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Restricted optimization: a clue to a fast and accurate implementation of the Common Reflection Surface Stack method

E. G. Birgin, R. Biloti, M. Tygel, and L. T. Santos[1]

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ABSTRACT

For a central ray in an isotropic elastic or acoustic media, traveltime moveouts of rays in its vicinity can be described in terms of a certain number of parameters that refer to the central ray only. In 2-D propagation, the traveltime expressions depend on three parameters directly related to the geometry of the unknown model in the vicinity of the central ray. We present a new method to extract these parameters out of coherency analysis applied directly to the data. It uses (a) fast one-parameter searches on different sections extracted from the data to derive initial values of the parameters, and (b) the application of a Spectral Projected Gradient optimization algorithm for the final parameter estimation. The results obtained so far indicate that the algorithm may be a feasible option to solve the corresponding, harder, full three-dimensional problem, in which eight parameters, instead of three, are required.

INTRODUCTION

Traveltimes of rays in the paraxial vicinity of a fixed central ray can be described by a certain number of parameters that refer to the central ray only. They are valid independently of any seismic configuration.

Assuming the central ray to be the primary zero-offset ray, the number of parameters are three and eight, for two- and three-dimensional propagation, respectively. For 2-D propagation, the parameters are the emergence angle of the normal ray and the wavefront curvatures of the normal and normal-incident-point eigenwaves, as introduced in Hubral (1983). All parameters are defined at the point of emergence of the central ray, called the central point. This point coincides with a common midpoint (CMP), where the simulated zero-offset trace is to be constructed.

[1] **email:** ernesto@ime.unicamp.br

The use of multi-parametric traveltime approximations for imaging purposes is a well-investigated subject. Main contributions are *Multifocusing* (see, e.g., Gelchinsky et al., 1997, for a recent description), *PolyStack*^{TM} (see, e.g., de Bazelaire et al., 1994) and the very recent *Common Reflection Surface* (CRS) method (see, e.g., Hubral et al., 1998). These methods vary in general on two aspects, namely, the multi-parametric traveltime moveout formula that is used, as well as in the strategy to extract the traveltime parameters from coherency analysis applied on the multi-coverage data.

The basic lines of the CRS approach are the choice of the hyperbolic traveltime function (see Tygel et al., 1997), and the strategy of breaking the original three-parameter estimation problem into simpler ones involving one or two unknowns. As shown in Müller (1999), quick estimations for the three parameters are obtained by simple one-parameter searches performed on CMP and CMP-stacked sections of the data.

To improve the accuracy of the estimations, as needed, e.g., for the construction of velocity models, a natural idea is to use the previously obtained parameter estimations as initial values for an optimization scheme directly applied to the multi-coverage data problem. Following this philosophy, Müller (1999) obtained significantly better results on synthetic data examples, however, at a high computational cost.

In this work, we present a new optimization strategy so as to achieve more accurate results than the ones derived by purely one-parameter searches, while maintaining the computational effort at a reasonable level. This becomes a crucial matter when real-data applications are envisaged. The method is illustrated by its application on a synthetic example, where the various aspects of the algorithm can be better understood.

HYPERBOLIC TRAVELTIME EXPANSION

As shown in Figure 1, let us assume a fixed target reflector Σ in depth, as well as a fixed *central point* X_0 on the seismic line, considered to be the location of a coincident source- and -receiver pair $S_0 = G_0 = X_0$. The corresponding zero-offset reflection ray, X_0 NIP X_0, will be called from now on the *central ray*. It hits the reflector at the *normal-incident-point* (NIP). For a source-receiver pair (S, G) in the vicinity of the central point, we consider the primary reflected ray SRG relative to the same reflector Σ. We use the horizontal coordinates x_0, x_S and x_G to specify the location of the central point X_0, the source S and the receiver G, respectively. We find it convenient to introduce the *midpoint* and *half-offset* coordinates $x_m = (x_G + x_S)/2 - x_0$ and $h = (x_G - x_S)/2$. We consider the hyperbolic traveltime expression as in Tygel et al. (1997)

$$T^2(x_m, h; \beta_0, K_N, K_{NIP}) = \left(t_0 + \frac{2x_m \sin\beta_0}{v_0}\right)^2 + \frac{2t_0 \cos^2\beta_0}{v_0}\,(K_N\, x_m^2 + K_{NIP}\, h^2)\,, \tag{1}$$

where t_0 is the zero-offset traveltime and β_0 is the angle of emergence at the zero-offset ray with respect to the surface normal at the central point. The quantities K_N and K_{NIP} are the wavefront curvatures of the *normal* N-wave and the NIP-wave, respectively, measured at the central point (Hubral, 1983).

