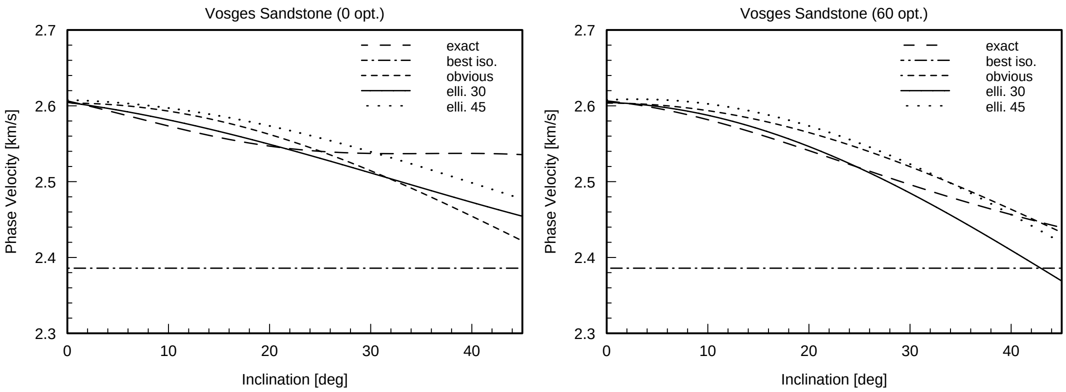

Figure 4: Phase velocities for triclinic sandstone along a 0 (left) and 60 deg. profile (right) optimized for a 30 deg. cone (*elli.* 30) and a 45 deg. cone (*elli.* 45). For further explanations see Fig. 1.

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