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Numerical Analysis of Two and One–Half Dimensional (2.5–D)True–Amplitude Diffraction Stack Migration

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ABSTRACT

By considering arbitrary source-receiver configurations the compressional primary reflections can be imaged into time or depth-migrated reflections so that the migrated wavefield amplitudes are a measured of angle-dependent reflection coeffients. In order to do this various migration algorithms were proposed in the recent past years based on Born or Kirchhoff approach. Both of them treats of a weighted diffraction stack integral operator that is applied to the input seismic data. As result we have a migrated seismic section where at each reflector point there is the source wavelet with the amplitude proportinal to the reflection coefficient at that point. Based on Kirchhoff approach, in this paper we derive the weight function and the diffraction stack integral operator for the two and one half (2.5-D) seimic model and apply it to a set of synthetic seismic data in noise enviroment. The result shows the accuracy and stability of the 2.5-D migration method as a tool for obtaining important informations about the reflectivity properties of the earth subsurface, which is of great interest for the amplitude versus offset (angle) analysis.

INTRODUCTION

In the recent past years we have seen through various published papers an increasing interest in true amplitude migration methods, in order to obtain more informations about the reflectivity properties of the earth subsurface. The most part of these works has treated of this thema either based on Born approximation as given by Bleistein (1987) and Bleistein et al. (1987), or on ray theoretical wavefield approximation as given by Hubral et al. (1991), Schleicher et al. (1993) and Martins et al. (1997).

This paper follows the alternative of working the seismic migration problem by using the ray theoretical approximation for the acoustic wavefield, and considering a

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seismic model where the velocity does not vary along one of the directions of coordinate axes, the so-called 2.5-D seismic model.

Starting from the three dimensional weighted modified diffraction stack operator as presented by Schleicher et al. (1993), we derive the appropriated method to perform a 2.5-D true-amplitude seismic migration, finding a necessary weight function to be applied to the amplitude of the 2.5-D seismic data.

In summary, the paper concists of theoretical development by which we present an expression for the 2.5-D weight as a function of parameters along each ray branch of the in-plane trajectory. Moreover, we show examples of application of the 2.5-D depth true-amplitude migration algorithm to 2.5-D synthetic seismic data in noise enviroment, in order to make a numerical analysis and to verify the stability and accuracy of the algorithm.

REVIEW OF 2.5-D RAY THEORY

The Seismic Model

In this paper we use the general Cartesian coordinate system being the position vector $\mathrm{x} = (x, y, z)$. Because one of the main concerns of this paper is to apply the ray field properties in a 2.5-D seismic model in order to study a true-amplitude seismic migration method, we think of the earth as a system of isotropic layers, where each layer is constituted by a velocity field $v = v(\mathrm{x})$, whose the first derivative with respect to the second component y vanishes, having smoothly curved surfaces as upper and lower bounds, where the upper bound surface Σ_o is the earth surface. Furthermore, we assume the curvature of each surface is zero along the second component y-axis. The intersection between the plane of symmetry $y = 0$ and the earth surface Σ_o defines the seismic line.

At our seismic experiment carried out on Σ_o, we consider to be registered only $P - P$ primary reflections at the source-receiver pairs (S, G) having position vectors denoted by

$$\mathrm{x}_s = \mathrm{x}_s(\boldsymbol{\xi}) \qquad \text{and} \qquad \mathrm{x}_g = \mathrm{x}_g(\boldsymbol{\xi}), \tag{1}$$

where $\boldsymbol{\xi} = (\xi_1, \xi_2)$ is a vector of parameters on Σ_o.

The high frequency primary reflection wavefield trajectory is then described by a ray that starts at the source point S on Σ_o, reaches the reflector Σ_r at the reflection point R, defined by a vector $\mathrm{x}_r = \mathrm{x}_r(\boldsymbol{\eta})$, $\boldsymbol{\eta} = (\eta_1, \eta_2)$ being a vector of parameters within Σ_r, and returns to the earth surface at G, the ray path SRG. By considering the 2.5-D case, the ray path SRG is assumed to be totally contained into the plane $y = 0$.