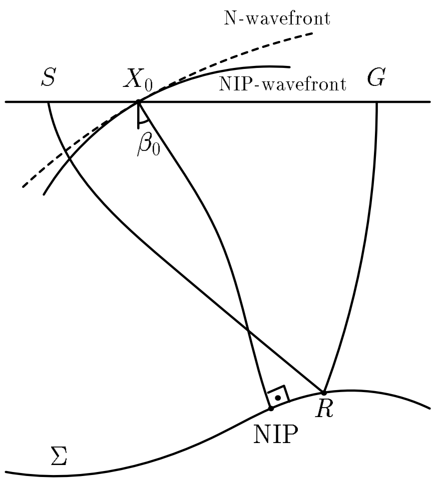

Figure 1: Physical interpretation of the hyperbolic traveltime formula parameters: Emergence angle, β_0, normal-wave curvature, K_N, and normal-incident-point-wave curvature, K_{NIP}.

For particular source-receiver gathers, the hyperbolic traveltime formula (1) can be simplified. The most used configurations are:

The common-midpoint configuration: Setting the fixed midpoint to coincide with the central point, the CMP-traveltime expression can be readily obtained from the hyperbolic traveltime (1) by simply placing $x_m = 0$ in that formula. We find the 1-D expression

$$T^2_{CMP}(h;q) = t_0^2 + \frac{2t_0\ h^2 q}{v_0}\ , \tag{2}$$

on the combined parameter $q = \cos^2\beta_0\,K_{NIP}$.

The zero-offset configuration: The zero-offset traveltime expression is readily obtained setting $h = 0$ in the hyperbolic traveltime (1). We find the 2-D expression

$$T^2_{ZO}(x_m;\beta_0,K_N) = \left(t_0 + \frac{2x_m\sin\beta_0}{v_0}\right)^2 + \frac{2t_0\cos^2\beta_0}{v_0}\,K_N\ x_m^2\ , \tag{3}$$

on the original parameters β_0 and K_N.

The common-shot configuration: Placing the common source to coincide with the central point, the common-shot traveltime expression is derived by setting $x_m = h$ in the hyperbolic traveltime (1). As a result, the traveltime expression becomes the 2-D formula

$$T^2_{CS}(h;\beta_0,\mu) = \left(t_0 + \frac{2h\sin\beta_0}{v_0}\right)^2 + \frac{2t_0\cos^2\beta_0}{v_0}\,\mu\ h^2\ , \tag{4}$$

depending on the original and combined parameters β_0 and $\mu = K_N + K_{NIP}$, respectively.

The common-offset configuration: The expression of the common-offset traveltime coincides with the general hyperbolic traveltime (1) upon the consideration of $h = constant$.

FORMULATION OF THE PROBLEM AND ITS SOLUTION

The multi-coverage data consists of a multitude of seismic traces $U(x_m, h, t)$ corresponding to source-receiver pairs located on a given seismic line by coordinate pairs (x_m, h), and recording time $0 < t < T$. Our problem is the following:

> Consider a dense grid of points (x_0, t_0), where x_0 locates a central point X_0 on the seismic line and t_0 is the zero-offset traveltime. For each central point X_0, let the medium velocity $v_0 = v(x_0)$ be known. From the given multi-coverage data, determine the corresponding parameters β_0, K_N and K_{NIP}, for any given point (x_0, t_0) and velocity v_0.

One approach to solve this problem could be the application of a multi-parameter coherency analysis to the data, using the traveltime formula (1) to a number of selected traces around X_0 and for a suitable time window around t_0. The desired values of the parameters are expected to be close to the ones for which the maximum coherence is achieved.

