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Statistical characterization of fractured rocks by seismic wavefield analysis

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ABSTRACT

Seismic characterization of fractured media is an important task in teleseismics and exploration seismology, as well as in non-destructive testing. The statistical properties of the wavefield that propagates in a fractured material are employed to characterize the material and to observe changes induced by external stress.

Ultrasonic wavefields records are obtained on fibre-reinforced composite samples that were subject to different levels of externally applied loading. They serve as a model for heterogeneous rocks with cracks.

The statistical wavefield fluctuation parameter ε is introduced, defined as the ratio of the incoherent to the coherent wavefield amplitude. Looking at intensities, we can identify higher fluctuation levels with increasing scattering of the wavefield on inhomogeneities. These inhomogeneities, e. g. cracks, are partially induced by external stress. The underlying theory is based on the Rytov approximation, assuming weak scattering and wavelengths shorter than the characteristic size of inhomogeneities.

The results for the averaged square of ε, $\langle\varepsilon^2\rangle$, show a clear dependence of the statistical wavefield fluctuation on the internal damage of the material. Different characteristic levels of fluctuation can be identified, yielding a valuable method for non-destructive testing as well as for the characterization of fractured zones in the Earth.

The quantitative analysis successfully reproduced theoretical predictions. It also presents the concept for a calculation of statistical medium parameters, such as the variance σ and the correlation length a.

INTRODUCTION

Seismic signals are distorted by medium inhomogeneities due to, among other effects, scattering. This is a commonly observed fact in various seismological applications,

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such as teleseismics or, for a different frequency range, non-destructive testing. Scattering mechanisms are inherently dependent on the frequency of the wave, medium contrasts and the size of inhomogeneities. What we desire is an inversion based on wavefield characteristics that yields information on medium properties, i. e. the size and kind of scattering objects. The effect of random inhomogeneities on the phase velocity has been studied by Shapiro et al. (1996). A treatise on wave attenuation caused by scattering can be found in Shapiro and Kneib (1993).
We present a new concept by defining a statistical parameter gained from the wavefield that relates incoherent to coherent intensity. Based on the Rytov approximation derived by Ishimaru (1978), a random acoustic medium is assumed, with parameters fluctuating weakly and large-scale inhomogeneities compared to the wavelength.
Considering meanfield theory (Ishimaru (1978)), we then verify the significance of the introduced statistical parameter with respect to the medium.

RANDOM MEDIA

Physical quantities that describe a random medium, such as density and the Lamé parameters λ and μ, can be conceived as stationary random fields in space. These are characterized by their statistical moments, which can be calculated by ensemble averaging over a great number of realizations. A statistical ensemble contains a set of realizations with identical moments. If the moments of an ensemble are equal to the moments of the realizations for a given argument, i. e. space, the medium described by the random field is called an ergodic medium.

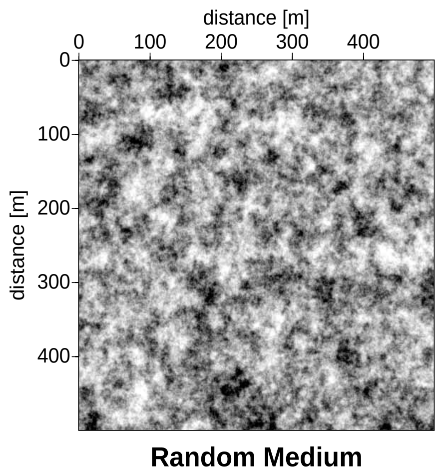

Figure 1: Example for a random medium created with an exponential correlation function. The correlation length is a=25 m.

Common autocorrelation functions that describe a random medium are the Gaussian, the exponential and the von-Karman function. An example for a random medium created with an exponential law is shown in Fig. 1. The correlation length a follows from the choice of autocorrelation function and is proportional to the size of inhomogeneities. It is a measure of how strongly the parameter varies in space. A complete treatment of random process theory can be found in Rytov et al. (1987).

WAVE PROPAGATION IN RANDOM MEDIA

At a point $\boldsymbol{r}$ in a random medium, the wavefield can be described as

$$u(\boldsymbol{r}, t) = \langle u(\boldsymbol{r}, t) \rangle + u_f(\boldsymbol{r}, t) \ . \tag{1}$$

$\langle u \rangle$ represents the coherent field (meanfield) and u_f is the fluctuation of u and called the incoherent field. The angular brackets $\langle \rangle$ denote statistical averaging. It is $\langle u_f \rangle = 0$.
We also define coherent, incoherent and total intensity as

$$\begin{aligned} I_c &= |\langle u \rangle|^2 \\ I_f &= \langle |u_f|^2 \rangle \\ I_t &= I_c + I_f \ . \end{aligned}$$

In the validity range of the Rytov approximation (Ishimaru (1978)), we consider an acoustic wavefield with neglected backscattering. This implies constant total intensity ($I_t = const.$).