We introduce other three local Cartesian coordinate systems with the first two having origins at the points S and G with components (x_{1s}, x_{2s}, x_{3s}) and (x_{1g}, x_{2g}, x_{3g}), respectively. The third coordinate system has origin at the point R with components (x_{1r}, x_{2r}, x_{3r}). The axes x_{1s} and x_{1g} are tangents to the seismic line, while x_{3s} and x_{3g} are downward normal to the Σ_o. The components (x_{1r}) and (x_{3r}) are defined in such way that the former is tangent to the reflector Σ_r within the symmetry plane $y = 0$, while the latter is upward normal to the reflector. The second components x_{2s}, x_{2g} and x_{2r} have the same direction of the y component in the general Cartesian coordinate system.

The Ray Theory

By considering only the 2.5-D wavefield propagation within the symmetry plane $y = 0$, we assume $\xi_2 = \eta_2 = 0, \xi_1 = \xi$ and $\eta_1 = \eta$, simplifying the notation, so that we have $\mathbf{x}_s = \mathbf{x}_s(\xi)$, $\mathbf{x}_g = \mathbf{x}_g(\xi)$ and $\mathbf{x}_r = \mathbf{x}_r(\eta)$. The principal component primary reflection of the acoustic seismic wavefield generated by a compressional point source located at $\mathbf{x}_s$ and registered at $\mathbf{x}_g$ is expressed in the zero-order (ZOr) ray approximation as given by Cerveny and Jech (1982)

$$U(\xi, t) = U_o W(t - \tau(\xi)). \tag{2}$$

The above cited principal component primary reflection describes the particle displacement into direction of the ray at the receiver point G. In equation (2), $W(t)$ represents the analytic point-source wavelet, i.e. this is a complex valued function whose the imaginary part is the Hilbert transform of the real source wavelet, and the real part is the wavelet itself. At the receiver position $\mathbf{x}_g$ within the surface Σ_o, the seismic trace is the superposition of the principal component primary reflections.

By taking into account the ray wavefield approximation within the plane $y = 0$, the reflection traveltime function $\tau = \tau(\mathbf{x})$ with $\mathbf{x} = (x, z)$, is proved by Cerveny and Jech (1982) to satisfy the eikonal equation

$$\nabla\tau \cdot \nabla\tau = 1/v^2(\mathbf{x}), \tag{3}$$

where the traveltime $\tau(\mathbf{x})$ is a continuous in-plane function. It is also proved that the amplitude factor U_o satisfies the ZOr transport equation

$$2\nabla\tau \cdot \nabla U_o + U_o \nabla^2 \tau = 0. \tag{4}$$

At this time, we need to introduce the fundamental *in-plane slowness vector*

$$\mathbf{p} = (p, q) = \frac{\mathbf{t}}{v(\mathbf{x})} = \nabla\tau(\mathbf{x}), \tag{5}$$

where the unitary vector $\mathbf{t} = \dfrac{d\mathbf{x}}{ds}$ is tangent to the ray trajectory, and s is the arclength of ray. In equation (**??**) the components p and q are the so-called horizontal and vertical slowness, respectively, which are related with each other by the expression

$$p = \pm\sqrt{\frac{1}{v^2} - q^2}. \tag{6}$$

The in-plane ray equations are alternatively described by, where $d\sigma = vds$,

$$\frac{dx}{d\sigma} = p, \tag{7}$$

$$\frac{dz}{d\sigma} = q, \tag{8}$$

$$\frac{dp}{d\tau} = -\frac{\partial \ln v(x,z)}{\partial x}, \tag{9}$$

$$\frac{dq}{d\tau} = -\frac{\partial \ln v(x,z)}{\partial z}. \tag{10}$$

In the 2.5-D case the slowness vector is orthogonal to the axis y, remaining in-plane. This means that the initial slowness vector $\mathbf{p}_o = (p_o, q_o)$ is reduced to the two components

$$p_o = \frac{\sin\beta_o}{v_o}, \qquad q_o = \frac{\cos\beta_o}{v_o}, \tag{11}$$

where β_o and v_o are, respectively, the start angle of the ray and the velocity at the source point S.

The solution of the transport equation (4) is the ZOr ray approximation of the principal component of the reflection wavefield given by Cerveny and Jech (1982), that corresponds to the displacement observed at the receiver point G after to have been reflected at the reflection point R in Σ_r, computed by

$$U_o = \frac{R_c\mathcal{A}}{\mathcal{L}}. \tag{12}$$

In the formula (12), R_c is the geometrical-optics reflection coefficient at the reflection point R as presented by Bleistein (1984). The factor $\mathcal{A}$ corresponds to the total loss energy dues to transmissions across all interfaces along the whole ray. In general this factor is considered to be equal one, what means that there is no loss. Finally, the amplitude factor $\mathcal{L}$ is the so-called divergence factor or *geometrical spreading*, whose expression will be given in the next section.