Given the seismic traces $U(x_m, h, t)$, and the vector of parameters $P = (\beta_0, K_N, K_{NIP})$, the coherency measure called *semblance* is given by

$$S = \frac{\sum [\sum U(x_m, h, T(x_m, h; P))]^2}{M \sum \sum [U(x_m, h, T(x_m, h; P))]^2} \, , \tag{5}$$

where $T(x_m, h; P) = T(x_m, h; \beta_0, K_N, K_{NIP})$ is the traveltime expression (1) and M is the total number of selected traces. The inner summation is performed over all selected traces, and the outer one is performed over a given time window around t_0. For each given pair (x_0, t_0), the objective is to find the global maximum of the semblance function (5) with respect to the parameters β_0, K_N and K_{NIP}. These parameters are restricted to the ranges $-\pi/2 < \beta_0 < \pi/2$ and $-\infty < K_N, K_{NIP} < \infty$.

To compute the global maximum of the semblance function, we propose the strategy described by the flow chart in Figure 2. In the first part we obtain initial values of the parameters. In the second part, an optimization process employs these parameters as initial values to produce the final estimations. Following the same lines as Müller

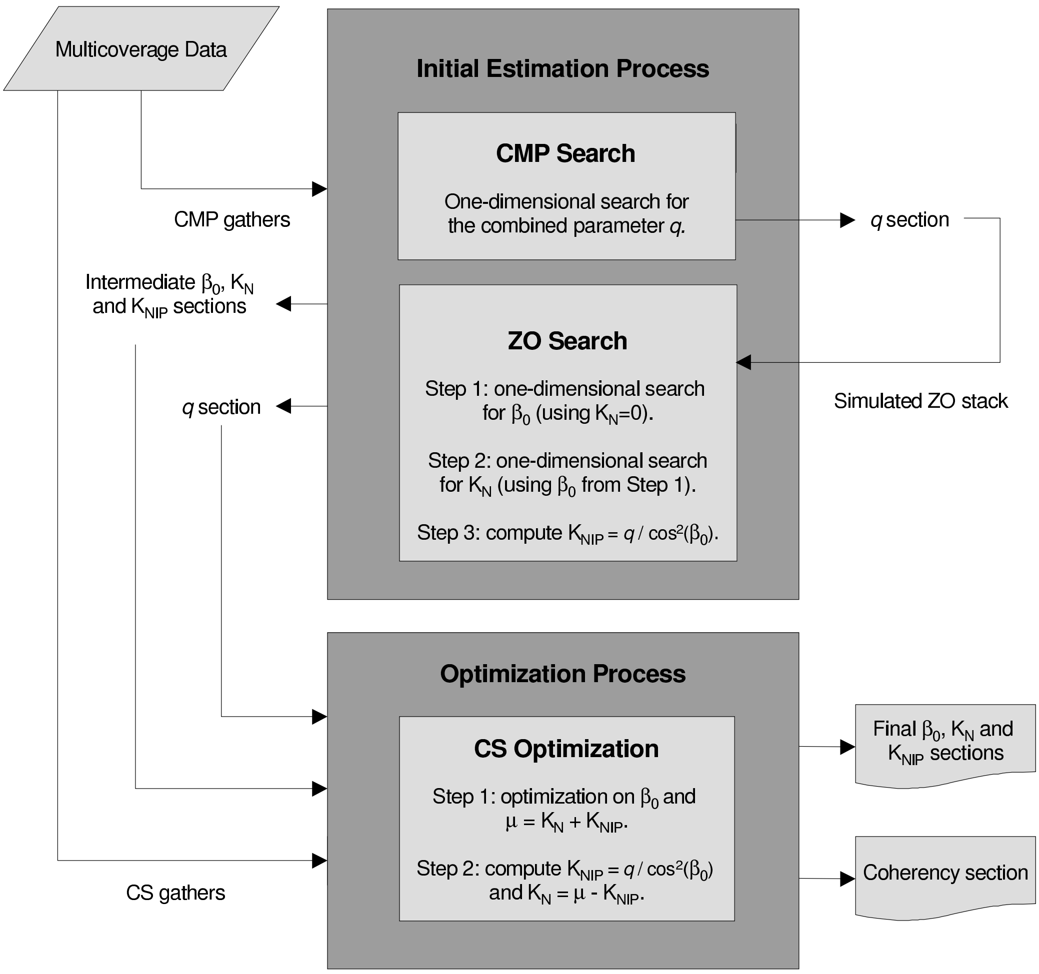

Figure 2: Flow-chart description of the parameter-estimation strategy. First part: Computation of initial estimations by one-parameter searches. Second part: Optimization method applied to common-shot sections for final parameter estimation.

(1999), the first part consists of two steps, namely, (a) a one-parameter search of the combined parameter q, performed on the CMP sections with the help of the traveltime expression (2), and (b) two one-parameter searches for β_0 and K_N, performed on the CMP-stacked section realized using the previous q-parameter. The CMP-stacked section is considered as an approximate zero-offset section, so the traveltime expression (3) is used.