We now introduce the fluctuation parameter ϵ by

$$\epsilon \equiv \frac{|u - \langle u \rangle|}{|\langle u \rangle|} = \frac{|u_f|}{|\langle u \rangle|} \tag{2}$$

or alternatively, in terms of intensities

$$\langle \epsilon^2 \rangle = \frac{I_f}{I_c} \ . \tag{3}$$

$\langle\epsilon^2\rangle$ represents a measure for the wavefield fluctuation. It depends on frequency, as higher frequencies are generally scattered more strongly than lower ones, and it is subject of interest what kind of scattering mechanism we have in the medium. Of course, $\langle\epsilon^2\rangle$ also depends on medium parameters, such as variance σ^2 and correlation length a, as will be shown in the following.
The region where $\langle\epsilon^2\rangle \ll 1$ is called the weak fluctuation region; for $\langle\epsilon^2\rangle \gg 1$, the incoherent intensity dominates and the wave propagates in a region of strong fluctuations (Shapiro and Kneib (1993)).
From (3), it follows for $\langle\epsilon^2\rangle \ll 1$

$$\langle\epsilon^2\rangle \approx 2\alpha_{\langle u\rangle} L \tag{4}$$

with $\alpha_{\langle u\rangle}$ being the meanfield scattering coefficient. For $\alpha_{\langle u\rangle}$, one obtains for harmonic waves and by using the Born approximation

$$\alpha_{\langle u\rangle} \sim \sigma^2 a k^2 \ . \tag{5}$$

Hence, combining (4) and (5), we get

$$\langle\epsilon^2\rangle = 2\sigma^2 a (\frac{2\pi}{c}\nu)^2 L \tag{6}$$

where ν denotes frequency and L traveldistance of the wave. Taking the logarithm, a simple linear expression follows

$$\ln\langle\epsilon^2\rangle = \ln(2\sigma^2 a (\frac{2\pi}{c}\nu_o)^2 L) + 2\ \ln(\frac{\nu}{\nu_o}) \tag{7}$$

which relates $\langle\epsilon^2\rangle$ and ν in an easily verifiable way ($\nu_o = 1$ Hz).

APPLICATION TO REAL DATA

In the field of non-destructive testing, ultrasonic measurements represent a versatile method for the investigation of materials with respect to their elastic properties. In this work, real data from such an experiment serve as test input for the theory. The measurement is carried out on a fibre-glass reinforced composite sample (see Fig. 2) that finds widespread use in engineering and construction applications. Fig. 3 shows a section comparable with a seismic zero-offset section.

The experimental goal is to characterize and distinguish the sample in terms of increasing degree of damage. Firstly, the measurement is carried out on the undamaged sample with an induced mean frequency of 10 MHz.

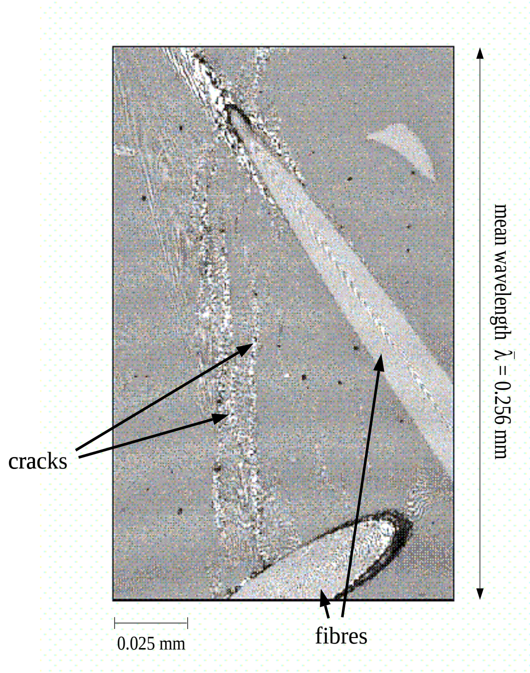

Figure 2: Enlarged photograph of a fibre-reinforced composite, a model for a rock with cracks.

Then, a strain of 1% is applied perpendicular to the wave propagation direction. Afterwards, the measurement is made on the released (presumably internally damaged) sample, and the procedure gets repeated for strains of 2% and 3% .
The externally applied strain induces cracks within the sample, as displayed in Fig. 2, and it is reasonable to assume that the number of cracks increases with strain. Microscopical examinations confirm that the crack width does not exceed the order of 10^{-5} m, as already suggested by Fig. 2. The mean wavelength of the signal is $\bar{\lambda} = 2.56 \cdot 10^{-4}$ m with $c = 2.56$ km/s .
A good coupling between source (ultrasonic piezo transducer) and sample is guaranteed by putting the sample in water during the measurement. For the computation of $\langle \epsilon^2 \rangle$, we use the transmitted signal (one-way through the sample) for data quality reasons rather than the reflected signal shown in the lower half of Fig. 3.