The Paraxial Ray Approximation

The paraxial ray approximation is based on the a priori knowledge of a ray trajectory also known as the *central ray*, which in our example is the ray that starts at the source point $S(\xi)$, reaches the reflector at the reflection point $R(\eta)$, and arrives at the receiver point $G(\xi)$. Thus, the *paraxial ray* is so defined as each ray that starts in the vicinity of S, at the point $S'(\xi')$, reflects at the point $R'(\eta')$ nearby the point R, and reaches the receiver point $G'(\xi')$ in the vicinity of G.

By applying the concept of paraxial ray, Cerveny and Jech (1982) derived the paraxial eikonal equation having as solution the two-point paraxial reflection traveltime from point S' at $\mathrm{x}'_s = \mathrm{x}_s(\xi')$ to the point G' at $\mathrm{x}'_g = \mathrm{x}_g(\xi')$ in the vicinity of points S and G, respectively. An equivalent second-order approximation solution was found also by Ursin (1982) and by Bortfeld (1989). At this paper we write using the same formalism as given by Schleicher et al. (1993), that is tailored for the in-plane ray trajectory, given by

$$\tau_R(s,g) \;=\; \tau_R(s=0,g=0) + p_G g - p_S s - sN_{SG}g + \frac{1}{2}N_S^G s^2 + \frac{1}{2}N_G^S g^2. \tag{13}$$

In the equation (13) the function $\tau_R(s=0,g=0)$ denotes the traveltime along the central ray SG, while s and g are linear distances in the axes x_{1s} and x_{1g}, the so-called *paraxial distances*. These distances are obtained using the following two-steps: (1) At the source/receiver points S' and G', the vectors x'_s and x'_g are orthogonally projected onto the respective axis x_{1s} and x_{1g}; (2) the distances s and g are then defined as having origin at the source/receiver points S and G with end at the projections of x'_s and x'_g, respectively. In the other hand, the so-called *local horizontal slowness* p_S and p_G are obtained as the orthogonal projections of the initial and final in-plane slowness vectors at source/receiver points S and G onto the respective axes x_{1s} and x_{1g}.

The quantities N_S^G and N_G^S are second-derivatives of the traveltime function (13) with respect to the source and receiver coordinates evaluated at $s=0$ and $g=0$, respectively. The other quantity N_{SG} is the second-order mixed-derivative of the same traveltime function (13) evaluated at $s=g=0$.

In the next section of this paper we will perform the 2.5-D true-amplutide migration by using a proper weighted modified diffraction stack. For that, we define for all points of parameters ξ on the earth surface for each point M within a specified volume of the macro-velocity model, the diffraction in-plane traveltime curve

$$\tau_D(\xi) = \tau(S,M) + \tau(M,G) = \tau_S + \tau_G. \tag{14}$$

Following Schleicher et al. (1993), we will refer to this curve as the *Huygens traveltime*. The traveltimes τ_S and τ_G denote, respectively, the traveltimes from the source point S to some arbitrary point M within the model, and from M to the receiver point G.

For obtaining the Huygens paraxial traveltime at a reflection point within Σ_r in the vicinity of R at $\mathrm{x}_r = \mathrm{x}_r(\eta)$, $M = R'$ in (14), with position vector $\mathrm{x}'_r = \mathrm{x}_r(\eta')$, we consider two equations of type (13) for the paraxial traveltime from S' to R'

$$\tau(s,r) = \tau(s=0,r=0) - p_S s + p_r r - sN_{SR}r + \frac{1}{2}N_S^R s^2 + \frac{1}{2}N_R^S r^2. \tag{15}$$

And from R' to G' we have

$$\tau(r,g) = \tau(r=0,g=0) - p_r r + p_G g - rN_{RG}g + \frac{1}{2}N_R^G r^2 + \frac{1}{2}N_G^R g^2. \qquad (16)$$

In the both formulas (15) and (16), the quantity r is the distance between $\mathbf{x}_r$ and the orthogonal projection of $\mathbf{x}'_r$ onto the axis x_{1r} tangent to the reflector at point R. The local horizontal slowness p_r is built by the projection of the in-plane slowness vector at $\mathbf{x}_r$ onto the x_{1r} axis.