The optimization process of the second part β_0 and $\mu = K_N + K_{NIP}$. For this purpose, we use the Spectral Projected Gradient (SPG) method (see Birgin et al. (1999)) applied to common-source sections. We use the traveltime expression (4) to obtain the original parameter β_0 and the combined parameter μ. Finally, using the relationships $K_{NIP} = q/\cos^2\beta_0$ and $K_N = \mu - K_{NIP}$ all the desired parameters can be determined.

A SYNTHETIC EXAMPLE

Referring to Figure 3, we consider the synthetic 2-D model of three smoothly curved reflectors separating different homogeneous acoustic media. Assuming unit density, the constant velocities are: $c_1 = 1400$m/s above the first reflector, $c_2 = 2000$m/s between the first and the second reflector, $c_3 = 3400$m/s between the second and the third reflector, and, finally, $c_4 = 5500$m/s below the deepest reflector. The input data

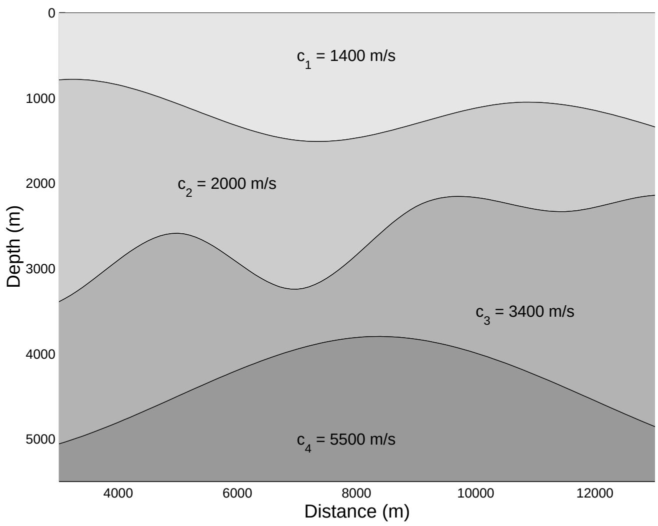

Figure 3: Synthetic two-dimensional model of four homogeneous acoustic layers separated by smooth curved interfaces. Unit density is assumed in all media.

for our experiment are 334 CMP seismic sections, centered at coordinates x_0 varying from 3010m to 13000m. Each CMP gather has 84 traces with half-offsets varying from

0m to 2490m. All traces are sampled within the range of 0s $\leq t \leq$ 6s, at a sample rate of 4ms. Noise was added to the data with a ratio signal:noise of 7:1.

Initial estimation – the combined parameter q: We start with the estimation of the combined parameter q, performed as a one-parameter search on the CMP gathers. The situation is similar to a conventional NMO-velocity analysis. For each fixed midpoint x_0, we determine, for each time sample t_0, the value of q that yields the best semblance in the CMP gather. For this computation, we use the CMP-traveltime formula (2) that depends on the q-parameter only. This leads to the construction of two auxiliary CMP-related sections, namely, the *q-section*, which consists of assigning to each (x_0, t_0) its corresponding q-parameter, and the *semblance section* in which the semblances are assigned. An extensive use of these auxiliary sections is described in Gelchinsky et al. (1997).