RESULTS FOR THE FLUCTUATION PARAMETER $\langle \epsilon^2 \rangle$

We now utilize the supplied real data sets as input for the theoretical considerations made above. We choose the transmitted signal and compute $\langle \epsilon^2 \rangle$ by the governing

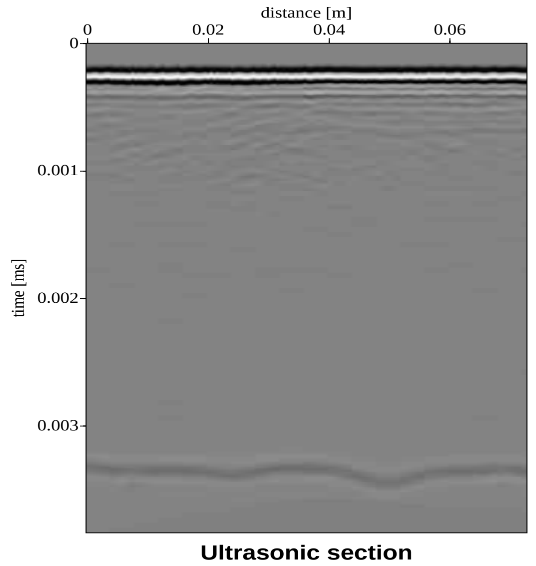

Figure 3: The early amplitude represents the input signal; the signal in the lower half of the section is the reflection at the bottom of the sample.

equation (3). The $\langle\epsilon^2\rangle$ - frequency relation is being evaluated and discussed as follows:

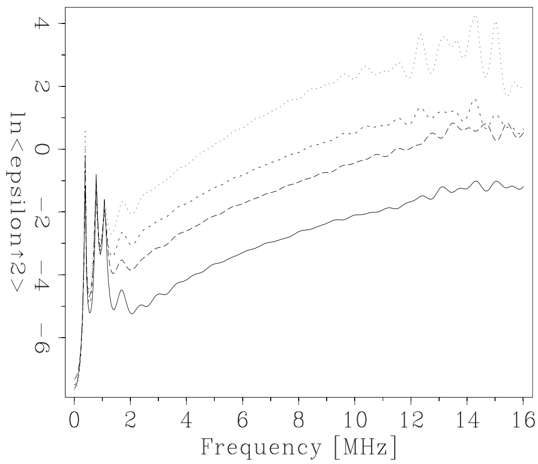

Figure 4: $\ln\langle\epsilon^2\rangle$ dependence on frequency for different strains applied: 0% (solid), 1% (large dashed), 2% (fine dashed), 3% (dotted).

Fig. 4 shows the overall dependence between $\ln\langle\epsilon^2\rangle$ and the frequency ν for experiments with different strains having been applied. The frequency ranges from 0 to 16 MHz, comprising the weak fluctuation region. For higher frequencies, the wavefield fluctuation tends to a constant at high level (saturation occurs), which is not subject of

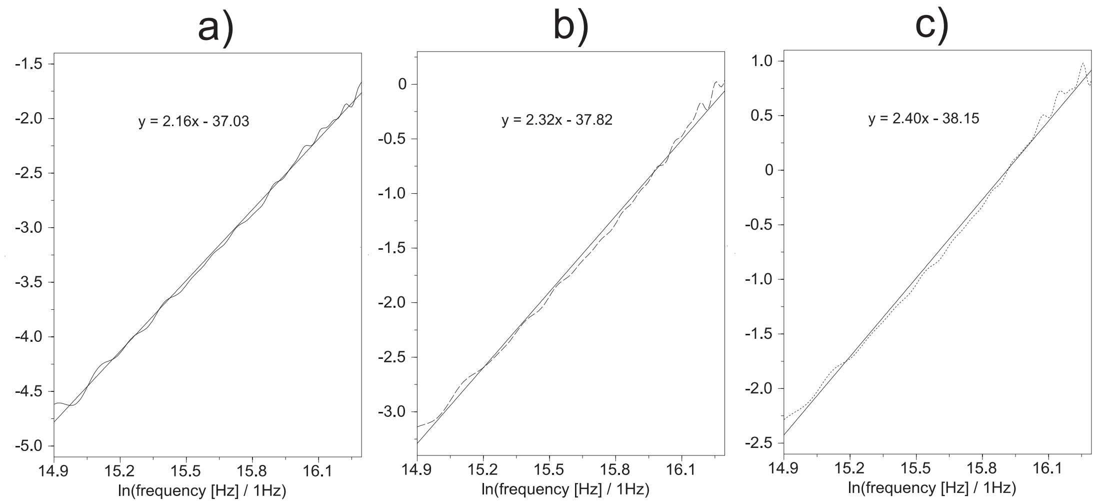

Figure 5: Linear approximations to the curves in Fig. 4 for a) 0%, b) 1%, and c) 2% maximum strain applied to the samples.