The quantities N_{SR} and N_{RG} are second order mixed-derivatives respective the traveltimes (15) and (16) calculated at $s = g = r = 0$, while N_S^R and N_R^S are the second-order derivatives of the traveltime function (15) on relation to s and r, respectively. Finally, the quantities N_R^G and N_G^R are the second-order derivatives of the traveltime function (16) on relation to r and g.

Following the authors Bleistein (1986), Liner (1991), Stockwell (1995) and Hanitzsch (1997), the expression of the geometrical spreading factor, when tailored for the 2.5-D ZOr ray approximation of the seismic wavefield is given by

$$\mathcal{L}_{2.5} = \frac{\sqrt{\cos\alpha_S \cos\alpha_G}}{v_s} \frac{\sqrt{\sigma_S + \sigma_G}}{\sqrt{|\mathcal{N}|}} \times \exp[-i\frac{\pi}{2}\kappa]. \qquad (17)$$

In the above formula (17), we have that α_S and α_G are the start and emergence angles of the central ray measured on relation to the normal at S and G on the earth surface, while v_s is the velocity at the source point S. The term $\mathcal{N}$ in the denominator is given by the ratio

$$\mathcal{N} = \frac{N_{SR}N_{GR}}{N_R^S + N_R^G}. \qquad (18)$$

In the other side, we have that σ_S and σ_G are two quantities related with each branch of the central ray SR and RG, and calculated by the expressions

$$\sigma_S = \int_S^R v(\mathbf{x})ds \qquad \text{and} \qquad \sigma_G = \int_R^G v(\mathbf{x})ds. \qquad (19)$$

The exponential term in (17) represents the phase shitf dues to the caustics along each branch of the central ray.

The 2.5-D spreading factor $\mathcal{L}_{2.5}$ can be expressed then as function of the 2-D spreading factor $\mathcal{L}_2$, given by

$$\mathcal{L}_{2.5} = \mathcal{L}_2\mathcal{F}_{2.5}, \qquad \mathcal{F}_{2.5} = \sqrt{\sigma_S + \sigma_G}, \qquad (20)$$

where $\mathcal{F}_{2.5}$ is called the out-of-plane factor.

Finally, the 2.5-D amplitude factor of the ZOr ray approximation is then rewrited as

$$(U_o)_{2.5} = \frac{R_c\mathcal{A}}{\mathcal{L}_{2.5}}. \qquad (21)$$

As result of the expression (20), the amplitude factor (21) can be alternatively written as

$$(U_o)_{2.5} = \frac{(U_o)_2}{\mathcal{F}_{2.5}}. \tag{22}$$

In the expression (22) we have that $(U_o)_2$ denotes the two-dimensional wavefield amplitude calculated in-plane. An equivalent relationship between 2-D and 2.5-D amplitude factors was found by Bleistein (1986). This means that if we know the 2-D amplitude factor, we need only to divide it by the out-of-plane factor $\mathcal{F}_{2.5}$ in order to obtain the 2.5-D amplitude.

THE 2.5-D RAY MIGRATION THEORY

By assuming s and g are linear function of ξ, we can write

$$s = (s = 0) + \Gamma_S \xi \qquad \text{and} \qquad g = (g = 0) + \Gamma_G \xi, \tag{23}$$

where $\Gamma_S = \dfrac{\partial s}{\partial \xi}$ and $\Gamma_G = \dfrac{\partial g}{\partial \xi}$, which are calculated at $\xi = 0$. In the same way, we consider r is a linear function of η so that

$$r = (r = 0) + \Gamma_r \eta, \qquad \text{where} \qquad \Gamma_r = \frac{\partial r}{\partial \eta}. \tag{24}$$

As a consequence of the above relations (23) and (24), we can express the traveltime functions $\tau_R = \tau_R(\xi)$ and $\tau_D = \tau_D(\xi, R)$. Moreover, we can define the function $\tau_F(\xi, R) = \tau_D(\xi, R) - \tau_R(\xi)$.