The q-search may be refined for greater accuracy. We consider the estimated q-parameters for which the current semblance values exceed a threshold that is interactively selected by the user. This provides an ensemble of q-values concentrated on a smaller range (in our case three orders of magnitude less than the original range search). It allows us to perform a new search, restricted to this smaller range divided into a much finer grid. Figure 4 shows the semblance section obtained after the refinement. The employed threshold semblance values were 0.13 and 0.15 for the time intervals $0s < t_0 < 2.5s$ and $2.5s < t_0 < 6s$, respectively. The very clear semblance section of Figure 4 can be looked upon as a simulated zero-offset section. The theoretical and estimated values of the combined parameter q along the reflectors are shown in Figure 5. The accurate results confirm the expectations of employing an exhaustive search to solve a 1-D problem. The obtained values of the q-parameter will be retained during the whole process.

Initial estimation – the parameters β_0, K_N and K_{NIP}: Using the just estimated q-values in the CMP-traveltime formula (2), we construct (like in conventional NMO-stacking) the corresponding CMP-stacked section. This will now be used as an approximation of a zero-offset section. To extract the emergence angles β_0 and the N-wave curvatures K_N, we proceed as follows: (a) Using the zero-offset traveltime expression (3), we first set $K_N = 0$ and perform, for each pair (x_0, t_0), a one-parameter search for β_0 between $-\pi/2$ and $\pi/2$; (b) Setting the obtained value of the β_0 parameter in the same zero-offset traveltime expression (3), we perform a further one-parameter search, this time for the parameter K_N. Use of the above results, together with the relationship $K_{NIP} = q/\cos^2\beta_0$, completes the initial estimations of the parameters β_0, K_N and K_{NIP}.

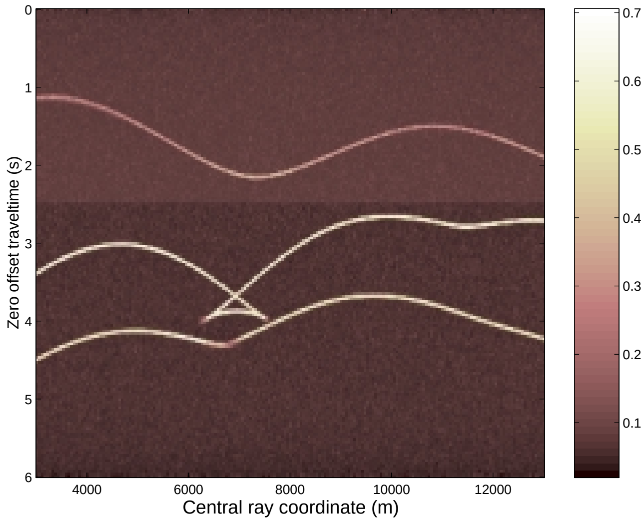

Figure 4: Semblance section obtained as a result of the one-parameter search of the combined parameter q. Note the excellent resolution of the section.

Optimization procedure – Final estimations: The second part of our method consists of the application of an optimization algorithm to common-shot sections. We use the common-shot traveltime formula (4), depending on the two parameters β_0 and $\mu = K_N + K_{NIP}$. From the previous initial estimation of the parameters, we apply the SPG optimization method (see Birgin et al., 1999) to achieve the final estimations.

Figures 6, 7 and 8 show the comparison between the theoretical and optimized parameters. We can recognize that the method provides generally accurate estimations in most of the section. We note, however, that the method also yields inaccurate results at various points within the range [6000m,8000m]. These points are characterized by small coherence measures and, for that matter, have not been displayed in Figures 6, 7 and 8. The reasons for those small coherence values may be (a) lack of illumination: use of end-on, common-shot gathers may not be the most adequate choice of illumination for the whole section. (b) Caustics: the same region contains a caustic due to the second reflector.

A possible improvement of the results could be obtained upon the combined use of traces that belong to different gathers (e.g, split-spread common-shot and common-offset gathers). The use of additional gathers may be recommended to overcome these difficulties. These aspects are under investigation.