further interest here.
Qualitatively, a clear distinction can be made between different states of the sample, depending on strain. In the range of 3.5 MHz to 12 MHz, the curves display a fairly smooth behaviour. For lower frequencies ($<$ 3 MHz), some high $\langle \epsilon^2 \rangle$ amplitudes occur, but they can be interpreted as artefacts due to the frequency content of the input signal.

Taking a closer look at the smooth part of the curves and displaying it on double-logarithmic axes, Fig. 5 shows single curves that are approximated by straight lines following a least squares fit. According to equation (7), one expects a linear relation, i. e. a straight line with a slope of 2, if the assumptions made on the medium are valid. The slopes are between $m = 2.16$ and $m = 2.40$ with an average larger than 2 and tending to increase with higher levels of fluctuation. We observe a rather good fit for the 0% and 1% cases and less matching of the straight line for 2%, which might be due to higher order scattering terms. Theory predicts a linear dependence of $\langle \varepsilon^2 \rangle$ on the squared frequency. A deviation from this relationship, as expressed by the slopes of the fitted straight lines, does not exceed 15%, if we restrict ourselves to the 0% and 1% cases.

As a result of the quantitative analysis of the statistical wavefield fluctuation, it is possible to draw conclusions on the statistics of the medium in which the waves propagate. It is necessary for this purpose to assume that the straight line approximation to

Evaluation of $\ln(2\sigma^2 a L(\frac{2\pi\nu_0}{c_0})^2)$ for $m = 2$			
	$\ln(\nu[\mathrm{Hz}]/1[\mathrm{Hz}])$	$\ln\langle\varepsilon^2\rangle$	$\sigma^2 a$ [m]
0%	16.1	-2.16	$2.48 \cdot 10^{-8}$
1%	16.1	-0.49	$1.32 \cdot 10^{-7}$
2%	16.1	0.49	$3.52 \cdot 10^{-7}$
$c_0 = 2.5 \cdot 10^3 \mathrm{m/s}, \quad L = 3.81 \cdot 10^{-3} \mathrm{m}, \quad \nu_0 = 1\mathrm{Hz}$			

Table 1: Calculation of statistical medium properties by evaluating the straight line fit from Fig. 5, assuming a quadratic power law between $\langle\varepsilon^2\rangle$ and frequency ν.

the fluctuation graphs is valid and matches real data aptly. This means relatively constant slopes of lines within one configuration of parameters. We assume that for high frequencies, the relation between $\langle\varepsilon^2\rangle$ and frequency approaches the quadratic power law, as Rayleigh scattering reduces. So, a point on the respective curves is chosen at the high frequency end, at 9.8 MHz (16.1 on the x-axis). Then a purely quadratic power law is assumed, resulting in a slope of 2, and the values for $\sigma^2 a$ are calculated. Table 1 displays the results.
Here, the tendency of the combination of statistical parameters $\sigma^2 a$ to increase with degree of strain can be seen clearly, which is due to the significant shift of the curves to higher y-values. The change in $\sigma^2 a$ is in the order of a magnitude when going up by one percent. It remains an open question though how to obtain σ^2 and a separately. Furthermore, the validity range of the quadratic power law is not clear yet. This requires larger frequencies to be investigated.

CONCLUSION

We have based our proceeding on the Rytov approximation for wavefields in random media. This involves large-scale inhomogeneities ($a > \lambda$) and smooth parameter variation. If, in our example, scattering happened at individual cracks, we should observe Rayleigh scattering with a ν^4 dependence, as $\lambda \gg a$. This is not confirmed by the results. On the contrary, the Rytov approximation works remarkably well. As a consequence, scattering must occur on large-scale objects. This observation gives rise to the assumption that regions of increased crack density that are themselves fairly homogeneous and large compared to the wavelength cause the scattering of the wave.

Since the wavefield fluctuation only depends on strain, other scattering mechanisms can certainly be excluded. The size of those regions is determined by the evaluation of $\sigma^2 a$.

Similar investigations using teleseismic data have been made by Ritter et al. (1997) in order to ascertain statistical inhomogeneities of the lithosphere.

To conclude, we have found a significant and robust parameter derived from the wavefield that allows to distinguish media with different scattering properties and to characterize media quantitatively by the computation of $\sigma^2 a$.

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