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Analysis of an acoustic wave equation for cylinder symmetric media (2.5D)

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ABSTRACT

In this work, we study three-dimensional (3D) acoustic wave propagation in a vertical plane representing a medium the properties of which do not depend on the orthogonal direction to the plane. This problem arises from the need to describe seismic data provenient from the standard acquisition geometry: the two-dimensional (2D) seismic line. Any modeling or inversion method based on such data must rely on some hypothesis permitting this geometric restriction. The simplest one is to consider the seismic wave propagation to occur only in the vertical plane containing the seismic line. This hypothesis implies the assumption that the medium parameters do not change in the orthogonal direction to the plane. In practice, of course, this means that the medium variations in the out-of-plane direction are so small that the resulting effects may be neglected. Such a medium is, in effect, a 2D medium or, from the 3D point of view, this medium has cylinder symmetry. The main goal of this thesis is to study the wave propagation in this vertical plane which contains the seismic line.

Due to the high cost of 3D modeling, there is a major interest in making use of the medium symmetry and applying a 2D modeling scheme to describe the situation. However, as is well-known (Bleistein, 1986), modeling by the 2D wave equation is not sufficient to describe the true 3D wave propagation, although the traveltimes of the waves propagating within the plane of consideration can be modeled correctly. The reason is that the geometrical spreading of a seismic wave generated by a point source is a 3D quantity that cannot be modeled by the 2D wave equation. Therefore, as the medium and the kinematics of the problem are 2D but the dynamics are 3D, the problem is generally referred to as 2.5D (Bleistein, 1986).

Because of the problems described above, 3D modeling seems unavoidable for the correct description of 2.5D wave propagation. However, several approaches exist in the literature to overcome these difficulties. On the one hand, 3D methods can be adapted to the 2.5D problem, i.e., they can be sped up making use of the medium

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symmetry analytically. Examples are ray theory and Kirchhoff modeling as described by Bleistein (1986). On the other hand, 2D methods can be corrected for the wrong amplitude treatment of 3D waves, as, e.g., the amplitude correction of the 2D finite difference solution also suggested by Bleistein (1986).

There is another 2D concept which tries to overcome the difficulty of the 2.5D problem. It is the concept of a partial differential equation called the 2.5-dimensional wave equation. Such an equation would correctly model the in-plane wave propagation, taking advantage of the kinematic part of 2D propagation (which is correct) but treating the amplitudes as required in 3D. From this ambivalence (2D traveltimes and 3D amplitudes) results the attribute 2.5D.

One can precisely define the 2.5D wave equation as a partial differential equation with the following properties:

- The 2.5D wave equation is in fact 2D, i.e., it depends on two spatial (x,z) and one temporal (t) coordinates;
- The equation has to simulate 3D in-plane waves, i.e., the waves propagating exclusively within the plane vertically below the receivers, but with a 3D amplitude behavior.

In short, we may say

> *"A 2.5D wave equation simulates 3D wave propagation with the cost of 2D modeling."*

A 2.5-dimensional wave equation has the following advantage over other methods: It is not necessary to consider elementary events separately. Reflected waves, refracted waves, diffractions, multiples, among other events are all considered in the 2.5D simulation. The contrary is true in all other 2.5D methods mentioned above, where one must define which events are to be modeled.

Note that the 2.5D wave equation describes wave propagation correctly in the 2.5D situation only. This equation is not correct in any plane imbedded in space that is not a symmetry plane.

To take advantage of the symmetry of the 2.5D problem, Liner (1991) devised an approximate 2.5D wave equation suitable to the finite difference method. For the derivation of his equation, Liner (1991) relies on two basic assumptions. He assumes the medium to be homogeneous in the close vicinity of the source and he assumes the source pulse to be short. Furthermore, he conjectures that the validity of the obtained equation can be extrapolated to the region farther from the source, where the medium is no longer homogeneous.

Williamson and Pratt (1995) show that Liner's equation is mathematically not correct. In this thesis, we study its numerical validity, i.e., whether its modeling results, although incorrect, remain a good approximation for the 3D waves in a 2D medium. In other words, it is our aim to verify whether 2.5D modeling by Liner's equation is still a worthwhile alternative to other 2.5D methods. A similar investigation was carried out by Bording and Liner (1993).