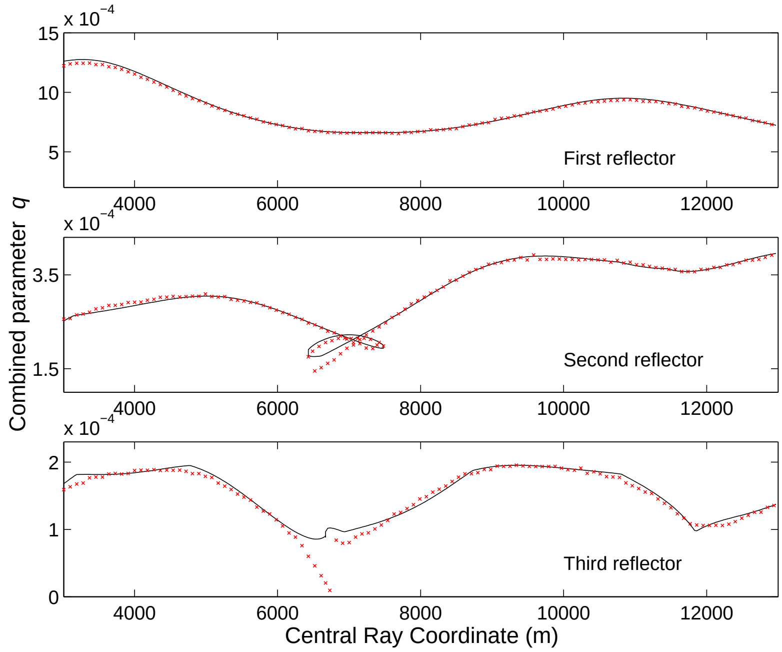

Figure 5: Combined parameter q: Theoretical curve (solid line) and estimated values (small x) obtained after the one-parameter search on the CMP sections.

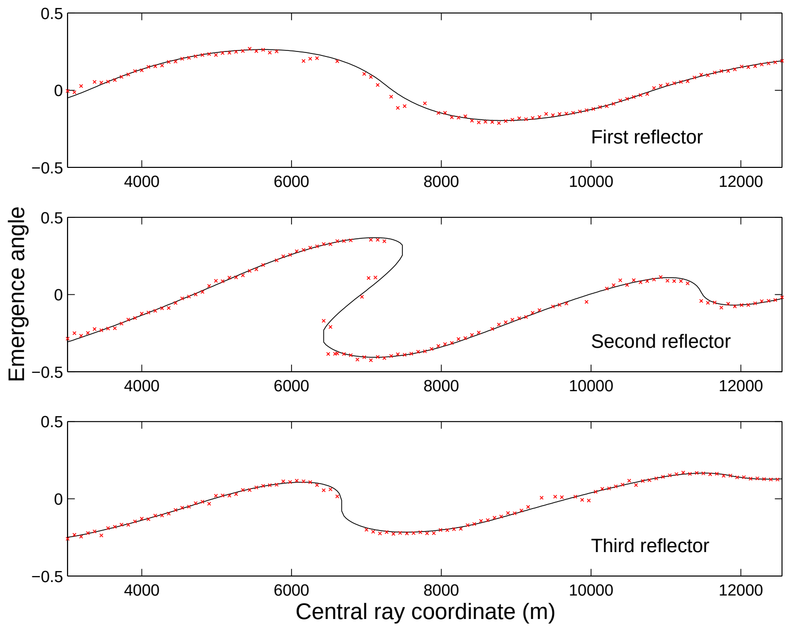

Figure 6: Emergence angle β_0: Theoretical curve (solid line) and estimated values (small x) obtained after the two-parameter optimization on the common-shot sections.