Starting from the result obtained in the Appendix by equation (A-9), we have the 2.5-D modified diffraction stack integral given by the stationary phase solution

$$\hat{V}(R,\omega) \approx \frac{\sqrt{-i\omega}}{\sqrt{2\pi}} \int_A d\xi [w(\xi, R)]_{2.5} [\hat{U}(\xi,\omega)]_{2.5} \text{exp}[i\omega\tau_D(\xi, R)]. \tag{25}$$

By considering that the observed wavefield is in-plane confined then we can insert the 2.5-D ZOr approximation of the primary reflection into the integral (25) rewriting

$$\hat{V}(R,\omega) \approx \frac{\sqrt{-i\omega}}{\sqrt{2\pi}} \int_A d\xi [w(\xi, R)]_{2.5} \frac{R_c \mathcal{A}}{\mathcal{L}_{2.5}} \hat{W}(\omega) \text{exp}[i\omega\tau_F(\xi, R)]. \tag{26}$$

The above integral (26) that represents the 2.5-D diffraction stack migration operator, is one more time calculated approximately by the stationary phase method. At this time we apply the stationary phase condition $\frac{\partial \tau_F}{\partial \xi}|_{\xi=\xi^*} = 0$. Thus we have

$$\begin{aligned}\hat{V}(R,\omega) &\approx \hat{W}(\omega)\frac{[w(\xi^*,R)]_{2.5}}{\sqrt{\tau''_F(\xi^*,R)}} \\ &\times \frac{R_c\mathcal{A}}{\mathcal{L}_{2.5}}\text{exp}[i\omega\tau_F(\xi^*,R) - \frac{i\pi}{4}(1-\text{Sgn}(\tau''_F(\xi^*,R))]. \end{aligned} \tag{27}$$

Where $\tau''_F(\xi^*,R) = \frac{\partial^2\tau_F(\xi,R)}{\partial\xi^2}\Big|_{\xi=\xi^*}$ is the second-order derivative of the Taylor expansion

$$\tau_F(\xi,R) = \tau_F(\xi^*,R) + \frac{1}{2}\tau''_F(\xi^*,R)(\xi-\xi^*)^2. \tag{28}$$

After some algebric manipulations involving the formulas (13), (15) and (16) we can express the second-order order derivative term by

$$\tau''_F = \frac{(\Gamma_S N_{SR} + \Gamma_G N_{GR})^2}{(N_R^S + N_R^G)}. \tag{29}$$

The Weight Function

The 2.5-D weight function $[w(\xi,M)]_{2.5}$ for an arbitrary point M inside the macrovelocity model in the integral operator (26) is defined such that the high frequency solution approximation of the diffraction stack integral, for a critical point ξ^* within the migration apperture A, equals the spectrum of the true-amplitude migration source wavelet multiplied with a phase shift operator, that represents the difference between the in-plane reflection and diffraction traveltime curves at the stationary point. Thus we have

$$\hat{V}(M,\omega) \approx \begin{cases} R_c\mathcal{A}\hat{W}(\omega)\text{exp}[i\omega\tau_F(\xi^*,M)] & : \quad \xi^* \in A \\ 0 & : \quad \xi^* \notin A \end{cases} \tag{30}$$

The 2.5-D weight function is then defined as

$$w(\xi^*,M)_{2.5} = \mathcal{L}_{2.5}\sqrt{\tau''_F(\xi^*,M)}\text{exp}[\frac{i\pi}{4}(1-\text{Sgn}(\tau''_F(\xi^*,M))]. \tag{31}$$

After replacing the appropriate definition of $\mathcal{L}_2$ as given by (17) and including the evaluation of τ''_F from the expression (29) we have the result

$$w(\xi^*,M)_{2.5} = \mathcal{F}_{2.5}\frac{\sqrt{\cos\alpha_S\cos\alpha_G}}{v_s}\left(\frac{\Gamma_S N_{SM} + \Gamma_G N_{GM}}{\sqrt{(N_{SM})(N_{GM})}}\right)\text{exp}\frac{-i\pi}{2}[\kappa_1+\kappa_2] \tag{32}$$

The above weight function is to be applied to the amplitude of the 2.5-D seismic data, that is generated when we have a situation of a point source lined up to a set of receivers in the plane $\xi_2 = 0$, by considering a seismic model where the velocity field does not depend on the second coordinate ξ_2. If the chosen point M inside the model coincides with a real reflection point R and $\xi = \xi^*$, the result of applying the difraction stack migration operator (26) to the seismic data is proportional to the reflection coefficient, and if it is put into the point R we have the so-called true amplitude depth migrated reflection data.