The numerical analysis was divided into two parts, considering separately the modeling of traveltimes and amplitudes. The dynamic comparison was restricted to peak amplitudes letting aside possible differences between the modeled pulse forms. In all cases, the traveltime analysis showed satisfactorial coincidence between all three methods. All errors remained below 0.5%.

Concerning amplitudes, Liner's equation yields good results in numerical experiments for relatively simple models, in particular for constant velocity. The difference between Liner's equation and ray theory varied between 3% for a planar reflector and 10% for a more complex one.

These results, although obtained for small models, can be without doubt extended to more realistic models since the theoretical analysis shows that for this case Liner's amplitudes are correct in the sense of a zero-order ray approximation.

For a constant vertical velocity gradient, the amplitude error went up to about 15%. The theoretical discussion using asymptotic expansions (Stockwell, 1995) explains this value as the ratio between the velocity near the source and the RMS-velocity along the ray. These results are depicted in greater detail in the Figure 1. It shows the relative error of modeling of a direct or a reflected vertical wave in a medium with a constant vertical velocity gradient. We see that the errors for a negative and positive velocity gradient are on opposite sides of the axis. This seems to suggest that the error of a reflected wave should have canceling contribution from the upgoing and downgoing ray. This is, however, not true. In fact, the error of the reflected wave is identical to that of direct wave.

In all experiments, the difference between ray theory and Liner's equation was greater than the one between the latter equation and the amplitude-corrected 2D wave equation. This might be due to the fact the both equations are solved with an identical FD scheme, thus giving rise to the same types of numerical errors.

CONCLUSION

In conclusion, Liner's equation is a worthwhile alternative for the modeling of 3D wave propagation in 2.5D situations. Although slightly erroneous in the resulting amplitudes, it remains a good approximation for models not too complex. Its amplitudes are an order of magnitude better approximations to the true 2.5D amplitudes than those

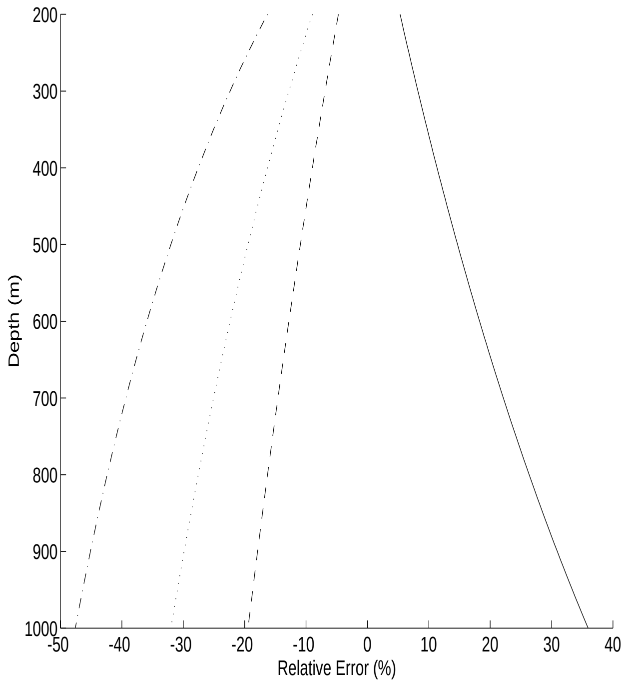

Figure 1: Relative error of a vertical wave in a medium with a constant vertical velocity gradient. Direct wave recorded at depth z or wave reflected at depth z and recorded at $z = 0$. Continuous line – gradient of -0.0005 ms^{-1}; dashed line – gradient of 0.0005 ms^{-1}; dotted line – gradient of 0.001 ms^{-1}; dashed-dotted line – gradient of 0.002 ms^{-1}.

of the 2D wave equation obtained with the same computational cost. If a good approximation for the ray parameter σ (Bleistein, 1986) can be obtained, the amplitudes of Liner's equation can be further corrected, without the need for an additional pulse-form correction as is necessary in the amplitude correction of the 2D wave equation. Future research is suggested on possible improvements of Liner's equation for inhomogeneous media.

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