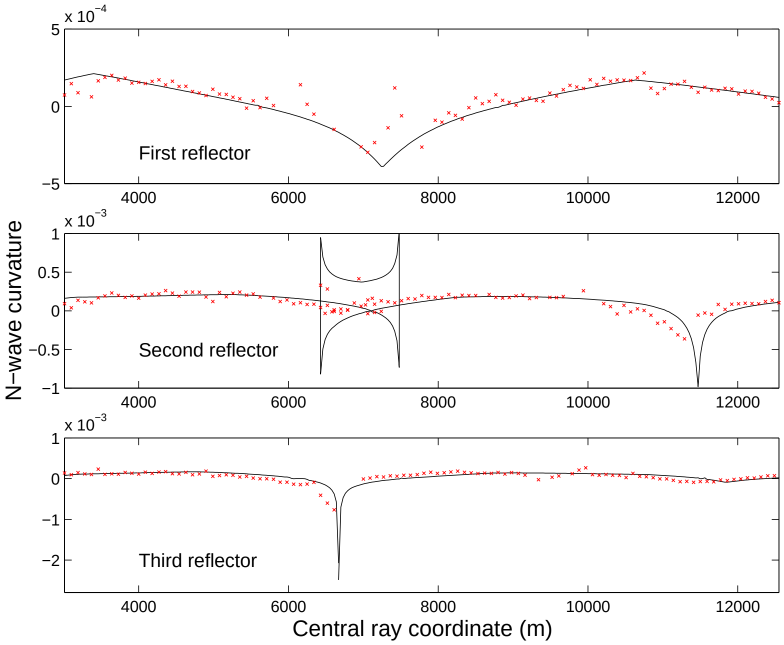

Figure 7: N-wave curvature K_N: Theoretical curve (solid line) and estimated values (small x) obtained after the two-parameter optimization on the common-shot sections.

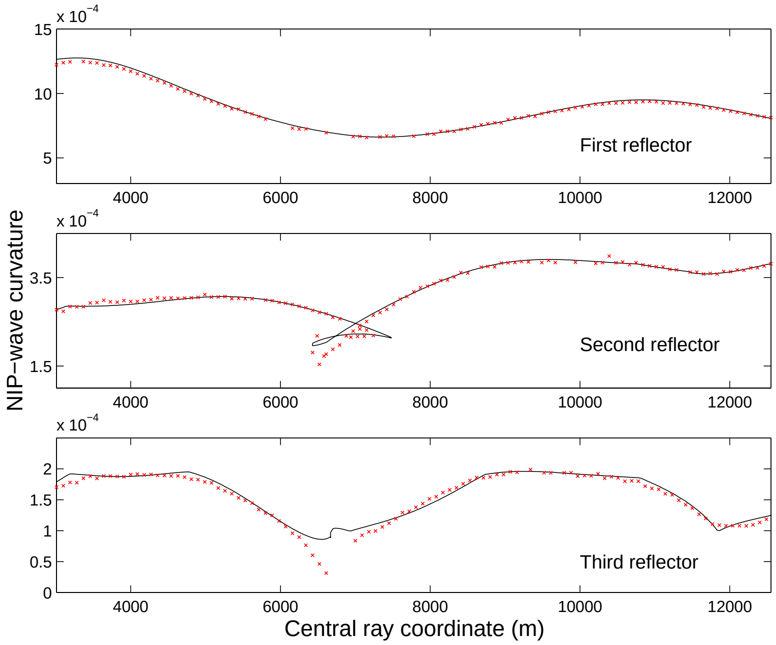

Figure 8: NIP-wave curvature K_{NIP}: Theoretical curve (solid line) and estimated values (small x) obtained after the two-parameter optimization on the common-shot sections.

CONCLUSIONS

We have proposed a new algorithm to determine the traveltime parameters out of coherency analysis applied to 2-D multi-coverage seismic data. Following the general philosophy of the CRS approach, we used the hyperbolic traveltime moveout together with a sequential application of one-parameter searches, followed by a two-parameter optimization scheme. The restriction of the two-parameter optimization to common-shot sections leads to a fast and generally accurate estimation of all three parameters.

We applied the algorithm to a three-reflector synthetic example. Although this is a simple model, it presents already some of the basic complications of more realistic situations. The obtained results were very encouraging, confirming our expectations concerning accuracy improvements at reasonable computational costs. Next steps will be to test the new algorithm on more complex models and to real data sets.

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PUBLICATIONS

Detailed results are in Birgin et al. (1999b).