In case of special configurations we can apply the weight function (32) as follow: (1) Common-offset: Γ_G=$\Gamma_S = 1$ for $S \neq G$; (2) Common-shot: $\Gamma_S = 0$ and $\Gamma_G = 1$ when the source point S is fixed; (3) Common-receiver: $\Gamma_S = 1$ and $\Gamma_G = 0$ when the receiver point G is fixed; and (4) Zero-offset: $\Gamma_S = \Gamma_G = 1$ for $S \equiv G$, and then $\alpha_S = \alpha_G$, $\kappa_1 = \kappa_2$ and $\sigma_S = \sigma_G$. In the common-midpoint configuration the weight function is not appropriated, because in this case the stationary phase solution is not valid.

EXAMPLES

In order to make a numerical analysis of the true-amplitude migration method, we have generated a set of synthetic seismic traces by using the ray theoretical modeling algorithm SEIS88. For that we use a seismic model constituted of two layers separated by a horizontal plane interface, with velocities 5 and 6 Km/s. The seismic line is considered to be coincident with the x axis. For calculating the data a common-shot configuration was considered with a point source positioned at $x = 0.1$ Km in the seismic line, while the 75 receivers are in-line within the interval 1.0 e 4.7 Km. The point-source wavelet is represented by a Gabor function with frequency of 40 Hz. The seismic trace has a sample interval of 1.0 ms. The free surface is not considered in this example. In order to simulate a noise enviroment, we add to the amplitude at each sample of the seismic traces a random number with a rate of 0.2 of the maximum amplitude observed in the seismic data as we can see in Figure (1).

The result of our expriment is observed by the Figures 2, 3, 4 and 5. In contrast with the test presented by Urban and Cruz (1998) in this volume, as a consequence of the addition of noise in the input data, the seismic migration algorithm does not correctly recover the original source wavelet. But even in noise enviroment we can see that the obtained seismic image represents the true reflector very good. The reflection coefficient determination comes to the samething when we have both the real and imaginary parts oscilating around and near the exact value. In case of noise in the data it is not so easy to determine where the so-called boundary effects begin to influence the migrated data.

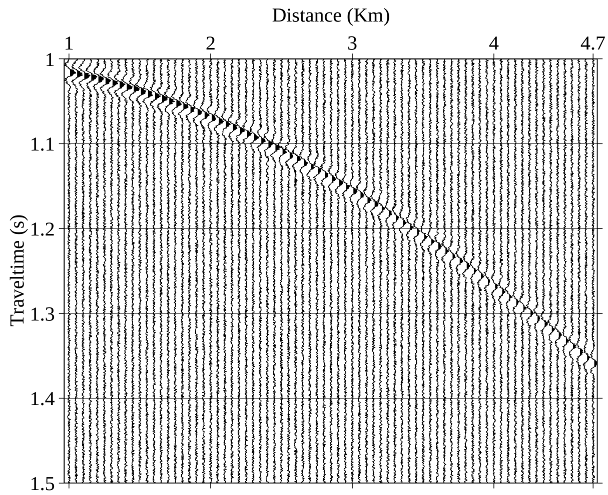

Figure 1: Synthetic seismic data used as input in the 2.5-D true-amplitude depth migration algorithm, with the signal to noise ratio equals to 1:0.2.

CONCLUSION

From the results obtained in this paper, we can affirm that the presented 2.5-D weight function when applied to the 2.5-D seismic data is abble to recover the reflection coeficeint even in noise enviroment. The 2.5-D true-amplitude migration algorithm is stable, when we have that small perturbation in the input data provides only slight deviation in the output migrated data.

APPENDIX A

The true-amplitude is here defined as the analytic primary $P-$wave reflection multiplied by $\mathcal{L}$ and shifted to $t = 0$. Thus, we can write

$$U_{TA}(t) = \mathcal{L}U(\boldsymbol{\xi}, t + \tau_R(\boldsymbol{\xi})) = R_c \mathcal{A} W(t). \tag{A-1}$$

Following Schleicher et al. (1993) the weighted modified diffraction stack is considered the appropriated method to perform a true-amplitude migration. For each point M in the macro-velocity model and all points (ξ_1, ξ_2) in the migration aperture A, the diffraction stacks are then performed by summation along the Huygens surface $\tau_D(\xi_1, \xi_2, M)$ for all points M into a region of the model. The true-amplitude migration is reached by summation using certainly Huygens surface and derived weight function, such that the stack output is proportional to the desired reflection coefficient.

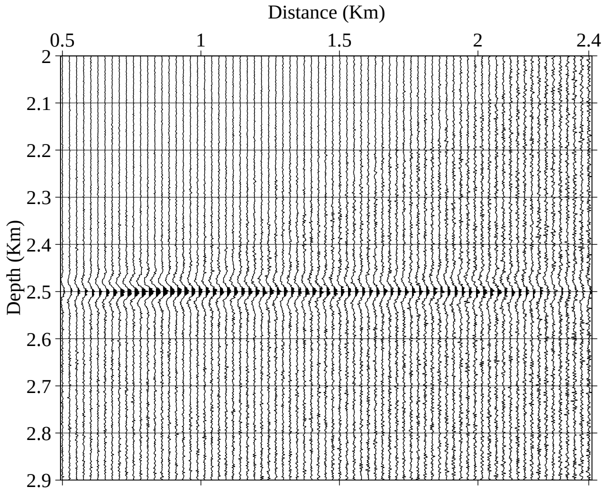

Figure 2: Migrated seismic data, real part.

Mathematically this operation is described by the two-dimensional integral

$$V(M,t) = \frac{-1}{2\pi}\int\int_A d\xi_1 d\xi_2 w(\boldsymbol{\xi}, M) \times \dot{U}(\boldsymbol{\xi}, t + \tau_D(\boldsymbol{\xi}, M)), \tag{A-2}$$

where the symbol $\dot{(\cdot)}$ means the first derivative with respect to time, and $w(\boldsymbol{\xi}, M)$ is the weight function used to stack.

By transforming the expression (A-2) into the frequency domain

$$\hat{V}(M,\omega) = \frac{-i\omega}{2\pi}\int\int_A d\xi_1 d\xi_2 w(\boldsymbol{\xi}, M)\hat{U}(\boldsymbol{\xi})\text{exp}[i\omega\tau_D(\boldsymbol{\xi}, M)]. \tag{A-3}$$

In order to specialize the 3-D formula (A-3) to the 2.5-D geometry, we start considering $M = R$, i.e. the reflection point itself. The migration integral needs solving asymptoticaly by the stationary phase method as found in Bleistein (1984) on relation to the coordinate ξ_2, by making use of the stationary condition as showed in Bleistein et al. (1987)

$$\frac{\partial\tau_D}{\partial\xi_2} = \frac{\partial\tau(S(\boldsymbol{\xi}), R)}{\partial\xi_2} + \frac{\partial\tau(R, G(\boldsymbol{\xi}))}{\partial\xi_2}\Big|_{\Sigma_o} = 0, \tag{A-4}$$

which can be expressed through the identity

$$\frac{\partial}{\partial\xi_2}\Big[\tau(S,M) + \tau(M,G)\Big]\Big|_{\Sigma_o} = p_{2s} + p_{2g}\Big|_{\Sigma_o} = 0. \tag{A-5}$$

By applying the in-plane ray condition $p_2 = p_{2o}$ into the 3-D ray equation as given by Cerveny and Jech (1982) we have

$$x_{2s} = \sigma_s p_{2s}\Big|_{\Sigma_o} \qquad \text{and} \qquad x_{2g} = \sigma_g p_{2g}\Big|_{\Sigma_o}, \tag{A-6}$$

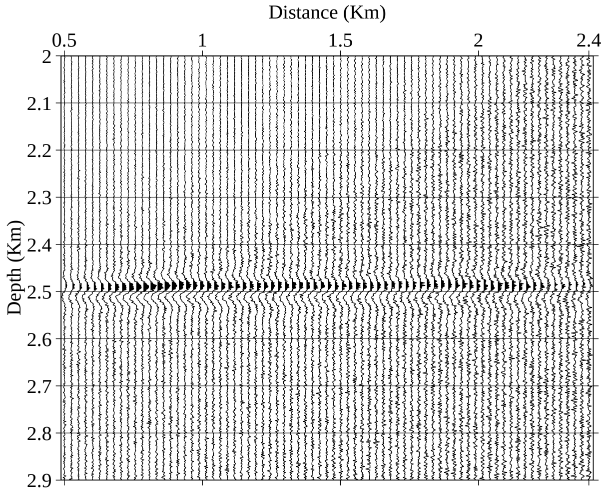

Figure 3: Migrated seismic data, imaginary part.

with σ_s and σ_g calculated along the ray paths SM and MG, respectively. By considering the 2.5-D geometry, $x_{2s} = x_{2g} = \xi_2$, we have finally the result

$$p_{2s} + p_{2g}\Big|_{\Sigma_o} = \Big(\frac{1}{\sigma_s} + \frac{1}{\sigma_g}\Big)\Big|_{\Sigma_o}\xi_2 = 0. \tag{A-7}$$

From equation (A-7) we conclude that the stationary phase condition is $\xi_2 = 0$. For completeness of our asymptotic analysis, we calculate the second derivative of the phase at $\xi_2 = 0$

$$\frac{\partial^2}{\partial \xi_2^2}\Big[\tau(S,R) + \tau(R,G)\Big]\Big|_{\xi_2=0} = \frac{1}{\sigma_S^o} + \frac{1}{\sigma_G^o}. \tag{A-8}$$

Being σ_S^o and σ_G^o the ray parameters for the ray branches RS and RG, when calculated on the earth surface Σ_o.

The above results yield the stationary phase solution

$$\hat{V}(R,\omega) \approx \frac{\sqrt{-i\omega}}{\sqrt{2\pi}}\int_A d\xi w(\xi,R)\left(\frac{1}{\sigma_S^o} + \frac{1}{\sigma_G^o}\right)^{-1/2}\hat{U}(\xi,\omega)\text{exp}[i\omega\tau_D(\xi,R)]. \tag{A-9}$$

As a consequence of the fact that $\hat{U}(\xi,\omega)$ is the in-plane observed point source wavefield amplitude factor, the 2.5-D weight function is defined as

$$[w(\xi,R)]_{2.5} = w(\xi,R)\left(\frac{1}{\sigma_S^o} + \frac{1}{\sigma_G^o}\right)^{-1/2}, \tag{A-10}$$

where $w(\xi,R)$ is the in-plane version of the three dimensional weight function of the 3-D modified diffraction stack Schleicher et al. (1993).

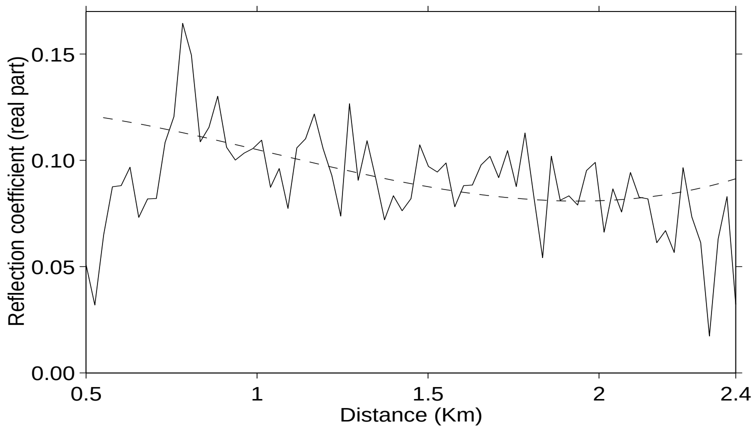

Figure 4: Reflection coefficients picked from the reflector position in the real part of the migrated data. The interrupted line corresponds to the exact value of the reflection coefficient.

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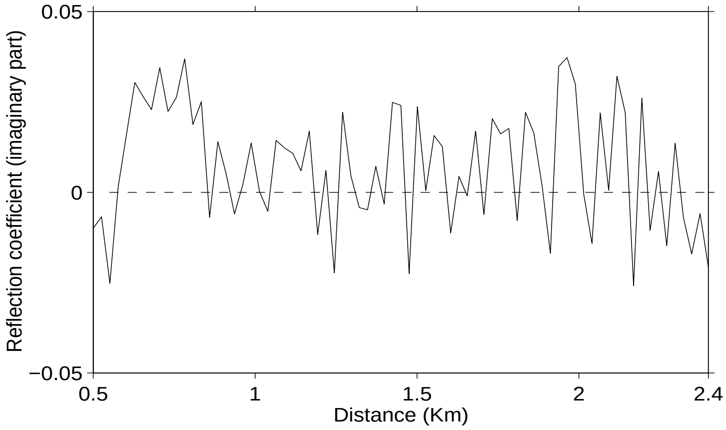

Figure 5: Reflection coefficients picked from the reflector position in the imaginary part of the migrated data. The interrupted line corresponds to the exact value of the reflection coefficient.

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